Matematika, 2001, Jilid 17, bil. 2, hlm. 71–76

Semiregular Property on Generalized Compact and Paracompact Spaces

Adem Kılıçman & Anwar Jabor Fawakhreh
Department of Mathematics
Universiti Putra Malaysia
43400 UPM, Serdang, Selangor
Malaysia
e-mail: akilic@fsas.upm.edu.my

Abstract We let (X, τ) be a topological space and (X, τ^*) its semiregularization. Then a topological property $\mathcal{P}$ is semiregular provided that τ has property $\mathcal{P}$ if and only if τ^* has the same property. In this work, we study semiregular property on some generalizations of compact and paracompact spaces; namely, paracompact, nearly paracompact, weakly compact and weakly paracompact spaces.

Keywords Semiregularity, semiregular property, compact and paracompact spaces.

Abstrak *Misalkan (X, τ) ruang topologi dan (X, τ^*) semiregularannya. Maka sifat topologi $\mathcal{P}$ adalah semiregularan asalkan τ mempunyai sifat $\mathcal{P}$ jika dan hanya jika τ^* mempunyai sifat yang serupa. Dalam kertas ini, kami menyelidiki sifat semiregularan di atas beberapa ruang padat dan para–padat teritlak, iaitu ruang para-padat, hampir padat, padat lemah dan para–padat lemah.*

Katakunci *Semiregularan, sifat semiregularan, ruang padat dan ruang para-padat.*

1 Introduction

Among the various covering properties of topological spaces, a lot of attention has been made to those covers that involve regularly open sets and regularly closed sets. In 1969 Singal and Matur [4] introduced and studied the notion of nearly compact spaces. Also Singal and Arya [3] introduced and studied nearly paracompact spaces on using regularly open covers. In 1981 Cammaroto and Lo Faro [1] studied weakly compact spaces that depend on another type of covers called regular covers, which we used to define weakly paracompact spaces. In fact regular covers are equivalent to shrinkable covers as defined in [5].

In this work, we study semiregular property on paracompact, nearly paracompact, weakly compact and weakly paracompact spaces.

In this paper, a space X means a topological space (X, τ) on which no separation axioms are assumed unless explicitly stated otherwise. The interior and the closure of any subset A of (X, τ) will be denoted by Int(A) or Int_τ(A) and Cl(A) or Cl_τ(A), respectively. By regular cover we mean the notion introduced in [1] and by regularly open cover of X we mean a cover of X by regularly open sets in (X, τ).

Recall that, a subset $A \subseteq X$ is called regularly open (regularly closed) if $A = \mathrm{Int(Cl(A))}$ ($A = \mathrm{Cl(Int(A))}$), respectively. The generated space by the regularly open subsets of the topological space (X, τ) is denoted by (X, τ^*) and, is called semiregularization of (X, τ). If $\tau = \tau^*$ then X is said to be semiregular.

Definition 1.1 Let (X, τ) be a topological space and (X, τ^*) its semiregularization. A topological property $\mathcal{P}$ is called semiregular provided that τ has the property $\mathcal{P}$ if and only if τ^* has the property $\mathcal{P}$. A set is called clopen set it it is both closed and open.

We will frequently make use of the following well known results, see [4].

Lemma 1.1 Let (X, τ) be a topological space and (X, τ^*) its semiregularization. Then

(a) regularly open sets of (X, τ) are the same of regularly open sets of (X, τ^*).

(b) regularly closed sets of (X, τ) are the same of regularly closed sets of (X, τ^*).

(c) clopen sets of (X, τ) are the same of clopen sets of (X, τ^*).

(d) $\mathrm{Cl}_\tau(\mathrm{A}) = \mathrm{Cl}_{\tau^*}(\mathrm{A})$ for every $A \in \tau$.

2 Semiregularizations of Compact and Paracompact Spaces

Compactness is a very important property in topology and analysis, mathematician studied it widely. In compact spaces one deals with open covers to get a finite subcover, e.g. A space X is compact if every open cover of the space X has a finite subcover. Later compactness was generalized as : nearly compactness, paracompactness, realcompactness, weakly compactness, metacompactness, etc.

The following two definitions were given in [4] and [3], respectively.

Definition 2.1 A topological space (X, τ) is said to be nearly compact if every open cover $\{U_\alpha : \alpha \in \Delta\}$ of X admits a finite subfamily such that

$$X = \bigcup_{k=1}^{n} \mathrm{Int(Cl(U_k))}$$

Definition 2.2 A topological space (X, τ) is said to be nearly paracompact if every regularly open cover of X admits an open locally finite refinement.

