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Electrochemical Cells as Experimental Verifications of n-ary Hyperstructures

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Abstract Algebraic hyperstructures represent a natural extension of classical algebraic structures and they have many applications in various sciences. The main purpose of this paper is to provide a new application of n-ary weak hyperstructures in Chemistry. More precisely, we present three different examples of ternary hyperstructures associated with electrochemical cells. In which we prove that our defined hyperstructures are ternary H_v-semigroups.

Keywords H_v-semigroup; H_v-ternary semigroup; galvanic cell; electrolytic cell.

Mathematics Subject Classification 76B15, 35Q53.

1 Introduction

Hyperstructure theory was known for the first time in 1934 at the eighth Congress of Scandinavian Mathematicians, when Marty [1] gave the definition of hypergroup as a generalization of the notion of the group. Where in a group, the operation's result of two elements is again an element while in a hypergroup, the hyperoperation's result of two elements is a non-void set. Moreover, he illustrated some applications and showed its utility in the study of groups, algebraic functions and relational fractions. Recently, the hyperstructures are studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics: geometry, topology, cryptography and code theory, graphs and hypergraphs, probability theory, binary relations, theory of fuzzy and rough sets, automata theory, economy, etc. In [2], Corsini and Leoreanu presented some of hyperstructures' applications to many of the mentioned subjects.

The largest class of hyperstructures is the one that satisfies weak axioms, i.e., the non-empty intersection replaces the equality. These are called H_v-structures and they were introduced in 1990 and different examples from Biology and Chemistry were studied to verify them. As a biological example of weak binary hyperstructures, in [3], the authors analyzed the second generation phenotypes of n-hybrid cross with a mathematical structure. They used the concepts of cyclic hypergroup and H_v-semigroup in the F_2-phenotypes with mating as a hyperoperation. Also, some chemical examples were studied in Chung, Davvaz *et al.*, and Davvaz and Nezhad [4-6], redox, chain and dismutation reactions and provided as different examples of weak binary hyperstructures. As a generalization of binary weak hyperstructure, n-ary weak hyperstructures were introduced. Several examples on the latter were studied by Al-Tahan *et al.* and Davvaz *et al.*[7-8].

In our paper, we consider the binary chemical hyperstructure using electrochemical cells defined and studied in details by the authors in Al-Tahan and Davvaz [9]. We define ternary hyperstructures associated with electrochemical cells and investigate their properties. This paper is organized as follows: after the Introduction Section, Section 2 presents some definitions that are used throughout the paper. Section 3 with two already defined binary hyperstructures by the authors related to both Galvanic cells and Electrolytic cells, defines ternary hyperstructures on them and proves that they are equivalent. Section 4 defines a ternary hyperstructure related to both Galvanic and Electrolytic cells at the same time and investigates its properties.

2 n-ary Weak Hyperstructures

In this section, we present some definitions related to n-ary hyperstructures (see [10-15]) that are used throughout the paper.

Definition 1 [10] Let H be a non-empty set. Then, a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ is called a *binary hyperoperation* on H, where $\mathcal{P}^*(H)$ is the family of all non-empty subsets of H. The couple $(H, \circ)$ is called a *hypergroupoid*.

In the above definition, if A and B are two non-empty subsets of H and $x \in H$, then we define:

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b, \; x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

H_v-structures were introduced by T. Vougiouklis as a generalization of the well-known algebraic hyperstructures. Some axioms of classical algebraic hyperstructures are replaced by their corresponding weak axioms in H_v-structures. Most of H_v-structures are used in the representation theory.

Definition 2 [16] A hypergroupoid $(H, \circ)$ is called an *H_v-semigroup* if $(x \circ (y \circ z)) \cap ((x \circ y) \circ z) \neq \emptyset$ for all $x, y, z \in H$.

Theorem 1 [9] Let $H = \{a, b, c, d\}$, $\oplus_1$ be the hyperoperation on H and consider Table 1 corresponding to $(H, \oplus_1)$. Then $(H, \oplus_1)$ is a commutative H_v-semigroup.

