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Alternative Approach to Deterministic Inventory Problem

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Abstract This paper addresses the problem of determining the best policy for an inventory replenishment with continuous deterministic time-varying demand over a finite planning horizon. We determine when to replenish and how much for each batch. We propose a genetic algorithm procedure which is based on Darwin's survival of the fittest principle and a method using spreadsheet modelling in Microsoft Excel Solver. Numerical results from our examples showed that both procedures produced optimal solutions reported in the literature.

Keywords Time-Varying Demand, Genetic Algorithm, Spreadsheet Modelling and Inventory.

Abstrak *Dalam kertas ini kita akan menentukan polisi terbaik bagi masalah penambahan perolehan inventori apabila kadar permintaannya tentu berubah dengan masa secara selanjar untuk sesuatu tempoh yang terhad. Kita tentukan bila dan jumlah kuantitinya untuk setiap kitaran. Kita gunakan prosedur genetik algoritma yang berasaskan kepada prinsipal kewujudan Darwin dan model hamparan di dalam Microsoft Excel Solver. Keputusan berangka dari beberapa contoh menunjukkan kedua-kedua prosedur mampu memberikan penyelesaian yang optimum.*

Katakunci *Permintaan berubah dengan masa, Genetik algoritma, Model hamparan dan Inventori.*

1 Introduction

Resh et al. [14] and, independently, Donaldson [4] were among the first to investigate the inventory model with linearly increasing demand patterns analytically. They derived the optimal number of replenishments and their replenishment times over a finite and an infinite time horizon.

The mathematical complexity of the above exact solutions has led to the development of much simpler heuristic procedures to obtain nearly optimal replenishment schedules. Silver

[15] showed how the Silver-Meal heuristic, which was developed for the discrete time-varying demand case, could be adapted to give an approximate solution procedure for continuous time-varying demand patterns. Instead of finding the lot sizes that minimise the total ordering and inventory holding costs up to the time horizon, the author determined each lot size sequentially, one at a time, by finding the first local minimum of the total inventory cost per unit time. Phelps [13] proposed a computationally easier procedure by restricting the replenishment intervals to be constant. Under this restriction he determined the optimal number of replenishments using an iterative algorithm over the time horizon. Mitra et al. [8] developed a simpler method by modifying the EOQ model to accommodate the case of linear trend and a finite horizon. Naddor [10] proposed heuristic solution procedures and obtained good results for linearly increasing demand. Hong et al. [6] extended Naddor's work by assuming the production rate is uniform and finite, and developed three different heuristic policies. Recently, Omar et.al. [12] considered how the existing methodology developed by Naddor can be adapted for the case of a finite input rate.

In this paper we propose two alternative methods to solve deterministic inventory problem. The first procedure is based on a genetic algorithm. The second is based on spreadsheet modelling supported in Microsoft Excel Solver. We illustrate these procedures using several examples and the results are compared with those found in the literature.

2 Mathematical Model

The mathematical model of the inventory replenishment problem is based on the following assumptions and notations:

1. An inventory schedule for a single item is required over a known and finite period of time $(0, H)$.
2. Replenishment occurs instantaneously at an infinite rate.
3. The demand rate, $f(t)$, is known and varies with time.
4. There is a fixed setup cost of c_1 for each batch replenishment.
5. There is a carrying inventory cost of c_2 per unit per unit time.
6. n is the total number of batch replenishments (and therefore we define $t_n = H$).

The total relevant cost for n-batches can be expressed as (see [4]) :

$$TRC(n) = nc_1 + c_2 \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} y(t)\, dt, \tag{1}$$

where $y(t)$ is an inventory level at time t and

$$\begin{aligned} y(t) &= \int_{t_i}^{t_{i+1}} f(t)\, dt - \int_{t_i}^{t} f(t)\, dt \\ &= \int_{t}^{t_{i+1}} f(t)\, dt \qquad t_i \le t \le t_{i+1} \end{aligned} \tag{2}$$

For linearly increasing or decreasing demand, $f(t) = a \pm bt$, we obtain

$$
\begin{aligned}
TRC(n) &= nc_1 + c_2 \sum_{i=0}^{n-1} \big\{ \frac{a}{2}(t_{i+1}^2 - t_i^2) \pm \frac{b}{3}(t_{i+1}^3 - t_i^3) - t_i[a(t_{i+1} - t_i) \\
&\pm \frac{b}{2}(t_{i+1}^2 - t_i^2)] \big\}. \qquad (3)
\end{aligned}
$$

The total relevant cost for the exponentially declining demand, $f(t) = a\, e^{-bt}$, is given by

$$
TRC(n) = c_1 + c_2 \frac{a}{b^2} \sum_{i=0}^{n-1} \big[e^{-bt_i} + e^{-bt_{i+1}}(bt_i - bt_{i+1} - 1) \big]. \qquad (4)
$$

The objective for this problem is to find the number of replenishment orders, n, placed during the planning horizon and the corresponding replenishment times, t_i for

$$i = 0, 1, \ldots, n-1,$$

that minimize the total relevant cost. The optimization problem is:

$$
\begin{aligned}
\text{Minimize :} \quad & nc_1 + c_2 \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} y(t)\, dt \\
\text{subject to :} \quad & t_i < t_{i+1}, \quad i = 0, 1, \ldots, n-1, \\
& t_0 = 0, \\
& t_n = H.
\end{aligned}
$$

For fixed n, the optimal replenishment times can be found by analytical, heuristic or numerical methods. In this paper, we used genetic algorithm and Microsoft Excel Solver, to find the optimal replenishment times which satisfy the above optimization condition.