We note that compactness is not a semiregular property whereas nearly compactness is a semiregular property.

Lemma 2.1 If $\{V_\gamma : \gamma \in \Gamma\}$ is an open locally finite refinement of a regularly open set $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ), then $\{\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) : \gamma \in \Gamma\}$ is an open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*).

Proof: Let $\{V_\gamma : \gamma \in \Gamma\}$ be an open locally finite refinement of a regularly open set $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ). Since

$$\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) \in \tau^* \quad \text{and} \quad \mathrm{V}_\gamma \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{U}_{\alpha(\gamma)})) = \mathrm{U}_{\alpha(\gamma)},$$

then $\{\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) : \gamma \in \Gamma\}$ is an open refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*). For locally finiteness, let $x \in X$ and $U_x \in \tau^*$ such that $x \in U_x$. Since $\tau^* \subseteq \tau$, $U_x \in \tau$. Since $\{V_\gamma : \gamma \in \Gamma\}$ is locally finite in (X, τ), $\{\gamma \in \Gamma : U_x \bigcap V_\gamma = \phi\}$ is finite. So $\{\gamma \in \Gamma : U_x \bigcap \mathrm{Cl}(\mathrm{V}_\gamma) = \phi\}$ is finite. But

$$\{\gamma \in \Gamma : U_x \bigcap \mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) = \phi\} \subseteq \{\gamma \in \Gamma : U_x \bigcap \mathrm{Cl}(\mathrm{V}_\gamma) = \phi\}$$

which implies that $\{\gamma \in \Gamma : U_x \bigcap \mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) = \phi\}$ is finite. Thus $\{\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) : \gamma \in \Gamma\}$ is locally finite in (X, τ^*). This implies that $\{\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) : \gamma \in \Gamma\}$ is an open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*), thus completing our proof of Lemma 2.1.

Theorem 2.1 Let (X, τ) be a topological space. If (X, τ) is paracompact, then (X, τ^*) is paracompact.

Proof: Assume that (X, τ) is paracompact, without loss of generality, let $\{U_\alpha : \alpha \in \Delta\}$ be any basic open cover of (X, τ^*). Then, we have, $U_\alpha = \mathrm{Int}(\mathrm{Cl}(\mathrm{U}_\alpha))$ for all $\alpha \in \Delta$. Since $\tau^* \subseteq \tau$, then $\{U_\alpha : \alpha \in \Delta\}$ is an open cover for the paracompact space (X, τ). So, by hypothesis, it has an open locally finite refinement in (X, τ) say, $\{V_\gamma : \gamma \in \Gamma \subseteq \Delta\}$ where $V_\gamma \subseteq U_{\alpha(\gamma)}$. But

$$V_\gamma \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{U}_{\alpha(\gamma)})) = \mathrm{U}_{\alpha(\gamma)}.$$

By Lemma 2.1 above, $\{\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) : \gamma \in \Gamma\}$ is an open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*) such that $\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) \in \tau^*$. This implies that (X, τ^*) is paracompact.

In general, the converse of the above theorem is not true since the left ray topology $(\mathbb{R}, \tau_{l.r})$ is not paracompact whereas $(\mathbb{R}, \tau^*_{l.r}) = \{\emptyset, \mathbb{R}\}$ is paracompact. Thus we conclude the following result.

Corollary 2.1 Paracompactness is not a semiregular property.

Theorem 2.2 Let (X, τ) be a topological space. Then (X, τ) is nearly paracompact if and only if (X, τ^*) is nearly paracompact.

Proof: Assume that (X, τ) is nearly paracompact and let $\{U_\alpha : \alpha \in \Delta\}$ be a regularly open cover of (X, τ^*). Then, using Lemma 1.1(a), we have $\{U_\alpha : \alpha \in \Delta\}$ is a regularly open cover of the nearly paracompact space (X, τ), so it has a locally finite refinement, say, $\{V_\gamma : \gamma \in \Gamma \subseteq \Delta\}$. But $V_\gamma \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma))$ and using Lemma 2.1, the family $\{\mathrm{Int}(\mathrm{Cl}(\mathrm{V}_\gamma)) :$

$\gamma \in \Gamma\}$ is still an open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*). This implies that (X, τ^*) is nearly paracompact.

