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The Silver-Meal Heuristic Method for Deterministic Time-Varying Demand

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Abstract Heuristic policies for time-varying deterministic processes with infinite input rate have been thoroughly explored in the literature. This paper addresses heuristic decision rules for the situation of a deterministic linearly increasing and decreasing demand patterns with a finite input rate. We determine the timing and sizing of replenishment so as to keep the total relevant cost as low as possible. We extended the Silver-Meal heuristic method and found the penalty cost is very low.

Keywords Production; Silver-Meal Method, Time-Varying Demand, Finite and Infinite Input Rate.

Abstrak *Polisi heuristik untuk masalah berketentuan berubah dengan masa dan penerimaan sekaligus telah banyak dibincangkan dan diselidiki dalam literature. Kertas ini memperihalkan kaedah heuristik untuk masalah berketentuan di mana permintaannya menoton naik dan menoton menurun dengan penerimaan tidak sekaligus. Masa dan saiz penambahan inventori akan ditentukan supaya jumlah kosnya minimum. Kita menggunakan kaedah Silver–Meal dan didapati penalti kosnya amatlah rendah.*

Katakunci *Pengeluaran, Kaedah Silver–Meal, Permintaan berubah dengan masa, sekaligus dan berkadaran.*

1 Introduction

Much of the heuristic method on determining approximation optimal production schedules to satisfy a deterministic time-varying demand process has assumed an infinite input rate. Silver [18] showed how the Silver-Meal heuristic, which was developed for the discrete time-varying demand case, could be adapted to give an approximate solution procedure for continuous time-varying demand patterns. Instead of finding the lot sizes that minimise the total ordering and inventory holding costs up to the time horizon, he determined each lot size sequentially, one at a time, by finding the first local minimum of the total inventory cost per unit time. Phelps [14] proposed a computationally easier procedure by restricted the replenishment intervals to be constant. Under this restriction he determined the optimal number of replenishments using an iterative algorithm over the time horizon. Mitra et al. [10] developed the simplest method by modifying the Economic Order Quantity (EOQ) model to accommodate the case of linear trend and a finite horizon. Ritchie [16] simplified the calculation of the optimal policy for linearly increasing demand and developed a heuristic version of Donaldson's method by extending the time horizon to infinity. Ritchie and Tsado [17] showed how the batch quantities for this policy relates to the EOQ formula and suggested that a cubic equation should be used iteratively in order to determine the duration of the replenishment cycles. Teng [20] proposed a heuristic solution to the problem of linear increasing demand by using an approximate total cost to find the number of replenishments and then by using Donaldson's analysis to obtain the optimal times for those replenishments. Naddor [11] developed heuristic solution procedures and produced good results for linearly increasing demand. Brosseau [1] considered the optimal policy for the case of linearly decreasing demand and derived an analytical method for determining the optimal number of replenishments, the times of those replenishment and the corresponding optimum cost. Hill [8] carried the analysis of Smith [19] and Brosseau back in time to the period of relatively level demand which is likely to precede the decline in demand. Hariga [5] extended Silver's model to cover the case of linearly decreasing demands. Hariga [5] also proposed the least-cost approach and the least-unit cost method, which were originally introduced for the case of discrete time-varying demand.

All of the above procedures assumed instantaneous replenishment and are only valid in the context of purchasing. In the context of production it is generally more appropriate to assume that the input rate is finite and at a constant rate. Hong et al.[9] extended Naddor's work by assuming the production rate is uniform and finite, and developed three different heuristic policies. Hill [7] considered how the existing methodology developed by Resh et al.[15], Donaldson [3] and others, derived for an increasing demand process, can be adapted when the production time for a batch is sufficiently long for a finite input rate to be appropriate.

Recently, Omar et al.[13] considered how the existing heuristic procedure developed by Naddor can be adapted when the input rate is finite. The purpose of this paper is to extend the Silver's heuristic to the case where the input rate is finite. In the next section, we outline the mathematical expressions and dynamic programming approach which was proposed by Omar et al. [13]. We then show how to extend the Silver's heuristic to the case of linearly increasing and decreasing demand with finite input rate by using the single stage cost expression (see Omar et al.[13]). In the following section, we conduct a numerical comparative study. Finally, we conclude the paper in the last section.