Theorem 2 [9] Let $H = \{a, b, c, d\}$, $\oplus_2$ be the hyperoperation on H and consider Table 2 corresponding to $(H, \oplus_2)$. Then $(H, \oplus_1)$ is a commutative H_v-semigroup.

Theorem 3 [9] Let $H = \{a, b, c, d\}$, $\oplus$ be the hyperoperation on H and consider Table 3 corresponding to $(H, \oplus)$. Then $(H, \oplus)$ is a commutative H_v-semigroup.

Table 1: $(H, \oplus_1)$

$\oplus_1$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$
b	$\{a,b\}$	b	$\{a,d\}$	$\{b,d\}$
c	$\{a,c\}$	$\{a,d\}$	c	$\{c,d\}$
d	$\{a,d\}$	$\{b,d\}$	$\{c,d\}$	d

Table 2: $(H, \oplus_2)$

$\oplus_2$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
b	$\{a,b\}$	b	$\{b,c\}$	$\{b,d\}$
c	$\{a,c\}$	$\{b,c\}$	c	$\{c,d\}$
d	$\{b,c\}$	$\{b,d\}$	$\{c,d\}$	d

Table 3: $(H, \oplus)$

$\oplus$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
b	$\{a,b\}$	b	$\{a,d\}$	$\{b,d\}$
c	$\{a,c\}$	$\{a,d\}$	c	$\{c,d\}$
d	$\{b,c\}$	$\{b,d\}$	$\{c,d\}$	d

An element $x \in H$ is called *idempotent* if $x^2 = x \circ x = x$ and an element $e \in H$ is called an *identity* of $(H, \circ)$ if $x \in x \circ e \cap e \circ x$, for all $x \in H$. The latter is called *strong identity* if $e \circ x = x \circ e \subseteq \{e, x\}$ for all $x \in H$.

Definition 3 [10] A hypergroupoid $(H, \circ)$ is called a:

1. *semihypergroup* if for every $x, y, z \in H$, we have $x \circ (y \circ z) = (x \circ y) \circ z$;
2. *quasi-hypergroup* if for every $x \in H$, $x \circ H = H = H \circ x$ (The latter condition is called the reproduction axiom);
3. *hypergroup* if it is a semihypergroup and a quasi-hypergroup.

Definition 4 [10] Two hypergroupoids $(H, \circ)$ and $(K, \star)$ are said to be *isomorphic hypergroupoids*, written as $H \cong K$, if there exists a bijective function $f : H \to K$ such that $f(x \circ y) = f(x) \star f(y)$ for all $x, y \in H$.

Definition 5 *n-ary hypergroupoid. Let H be a nonempty set. Then a map $f : H^n \to \mathcal{P}^*(H)$ is called an n-ary hyperoperation on H and the pair (H, f) is called an n-ary hypergroupoid. Here,* $H^n = \underbrace{H \times \ldots \times H}_{n \text{ times}}$.

If $A_1, \ldots, A_n$ are nonempty subsets of H, then we define:

$$f(A_1, \ldots, A_n) = \bigcup_{a_i \in A_i} f(a_1, \ldots, a_n).$$

$f(x_1, \ldots, x_i, y_{i+1}, \ldots, y_j, z_{j+1}, \ldots, z_n)$ can be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^n)$ where $x_1^i, y_{i+1}^j, z_{j+1}^n$ are the sequences given by $\{x_1, \ldots, x_i\}$, $\{y_{i+1}, \ldots, y_j\}$ and $\{z_{j+1}, \ldots, z_n\}$ respectively.

Definition 6 [14] An n-ary hypergroupoid (H, f) is called a

1. *n-ary semihypergroup* if (H, f) is associative, i.e., for every $x_1, \ldots, x_{2n-1} \in H$ and $i, j \in \{1, \ldots, n\}$ we have
$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1}).$$
2. *n-ary H_v-semigroup* if (H, f) is weak associative, i.e., for every $x_1, \ldots, x_{2n-1} \in H$ and $i, j \in \{1, \ldots, n\}$ we have
$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) \cap f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1}) \neq \emptyset.$$

For example, in the case of $n = 3$ (ternary), we have (H, f) a ternary semihypergroup if: For every $a_1, a_2, a_3, a_4, a_5 \in H$ we have

$$f(f(a_1, a_2, a_3), a_4, a_5) = f(a_1, f(a_2, a_3, a_4), a_5) = f(a_1, a_2, f(a_3, a_4, a_5)).$$