Conversely, suppose that (X, τ^*) is nearly paracompact and let $\{U_\alpha : \alpha \in \Delta\}$ be a regularly open cover of (X, τ). So $U_\alpha \in \tau^*$ for every $\alpha \in \Delta$. Thus, by Lemma 1.1(a), we have $\{U_\alpha : \alpha \in \Delta\}$ is a regularly open cover of the nearly paracompact space $(X, \tau^{*)}$. So it has an open locally finite refinement $\{V_\gamma : \gamma \in \Gamma \subseteq \Delta\}$ in (X, τ^*). However $\tau^* \subseteq \tau$ implies that $\{V_\gamma : \gamma \in \Gamma\}$ is an open refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ). To show it is locally finite in (X, τ), let $x \in X$ and $U_x \in \tau$ such that $x \in U_x$. Since $U_x \subseteq \mathrm{Int(Cl(U_x))}$, $\{\gamma \in \Gamma : U_x \bigcap V_\gamma = \phi\} \subseteq \{\gamma \in \Gamma : \mathrm{Int(Cl(U_x))} \bigcap \mathrm{V}_\gamma = \phi\}$. Since $\mathrm{Int(Cl(U_x))} \in \tau^*$ and $\{V_\gamma : \gamma \in \Gamma\}$ is locally finite in $(X, \tau^{*)}$, $\{\gamma \in \Gamma : \mathrm{Int(Cl(U_x))} \bigcap \mathrm{V}_\gamma = \phi\}$ is finite. So $\{\gamma \in \Gamma : U_x \bigcap V_\gamma = \phi\}$ is finite. Thus $\{V_\gamma : \gamma \in \Gamma\}$ is locally finite in (X, τ). This shows that (X, τ) is nearly paracompact, thus completing our proof of Theorem 2.2.

The folllowing corollary follows immediately from Theorem 2.2.

Corollary 2.2 Nearly paracompactness is a semiregular property.

3 Semiregularizations of Weakly Compact and Weakly Paracompact Spaces

Recall that, an open cover $\{U_\alpha : \alpha \in \Delta\}$ of a topological space X is called regular cover if, for every $\alpha \in \Delta$, there exists a nonempty regularly closed subset C_α of X such that $C_\alpha \subseteq U_\alpha$ and $X = \bigcup_{\alpha\in\Delta} \mathrm{Int(C_\alpha)}$ (see[1]).

The following definitions are given in [2].

Definition 3.1 A topological space (X, τ) is said to be weakly compact if every regular cover $\{U_\alpha : \alpha \in \Delta\}$ of X admits a finite subfamily $\{U_{\alpha_i} : i = 1, \ldots, n\}$ such that $X = \bigcup_{i=1}^n \mathrm{Cl(U_{\alpha_i})}$

Definition 3.2 A topological space (X, τ) is said to be weakly paracompact if every regular cover of X has an open locally finite refinement.

Theorem 3.1 Let (X, τ) be a topological space. Then (X, τ) is weakly compact if and only if (X, τ^*) is weakly compact.

Proof: Suppose that (X, τ) is a weakly compact space and let $\{U_\alpha : \alpha \in \Delta\}$ be a regular cover of (X, τ^*). Then there exists a nonempty regularly closed subset C_α of X such that $C_\alpha \subseteq U_\alpha$ and $X = \bigcup_{\alpha\in\Delta} \mathrm{Int}_{\tau^*} C_\alpha$. Since $\tau^* \subseteq \tau$, $X = \bigcup_{\alpha\in\Delta} \mathrm{Int}_{\tau^*}(\mathrm{C}_\alpha) \subseteq \bigcup_{\alpha\in\Delta} \mathrm{Int}_\tau(\mathrm{C}_\alpha)$. By Lemma 1.1(b), we have, $\{U_\alpha : \alpha \in \Delta\}$ is a regular cover of the weakly compact space (X, τ). So it has a finite subset $\{U_{\alpha_i} : i = 1, \ldots, n\}$ such that $X = \bigcup_{i=1}^n \mathrm{Cl}_\tau(\mathrm{U}_{\alpha_i})$. Thus, by Lemma 1.1(d), $X = \bigcup_{i=1}^n \mathrm{Cl}_{\tau^*}(\mathrm{U}_{\alpha_i})$, which implies that (X, τ^*) is weakly compact.