2 Model assumptions and definitions

The following terminology is used:

- An inventory schedule is required for the period of $(0, H)$.
- The demand rate at time t, $f(t)$, in $(0, H)$ is linearly increasing or decreasing.
- There is a fixed setup cost of c_1 for each batch replenishment.
- There is a carrying inventory cost of c_2 per unit per unit time.
- t_i is the time at which we start to manufacture the $(i+1)$st batch ($t_0 = 0$).
- Q_i is the size of the ith batch.
- n is the total number of batch replenishment (and therefore we define $t_n = H$).
- The finite production rate is P units per unit time.
- Finally, we assume that $P > f(t), \quad t \in (0, H)$.

The expression for the time-weighted stockholding with the input rate of $P(TWS^P)$, during any period (t_i, t_{i+1}) is (see, Omar et al.[13])

$$TWS^P(t_i, t_{i+1}) = \int_{t_i}^{t_{i+1}} y_1(t)\,dt - \int_{t_i}^{t_i^*} \big[y_1(t) - y_2(t)\big]\,dt. \tag{1}$$

where

$$\begin{aligned} y_1(t) &= \int_{t_i}^{t_{i+1}} f(t)\,dt - \int_{t_i}^{t} f(t)\,dt \\ &= \int_{t}^{t_{i+1}} f(t)\,dt \qquad t_i \le t \le t_{i+1} \end{aligned} \tag{2}$$

and

$$y_2(t) = P(t - t_i) - \int_{t_i}^{t} f(t)\,dt \qquad t_i \le t \le t_i^*. \tag{3}$$

t_i^* is the time for which the $(i+1)$st batch last. Equation (2) gives the inventory level during the period of (t_i, t_{i+1}) with infinite input rate while equation (3) gives the inventory level during the production period with a finite and constant input rate. The quantity produced during this period,(t_i, t_i^*) must balance the quantity demanded during (t_i, t_{i+1}), hence

$$P(t_i^* - t_i) = \int_{t_i}^{t_{i+1}} f(t)\,dt. \tag{4}$$

Define $SSC^P(t_i, t_{i+1})$ as the single stage cost of meeting all demand from time t_i to time t_{i+1} by a single replenishment at time t_i, when stock is zero with the production rate is finite and at rate P. Thus

$$SSC^P(t_i, t_{i+1}) = c_1 + c_2 TWS^P(t_i, t_{i+1}). \tag{5}$$

It follows that the total relevant cost with the finite input rate of $P(TRC^P(n))$, for n-batches is

$$TRC^P(n) = nc_1 + c_2 \sum_{i=0}^{n-1} TWS^P(t_i, t_{i+1}). \tag{6}$$

If $f(t) = a + bt$, thus

$$\begin{aligned} TRC^P(n) &= nc_1 + c_2 \sum_{i=0}^{n-1} \frac{(t_{i+1} - t_i)^2}{2} \big\{[a + \frac{b}{3}(2t_{i+1} + t_i)] \\ &\quad - \frac{1}{P}[a + \frac{b}{2}(t_{i+1} + t_i)]^2\big\}. \end{aligned} \tag{7}$$

Let $C^P(n, t)$ be a minimum total cost of meeting all future demand, starting with no stock at time t and making exactly n batches with the finite input rate of P. The dynamic programming formulation for this problem is:

$$\begin{aligned} C^P(0, H) &= 0 \\ C^P(0, t) &= \infty \qquad 0 \le t \le H \\ C^P(n, t) &= \min_{t \le x \le H} \big\{SSC^P(t, x) + C^P(n-1, x)\big\} \end{aligned} \tag{8}$$

where $n = 1, 2, 3, ...,$ and $\quad 0 < t < H$.

We proceed the above calculation by calculating values $C^P(n, t)$ backwards in time from the horizon H for a finely-graded spectrum values of t. This procedure will be evaluated all possible solutions. For the optimal n-batch policy we need $C^P(n, 0)$, the optimal solution with n-batches starting from time zero.

3 Extended Silver's heuristic procedure

Silver [18] modified the Silver-Meal heuristic to produce an approximation for the case of continuous linear increasing demand over a finite time horizon. He determined each lot size sequentially, one at a time, by minimizing the total variable costs per unit time. The criterion is appealing but is sub-optimal, particularly when there is a well-defined ending point in the demand pattern.

From (5), the total relevant cost per unit time ($TRCUT$) for a batch made at time 0 which meets all demand up to time T for a finite input rate, P, is

$$TRCUT^P(T) = \frac{c_1 + c_2\big\{\int_0^T y_1(t)\,dt - \int_0^{t^*} [y_1(t) - y_2(t)]\,dt\big\}}{T}, \tag{9}$$

where $t^* = \frac{1}{P}\int_0^T f(t)\,dt$.