Definition 7 *An n-ary hypergroupoid (H, f) is commutative if for every $x_1, \ldots, x_n \in H$ and $\sigma \in S_n$ (S_n is the symmetric group on n letters) we have*

$$f(x_1, \ldots, x_n) = f(x_{\sigma(1)}, \ldots, x_{\sigma(n)}).$$

Definition 8 [15] A subset K of an n-ary H_v-semigroup (H, f) is called an *n-ary H_v-subsemigroup* if (K, f) is an n-ary H_v-semigroup.

Definition 9 *Weak neutral element. Let (H, f) be an n-ary hypergroupoid. An element $e \in H$ is called a weak neutral element of H if for every $x \in H$ and $i \in \{1, \ldots, n\}$ we have*

$$x \in f(\underbrace{e, \ldots, e}_{i-1}, x, \underbrace{e, \ldots, e}_{n-i}).$$

For the case $n = 3$ the map $f : H \times H \times H \to \mathcal{P}^*(H)$ is called a *ternary hyperoperation* on H and the pair (H, f) is called a *ternary hypergroupoid*. If A, B, C are nonempty subsets of H, then we define:

$$f(A, B, C) = \bigcup_{\substack{a \in A \\ b \in B, c \in C}} f(a, b, c).$$

A ternary hypergroupoid is called commutative if for every $a_1, a_2, a_3 \in H$ and $\sigma \in S_3$ we have

$$f(a_1, a_2, a_3) = f(a_{\sigma(1)}, a_{\sigma(2)}, a_{\sigma(3)}).$$

And it is called a *ternary quasi-hypergroup* if for every $x, y \in H$ we have

$$f(H, x, y) = f(x, H, y) = f(x, y, H) = H.$$

An element $e \in H$ is called a *neutral element* of a ternary hypergroupoid (H, f) if for all $a \in H$,

$$f(e, e, a) = f(e, a, e) = f(a, e, e),$$

and is called *weak neutral element* if for all $a \in H$,

$$f(e, e, a) \cap f(e, a, e) \cap f(a, e, e) \neq \emptyset.$$

3 Ternary Hyperstructures in Galvanic Cells

Galvanic cell consists of two half-cells, such that the electrode of one half-cell is composed of metal A (with larger electronegativity) and the electrode of the other half-cell is composed of metal B (with smaller electronegativity). The redox reactions for the two separate half-cells are given as follows:

$$A^{n+} + ne^- \longrightarrow A,$$
$$B \longrightarrow B^{m+} + me^-.$$

The two metals A and B can react with each other according to the following balanced equation:

$$nB + mA^{n+} \longrightarrow mA + nB^{m+}.$$

The authors in Al-Tahan and Davvaz [9] considered the set $H = \{A, B, A^{n+}, B^{m+}\}$ and defined a binary hyperoperation $\oplus_1$ on H as follows:

> $x \oplus_1 y$ is the result of a possible reaction between x and y in a Galvanic cell. If x and y do not react in a Galvanic cell then we set $x \oplus_1 y = \{x, y\}$.

All possible spontaneous redox reactions of $\{A, B, A^{n+}, B^{m+}\}$ in a Galvanic cell are summarized in Table 4.

Table 4: Redox Reactions in Galvanic Cell

$\oplus_1$	A	B	A^{n+}	B^{m+}
A	A	$\{A, B\}$	$\{A, A^{n+}\}$	$\{A, B^{m+}\}$
B	$\{A, B\}$	B	$\{B^{m+}, A\}$	$\{B, B^{m+}\}$
A^{n+}	$\{A, A^{n+}\}$	$\{B^{m+}, A\}$	A^{n+}	$\{A^{n+}, B^{m+}\}$
B^{m+}	$\{A, B^{m+}\}$	$\{B^{m+}, B\}$	$\{A^{n+}, B^{m+}\}$	B^{m+}