Conversely, suppose (X, τ^*) is weakly compact and let $\{U_\alpha : \alpha \in \Delta\}$ be a regular cover of (X, τ). Since $U_\alpha \subseteq \mathrm{Int(Cl(U_\alpha))}$, by Lemma 1.1(a),(b) we have $\{\mathrm{Int(Cl(U_\alpha))} : \alpha \in \Delta\}$ is a regular cover of the weakly compact space (X, τ^*). So it has a finite subfamily such that

$$X = \bigcup_{i=1}^n \mathrm{Cl}_{\tau^*}(\mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{U}_{\alpha_i}))) \subseteq \bigcup_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Cl}_{\tau^*}(\mathrm{U}_{\alpha_i}) = \bigcup_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Cl}_\tau(\mathrm{U}_{\alpha_i}).$$

This shows that (X, τ) is weakly compact and completes our proof of Theorem 3.1.

Then we state the following corollary as a result of Theorem 3.1

Corollary 3.1 Weakly compactness is a semiregular property.

In the next, we have the following theorem for the weakly paracompact spaces.

Theorem 3.2 Let (X, τ) be a topological space. Then (X, τ) is weakly paracompact if and only if (X, τ^*) is weakly paracompact.

Proof: Suppose that (X, τ) is a weakly paracompact space and let $\{U_\alpha : \alpha \in \Delta\}$ be a regular cover of (X, τ^*). Since $U_\alpha \in \tau^*$, $U_\alpha = \bigcup_{\gamma\in\Gamma} W_{\alpha(\gamma)}$ where $W_{\alpha(\gamma)} = \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{W}_{\alpha(\gamma)}))$. Since $\{U_\alpha : \alpha \in \Delta\}$ is a regular cover of (X, τ^*), for every $\alpha \in \Delta$, there exists a regularly closed set C_α in (X, τ^*) such that $C_\alpha \subseteq U_\alpha$ and $\bigcup_{\alpha\in\Delta} \mathrm{Int}_{\tau^*}(\mathrm{C}_\alpha) = \mathrm{X}$. Since $\tau^* \subseteq \tau$ and by Lemma 1.1(b), for every $\alpha \in \Delta$, there exists the regularly closed set C_α such that $C_\alpha \subseteq U_\alpha$ and $X = \bigcup_{\alpha\in\Delta} \mathrm{Int}_{\tau^*}(\mathrm{C}_\alpha) \subseteq \bigcup_{\alpha\in\Delta} \mathrm{Int}_\tau(\mathrm{C}_\alpha)$. Thus $X = \bigcup_{\alpha\in\Delta} \mathrm{Int}_\tau(\mathrm{C}_\alpha)$. This implies that $\{U_\alpha : \alpha \in \Delta\}$ is a regular cover of (X, τ). Since (X, τ) is weakly paracompact, then there exists an open locally finite refinement $\{V_\lambda : \lambda \in \Lambda\}$ of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ). So, for every $\lambda \in \Lambda, V_\lambda \subseteq U_{\alpha(\lambda)}, V_\lambda \in \tau$ and $\{\lambda \in \Lambda : U_x \bigcap V_\lambda = \phi\}$ is finite for any open set $U_x \in \tau$ contains x.

Now we prove that $\{W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda)) : \alpha \in \Delta, \gamma \in \Gamma, \lambda \in \Lambda\}$ is an open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*). It is open since the intersection of two regularly open sets is regularly open, so $W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda)) \in \tau^*$. It is also a refinement since $W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda)) \subseteq \mathrm{W}_{\alpha(\gamma)} \subseteq \mathrm{U}_\alpha$. For locally finiteness, let $x \in X$ and $U_x \in \tau^*$ such that $x \in U_x$. Since

$$U_x \bigcap (W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda))) \subseteq U_x \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda)) \subseteq U_x \bigcap \mathrm{Cl}_\tau(\mathrm{V}_\lambda),$$

we have

$$\begin{aligned} &\{\alpha \in \Delta, \gamma \in \Gamma, \lambda \in \Lambda : U_x \bigcap (W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda))) = \phi\} \\ &\subseteq \{\lambda \in \Lambda : U_x \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda)) = \phi\} \\ &\subseteq \{\lambda \in \Lambda : U_x \bigcap \mathrm{Cl}_\tau(\mathrm{V}_\lambda) = \phi\}. \end{aligned}$$

However $\{V_\lambda : \lambda \in \Lambda\}$ is locally finite in (X, τ), so $\{\mathrm{Cl}_\tau(\mathrm{V}_\lambda) : \lambda \in \Lambda\}$ is locally finite in (X, τ). This implies that $\{\lambda \in \Lambda : U_x \bigcap \mathrm{Cl}_\tau(\mathrm{V}_\lambda) = \phi\}$ is finite. Thus

$$\{\alpha \in \Delta, \gamma \in \Gamma, \lambda \in \Lambda : U_x \bigcap (W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda))) = \phi\}$$

is finite, which implies that

$$\{W_{\alpha(\gamma)} \bigcap \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{V}_\lambda)) : \alpha \in \Delta, \gamma \in \Gamma, \lambda \in \Lambda\}$$

is an open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$ in (X, τ^*). This shows that (X, τ^*) is weakly paracompact.