The necessary condition for $TRCUT^P(T)$ to be minimum is that $d\big(TRCUT^P(T)\big)/dT = G(T) = 0$, and, if $f(t) = a + bt$, this gives

$$G(T) = (\frac{a}{2} - \frac{a^2}{2P})T^2 + (\frac{2b}{3} - \frac{ab}{P})T^3 - \frac{3b^2T^4}{8P} - \frac{c_1}{c_2}. \tag{10}$$

For all values of b, a root of $G(T) = 0$ can be found. When $b > 0$, the root of (10) is unique and corresponds to a global minimum. For this case it is easier to solve equation (10) by numerical method.

However when $b < 0$, the analysis is more complicated. Numerical works agreed with Hariga's result [5] for the case of infinite input rate. Let T_0, $T_0 \neq 0$ is the solution for $G'(T) = 0$. If $G(T_0) > 0$, there exist two solutions T_1 and T_2 $(T_1 < T_2)$ to (10) which correspond to a local minimum and a local maximum to (9) respectively. The optimal solution is T_1 if $TRCUT^P(T_1) < TRCUT^P(H)$, and H otherwise. However when $G(T_0) < 0$, the optimal solution will be H.

4 Numerical results

To demonstrate the effectiveness of this heuristic, we present two examples. We consider linearly increasing and decreasing demand patterns. The results below show the optimal cost, found from equation (7), and corresponding optimal number of orders. Silver solution cost follows with the penalty.

Example 1

For this example, demand is linearly increasing with $f(t) = 6 + t, c_1 = 180$, $c_2 = 2.0$ and $H = 11$. Some results are given in Table 1 for different values of P. In that table, OC means Optimal Cost, $ESSC$ means Extended Silver Solution Cost and $Pen.$ is the value of $100(\frac{SSC-OC}{OC})$.

Table 1: Comparison between optimal method and heuristic procedure (Example 1)

	Optimal Method		**Silver Heuristic**		
P	n	OC	n	$ESSC$	$Pen.$
30	2	828.65	3	871.83	5.21
50	3	912.06	3	967.03	0.74
100	3	967.03	3	967.33	0.03
1000	3	1016.23	3	1017.88	0.16
∞	3	1021.69	3	1023.84	0.21

In Table 1, the results for $P = 1000$ and $P = \infty$ are the results after adjustment. For example, when $P = 1000$, the actual values of t_i and T are

$P = 1000$				
i	0	1	2	3
t_i	0	4.0015	7.5230	10.7285
T	4.0015	3.5215	3.2055	0.2715

As in the case of infinite input rate, the boundary at $H = 11$ has a severe effect. The T value of the fourth replenishment would be 2.9765, not just 0.2715. The total relevant cost of this heuristic policy is 1168.97 and 15.03% above the optimal value. However with simple adjustment that is slightly increase the period of time for the third replenishment from $T = 3.2055$ to $T = 3.4770$, we get the better heuristic solution with penalty only 0.16%. The optimal replenishment for this case are at times 0, 4.20 and 7.79 with the total relevant cost is 1016.23. Following this adjustment, the penalty for the case of P is infinity was reduced from 14.44% to only 0.21%. For all cases, equation (4) give us the production period for each batch.

Example 2

For this example, demand is linearly decreasing with $f(t) = 100 - 20t$, $c_1 = 100$, $c_2 = 7.5$ and $H = 5$. The results appear in Table 2.

Table 2: Comparison between optimal method and heuristic procedure (Example 2)

	Optimal Method		**Silver Heuristic**		
P	n	OC	n	$ESSC$	$Pen.$
150	4	921.80	4	926.32	0.50
200	5	1013.40	4	1052.31	3.84
400	5	1137.10	5	1138.41	0.12
600	6	1172.3	5	1184.56	1.05
800	6	1189.25	5	1199.37	0.85
1000	6	1199.37	5	1224.97	2.13
∞	6	1239.82	5	1290.55	4.09

Tables 1 and 2 give the Optimal Cost (OC) and the Extended Silver Solution Cost ($ESSC$) for differents input rates. As expected, the slower the input rate (P) the smaller the optimal number of batches (n) and the lower the corresponding cost. Finally $Pen.$ gives the percentage cost increase from using that Silver heuristic instead of the optimal approach.

5 Conclusions

In this paper we have extended Siver Meal heuristic for time-varying demand processes to the case of a finite input rate. We have used a dynamic programming approach to obtain the optimal solution. Numerical work suggests that the cost penalty of using the unadjusted Extended Silver Meal heuristic is relatively small except when the demand termination point happen just after the last replenishment.

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