Electrolytic cells consist of two half-cells, such that the electrode of one half-cell is composed of metal A (with larger electronegativity) and the electrode of the other half-cell is composed of metal B (with smaller electronegativity). The redox reactions for the two separate half-cells are given as follows:

$$A \longrightarrow A^{n+} + ne^-,$$
$$B^{m+} + me^- \longrightarrow B.$$

The two metals A and B can react with each other according to the following balanced equation:

$$mA + nB^{m+} \longrightarrow nB + mA^{n+}.$$

For more details about electrocemical cells, see Zumdahl [17]. The authors in Al-Tahan and Davvaz [9] considered the set $H = \{A, B, A^{n+}, B^{m+}\}$ and defined another hyperoperation $\oplus_2$ on H as follows:

> $x \oplus_2 y$ is the result of a possible reaction between x and y in an Electrolytic cell. If x and y do not react in an Electrolytic cell then we set $x \oplus_2 y = \{x, y\}$.

All possible non-spontaneous redox reactions of $\{A, B, A^{n+}, B^{m+}\}$ in an Electrolytic cell are summarized in Table 5.

Table 5: Redox reactions in Electrolytic cell

$\oplus_2$	A	B	A^{n+}	B^{m+}
A	A	$\{A, B\}$	$\{A, A^{n+}\}$	$\{A^{n+}, B\}$
B	$\{A, B\}$	B	$\{A^{n+}, B\}$	$\{B, B^{m+}\}$
A^{n+}	$\{A, A^{n+}\}$	$\{A^{n+}, B\}$	A^{n+}	$\{A^{n+}, B^{m+}\}$
B^{m+}	$\{A^{n+}, B\}$	$\{B^{m+}, B\}$	$\{A^{n+}, B^{m+}\}$	B^{m+}

The authors proved Theorems 1 and 2 by changing the names from A, B, A^{n+}, B^{m+} to a, b, c, d respectively.

Remark 1 The authors proved in [9] that $(H, \oplus_1)$ and $(H, \oplus_2)$ are isomorphic H_v-semigroups.

First, we present a ternary hyperstructure associated with Galvanic cells. Consider $H = \{a, b, c, d\}$ and define the ternary hyperoperation f obtained from $\oplus_1$ as $f(x, y, z) = x \oplus_1 (y \oplus_1 z)$. We may think of $f(x, y, z)$ as the products resulting from a possible reaction between x and one of the products resulting from a possible reaction between y and z in a Galvanic cell. Since $f(x, y, z) = f(x, z, y)$ as $y \oplus_1 z = z \oplus_1 y$ then we may present (H, f) by the four symmetric tables: Table 6, Table 7, Table 8 and Table 9.

Table 6: $f(a, -, -)$

$f(a, -, -)$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
b		$\{a, b\}$	$\{a, d\}$	$\{a, b, d\}$
c			$\{a, c\}$	$\{a, c, d\}$
d				$\{a, d\}$

Table 7: $f(b, -, -)$

$f(b, -, -)$	a	b	c	d
a	$\{a, b\}$	$\{a, b\}$	$\{a, b, d\}$	$\{a, b, d\}$
b		b	$\{a, b, d\}$	$\{b, d\}$
c			$\{a, d\}$	$\{a, b, d\}$
d				$\{b, d\}$

Table 8: $f(c,-,-)$

$f(c,-,-)$	a	b	c	d
a	$\{a,c\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c,d\}$
b		$\{a,d\}$	$\{a,c,d\}$	$\{a,c,d\}$
c			c	$\{c,d\}$
d				$\{c,d\}$

Table 9: $f(d,-,-)$

$f(d,-,-)$	a	b	c	d
a	$\{a,d\}$	$\{a,b,d\}$	$\{a,c,d\}$	$\{a,d\}$
b		$\{b,d\}$	$\{a,d\}$	$\{b,d\}$
c			$\{c,d\}$	$\{c,d\}$
d				d