Conversely, suppose (X, τ^*) is weakly paracompact and let $\{U_\alpha : \alpha \in \Delta\}$ be a regular cover of (X, τ). Since, for every $\alpha \in \Delta$, there exists a regularly closed set C_α such that $C_\alpha \subseteq$

$U_\alpha \subseteq \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{U}_\alpha))$ and $\bigcup_{\alpha\in\Delta} \mathrm{Int}_\tau(\mathrm{C}_\alpha) = \mathrm{X}$. By Lemma 1.1(a),(b) and since $\mathrm{Int}_{\tau^*}(\mathrm{C}_\alpha) = \mathrm{Int}_\tau\big(\mathrm{Cl}_\tau(\mathrm{Int}_{\tau^*}(\mathrm{C}_\alpha))\big) = \mathrm{Int}_\tau\big(\mathrm{Cl}_{\tau^*}(\mathrm{Int}_{\tau^*}(\mathrm{C}_\alpha))\big) = \mathrm{Int}_\tau(\mathrm{C}_\alpha)$, then we have $\{\mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{U}_\alpha)) : \alpha \in \Delta\}$ is a regular cover of the weakly paracompact space (X, τ^*). So it has an open locally finite refinement $\{V_\gamma : \gamma \in \Gamma \subseteq \Delta\}$ in (X, τ^*) with $V_\gamma \subseteq \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{U}_{\alpha(\gamma)}))$. Now it is obvious that the set $\{V_\gamma \bigcap U_{\alpha(\gamma)} : \gamma \in \Gamma, \alpha \in \Delta\}$ is a τ-open refinement of $\{U_\alpha : \alpha \in \Delta\}$. To show it is locally finite, let $x \in X$ and $U_x \in \tau$ such that $x \in U_x$. Since $\{V_\gamma : \gamma \in \Gamma\}$ is locally finite in (X, τ^*) then

$$\{\gamma \in \Gamma : \mathrm{Int}_\tau(\mathrm{Cl}_\tau(\mathrm{U_x})) \bigcap \mathrm{V}_\gamma = \phi\}$$

is finite. But, for every $\alpha \in \Delta$, we have

$$\{\gamma \in \Gamma : U_x \bigcap (V_\gamma \bigcap U_\alpha) = \phi\} \subseteq \{\gamma \in \Gamma : \mathrm{Int}_\tau(\mathrm{Cl}_\tau(U_x)) \bigcap (V_\gamma \bigcap U_\alpha) = \phi\}$$

$$\subseteq \{\gamma \in \Gamma : \mathrm{Int}_\tau(\mathrm{Cl}_\tau(U_x)) \bigcap V_\gamma = \phi\}.$$

Since the last term is finite, $\{\gamma \in \Gamma : U_x \bigcap (V_\gamma \bigcap U_\alpha) = \phi\}$ is finite. Thus $\{V_\gamma \bigcap U_{\alpha(\gamma)} : \gamma \in \Gamma, \alpha \in \Delta\}$ is a τ-open locally finite refinement of $\{U_\alpha : \alpha \in \Delta\}$. This shows that (X, τ) is weakly paracompact and completes our proof of theorem 3.2.

Corollary 3.2 Weakly paracompactness is a semiregular property.

References

[1] F. Cammaroto and G.Lo Faro,*Spazi Weakly Compact*, Riv. Mat. University Parma, 14(7)(1981), 383–395.

[2] M. K. Singal and S.P. Arya,*On Almost Regular Spaces*, Glasnik Math. Ser. III, 4(24)(1969), 89–99.

[3] M. K. Singal and S.P. Arya, *On Nearly Paracompact Spaces*, Math. Vesnik, 6(21)(1969), 3–16.

[4] M. K. Singal and A. Matur, *On Nearly Compact Spaces*, Boll. Un. Mat. Ital., 4(2)(1969), 702–710.

[5] S. Willard, General Topology, Addison-Wesley, Canada, 1970.