Theorem 4 *(H,f) is a ternary H_v-semigroup.*

Proof It is clear from Tables 6, 7, 8 and 9 that (H,f) is a ternary hypergroupoid. We need to show that (H,f) is weak associative. Let x_1, x_2, x_3, x_4 and $x_5 \in H$. It is clear from Tables 6, 7, 8 and 9 that if $x_1 = a$ or d then $x_1 \in f(f(x_1,x_2,x_3),x_4,x_5) \cap f(x_1,f(x_2,x_3,x_4),x_5) \cap f(x_1,x_2,f(x_3,x_4,x_5))$. Simple computations show that if $x_1 = b$ or c then $f(f(x_1,x_2,x_3),x_4,x_5) \cap f(x_1,f(x_2,x_3,x_4),x_5) \cap f(x_1,x_2,f(x_3,x_4,x_5)) \neq \emptyset$. Therefore, (H,f) is a ternary H_v-semigroup. □

Proposition 1 *(H,f) admits two neutral elements.*

Proof Let $x \in H$. Having that

$$f(a,a,x) = f(a,x,a) = a \oplus_1 (a \oplus_1 x) = a \oplus_1 \{a,x\} = \{a,x\},$$

$$f(x,a,a) = x \oplus_1 (a \oplus_1 a) = x \oplus_1 a = \{a,x\},$$

$$f(d,d,x) = f(d,x,d) = d \oplus_1 (d \oplus_1 x) = d \oplus_1 \{d,x\} = \{d,x\},$$

$$f(x,d,d) = x \oplus_1 (d \oplus_1 d) = x \oplus_1 d = \{d,x\}$$

implies that a,d are two neutral elements of (H,f). One can easily see that b,c are not neutral elements of (H,f).

Proposition 2 *(H,f) is weak commutative.*

Proof Let $x, y, z \in H$. Since $f(x,y,z) = f(x,z,y)$, it suffices to show that $f(x,y,z) \cap f(y,x,z) \cap f(z,x,y) \neq \emptyset$. We have four cases for x: $x = a$, $x = b$, $x = c$ and $x = d$. It is clear from the tables of (H,f) (Tables 6, 7, 8 and 9) that if $x = a$ or $x = d$ then $x \in f(x,y,z) \cap f(y,x,z) \cap f(z,x,y) \neq \emptyset$. Simple computations show that $f(b,y,z) \cap f(y,b,z) \neq \emptyset$ and $f(c,y,z) \cap f(y,c,z) \neq \emptyset$. □

Remark 2 (H,f) is not commutative as $f(a,b,c) = \{a,d\} \neq f(b,a,c) = \{a,b,d\}$.

Remark 3 (H,f) is neither a ternary semihypergroup as $f(f(a,a,a),b,c) = \{a,d\} \neq f(a,f(a,a,b),c) = \{a,c,d\}$ nor a ternary quasi-hypergroup as $f(H,b,b) = \{a,b,d\} \neq H$.

Proposition 3 *(H,f) has three proper ternary H_v-subsemigroups up to isomorphism.*

Proof It is clear that $(\{a\},f)$, $(\{a,b\},f)$ and $(\{a,c,d\},f)$ are the only proper ternary H_v-subsemigroups of (H,f) up to isomorphism. □

Remark 4 The ternary H_v-subsemigroups presented in the proof of Proposition 3 are ternary hypergroups.

Next, we present a ternary hyperstructure associated with Electrolytic cells.
Consider $H = \{a,b,c,d\}$ and define the ternary hyperoperation f_1 obtained from $\oplus_2$ as $f_1(x,y,z) = x \oplus_2 (y \oplus_2 z)$. We may think of $f_1(x,y,z)$ as the products resulting from a possible reaction between x and one of the products resulting from a possible reaction between y and z in an Electrolytic cell.

Theorem 5 *(H,f_1) is a ternary H_v-semigroup that is isomorphic to (H,f).*

Proof The proof results from having $(H,\oplus_1) \cong (H,\oplus_2)$ and having (H,f) a ternary H_v-semigroup. □

4 Ternary Hyperstructures in Galvanic/Electrolytic Cells

In this section, we present a ternary hyperstructure related to Galvanic/Electrolytic cells and investigate its properties. The authors in Al-Tahan and Davvaz [9] considered the set $H = \{A, B, A^{n+}, B^{m+}\}$ and defined a binary hyperoperation $\oplus$ on H as follows:

> $x \oplus y$ is the result of a possible reaction between x and y in either a Galvanic cell or in an Electrolytic cell. If x and y neither react in a Galvanic cell nor in an Electrolytic cell then we set $x \oplus y = \{x,y\}$.

All possible spontaneous/non-spontaneous redox reactions of $\{A, B, A^{n+}, B^{m+}\}$ in a Galvanic/Electrolytic cell are summarized in Table 10.

The authors proved Theorem 3 by changing the names from A, B, A^{n+}, B^{m+} to a, b, c, d respectively.

Consider $H = \{a,b,c,d\}$ and define the ternary hyperoperation g obtained from $\oplus$ as $g(x,y,z) = x \oplus (y \oplus z)$. We may think of $g(x,y,z)$ as the products resulting from a possible reaction between x and one of the products resulting from a possible reaction between y and z either in a Galvanic cell or in an Electrolytic cell.

Since $g(x,y,z) = g(x,z,y)$ as $y \oplus z = z \oplus y$ then we may present (H,g) by the four symmetric tables: Table 11, Table 12, Table 13 and Table 14.

Table 10: Redox Reactions in Galvanic/Electrolytic Cells

$\oplus$	A	B	A^{n+}	B^{m+}
A	A	$\{A, B\}$	$\{A, A^{n+}\}$	$\{A^{n+}, B\}$
B	$\{A, B\}$	B	$\{A, B^{m+}\}$	$\{B, B^{m+}\}$
A^{n+}	$\{A, A^{n+}\}$	$\{A, B^{m+}\}$	A^{n+}	$\{A^{n+}, B^{m+}\}$
B^{m+}	$\{A^{n+}, B\}$	$\{B^{m+}, B\}$	$\{A^{n+}, B^{m+}\}$	B^{m+}

Table 11: $g(a, -, -)$

$g(a, -, -)$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, b, c\}$
b		$\{a, b\}$	$\{a, b, c\}$	$\{a, b, c\}$
c			$\{a, c\}$	$\{a, b, c\}$
d				$\{b, c\}$

Theorem 6 *(H, g) is a ternary H_v-semigroup.*

Proof It is clear from the tables of (H, g) (Tables 11, 12, 13 and 14) that (H, g) is a ternary hypergroupoid. Simple computations show that for all $x_1, x_2, x_3, x_4, x_5 \in H$,

$$g(g(x_1, x_2, x_3), x_4, x_5) \cap g(x_1, g(x_2, x_3, x_4), x_5)) \cap g(x_1, x_2, g(x_3, x_4, x_5) \neq \emptyset.$$

As a simple example of how to calculate weak associativity, we present the following case:

$$g(g(d, a, a), a, a) = g(\{b, c\}, a, a) = \{a, b, c\},$$

$$g(d, g(a, a, a), a) = g(d, a, a) = \{b, c\}$$

and

$$g(d, a, g(a, a, a)) = g(d, a, a) = \{b, c\}$$ □

Proposition 4 *(H, g) admits no weak neutral elements.*

Proof We have that a, b, c, d are not weak neutral elements of (H, g) as d, c, b, a are not elements in $g(a, a, d)$, $g(b, b, c)$, $g(c, c, b)$, $g(d, d, a)$ respectively. □

Table 12: $g(b,-,-)$

$g(b,-,-)$	a	b	c	d
a	$\{a,b\}$	$\{a,b\}$	$\{a,b,d\}$	$\{a,b,d\}$
b		b	$\{a,b,d\}$	$\{b,d\}$
c			$\{a,d\}$	$\{a,b,d\}$
d				$\{b,d\}$

Table 13: $g(c,-,-)$

$g(c,-,-)$	a	b	c	d
a	$\{a,c\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c,d\}$
b		$\{a,d\}$	$\{a,c,d\}$	$\{a,c,d\}$
c			c	$\{c,d\}$
d				$\{c,d\}$

Table 14: $g(d,-,-)$

$g(d,-,-)$	a	b	c	d
a	$\{b,c\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$
b		$\{b,d\}$	$\{b,c,d\}$	$\{b,d\}$
c			$\{c,d\}$	$\{c,d\}$
d				d

Proposition 5 (H,g) *is weak commutative.*

Proof The proof is similar to that of Proposition 2. □

As a simple example of how to calculate weak commutativity, we consider the following case:

$$g(a,c,d) = \{a,b,c\}, g(c,a,d) = \{a,c,d\} \text{ and } g(d,a,c) = \{b,c,d\}.$$

Remark 5 (H,g) is not commutative as $g(a,b,c) = \{a,b,c\} \neq g(b,a,c) = \{a,b,d\}$.

Remark 6 (H,g) is neither a ternary quasi-hypergroup as $g(H,a,a) = \{a,b,c\} \neq H$ nor a ternary semihypergroup as $g(a,a,a),b,c) = \{b,c\} \neq g(a,g(a,a,b),c) = \{a,b,c\}$.

Remark 7 Using Proposition 1 and Theorem 5, one can easily see that (H,g) is not isomorphic to (H,f) nor to (H,f_1).

Proposition 6 (H,g) *has two proper ternary* H_v*-subsemigroups up to isomorphism.*

Proof It is clear that $(\{a\}, g)$ and $(\{a, b\}, g)$ are the only proper ternary H_v-subsemigroups of (H, g) up to isomorphism. □

Remark 8 The ternary H_v-subsemigroups presented in the proof of Proposition 6 are ternary hypergroups.

5 Conclusion

Chemical reactions are examples of the phenomena when composition of two elements is a set of elements. This paper provided a new ternary chemical hyperstructure associated with electrochemical cells that is not equivalent to any of the studied chemical hyperstructures before. We observed that electrochemical cells are experimental verifications of ternary hyperstructures.

References

[1] Marty, F. Sur une generalization de la notion de group. In *8th Congress Math* Scandenaves. 1934. 45-49.

[2] Corsini, P., Leoreanu, V. *Applications of hyperstructures theory*. Springer Science and Business Media. 2013.

[3] Al- Tahan, M., Davvaz, B. Algebraic hyperstructures associated with biological inheritance. *Mathematical Biosciences*. 2017. 285: 112-118.

[4] Chung, S. Chemical hyperstructures for Vanadium. *Journal of the Chungcheong Mathematical Society*. 2014. 27(2): 309-317.

[5] Davvaz, B., Nezhad, A.D., Benvidi, A. Chemical hyperalgebra: Dismutation reactions. *Match-Communications in Mathematical and in Computer Chemistry*. 2012. 67(1): 55.

[6] Davvaz, B., Nezhad, A. D. Chemical examples in hypergroups. *Ratio Mathematica*. 2003. 14: 71-74.

[7] Al- Tahan, M., Davvaz, B. n- ary hyperstructures associated with the genotypes of F_2-offspring. *Int J. of Biomathematics*. 2017. 10(8): 17.

[8] Davvaz, B., Nezhad, A.D., Benvidi, A. Chain reactions as experimental examples of ternary algebraic hyperstructures. *MATCH Communications in Mathematical and in Computer Chemistry*. 2011. 65 (2): 491-499.

[9] Al- Tahan, M., Davvaz, B. Weak chemical hyperstructures associated to electrochemical cells. *Iranian Journal of Mathematical Chemistry*. 2018. 9(1): 65-75.

[10] Corsini,P. *Prolegomena of hypergroup theory*. Second edition, Aviani Editore, Italy. 1993.

[11] Davvaz, B. *Polygroup theory and related systems*. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ. 2013

[12] Davvaz, B. *Semihypergroup Theory*. Elsevier. 2016.

[13] Davvaz, B., Leoreanu-Fotea, V. *Hyperring theory and applications*. International Academic Press, USA. 2007.

[14] Davvaz, B., Vougiouklis, V.n-Ary hypergroups. *Iranian Journal of Science and Technology, Transaction A*. 2006. 30(A2): 165-174.

[15] Ghadiri, M., Waphare, B.N., Davvaz, B. n-Ary H_v-structures. *Southeast Asian Bulletin of Mathematics*. 2010. 34: 243255.

[16] Vougiouklis, T. *Hyperstructures and their representations*. Hadronic Press, Palm Harbor, USA. 1994.

[17] Zumdahl, S. *Chemistry*. Seventh Edition, Houghton Mifflin Compan, New York. 2007.