3 Genetic Algorithms

In the last decade we have seen a growing interest in biologically motivated approaches such as Neural Networks, Evolutionary Strategies and Genetic Algorithms (GAs) being applied to many complex optimisation problems. The processes occurring in the Natural Systems such as the intricate networks of human nervous systems, the behaviour of ants looking for food and the process of evolution, have inspired the development of these algorithms. Evolutionary Strategies and GAs are constructed based on the observation of evolutionary processes such as adaptation, selection, reproduction, mutation and competition. These processes are closely studied and translated into the form of computer simulations. Although these algorithms are a crude simplification of the natural processes, they have been successfully applied to many complex problems that were once intractable (see for example Mitchell [7]).

As with other evolutionary-based algorithms, GA is a stochastic search technique that closely mimics the metaphor of natural biological evolution. GA explores the problem domain by maintaining a population of individuals, which represents a set of potential solutions in the search space. The survival of each individual into the next generation is determined by its fitness. The fitness of an individual is a performance measure based on an objective function that describes the problem. At each iteration, new individuals (offspring) are created by selecting individuals according to their fitness and breeding them using genetic operators similar to natural genetics. The selection is carried out based on the principle of the survival-of-the-fittest where stronger individuals are allowed to participate more in the reproduction of new individuals than the weaker ones, who may not even contribute at all. Using genetic operators, GA attempts to combine the good features found in each individual using a structured yet randomised information exchange in order to construct individuals which are better suited to their environment than the individuals that they were created from. Through the evolution of better individuals, it is hoped that the desired solution will be found.

GA emerged in the mid-1960s and the book written by Holland [5], that gave the first rigorous description of GA, has generated an overwhelming interest within the scientific research community. GA has since been applied to a wide range of research fields including machine learning, pattern recognition, function optimisation, optimal control, control system engineering, scheduling, wire routing, etc. GA is an active research area and is continuously expanding as indicated by the vast amount of work and the number of conferences devoted specifically to GA.

4 Microsoft Excel Solver

Microsoft Excel Solver is one of the facilities available in the spreadsheet Microsoft Excel. It can be used to solve linear and nonlinear mathematical programming models with continuous and/or integer variables. Linear and integer problems use the simplex method with bounds on the variables and the branch-and-bound method. The nonlinear optimization model is solved using the Generalized Reduced Gradient (GRG2) method implemented by Lasdon and Waren. The details of the Solver can be found in Anderson et al. [1]

The Microsoft Excel spreadsheet version of the inventory model for linearly decreasing demand, $f(t) = 100 - 20t$ with $c_1 = 100, c_2 = 7.5$ and $H = 5$ appears in Table 1. It shows the nonoptimal solution for the problem. The Solver will minimize the target cell, B15, by changing the values of cells C9, D9, E9, F9 and G9 subject to the given constraints. In the Solver Options, we click the box Assume Non-Negative so that the changing cells will be nonnegative. The optimal solution is given in Table 2.

5 Numerical Results and Discussion

We have adapted GA based on parameter optimization to solve the inventory problem. Each chromosome represents the possible values of the time varying variables, t_i. Each variable is represented using a floating point representation as it offers high degree of precision and is capable of representing quite large domains which is the essence of many inventory problems.

The objective values of each chromosome is evaluated using equations 1–4.

Table 1: Non-optimal solution

	A	B	C	D	E	F	G	H
1	Inventory policy							
2								
3	Input data							
4	Demand parameters	$a =$	100		$b =$	-20		
5	Cost parameters	$c_1 =$	100		$c_2 =$	7.5		
6	Time horizon	$H =$	5					
7	Number of replenishment, n	6						
8	Batch time	t_0	t_1	t_2	t_3	t_4	t_5	t_6
9		0	0.5000	1.0000	2.0000	2.5000	4.0000	5.0000
10	Inventory per unit time per unit time		11.6667	10.4167	33.3333	6.6667	33.7500	3.3333
11								
12								
13	Total inventory cost	743.7500						
14								
15	Total cost	1343.7500						
16								
17	Batch time constraints	0.5000	<=	1.0000				
18		1.0000	<=	2.0000				
19		2.0000	<=	2.5000				
20		2.5000	<=	4.0000				
21		4.0000	<=	5.0000				

Table 2: Optimal solution

	A	B	C	D	E	F	G	H
1	Inventory policy							
2								
3	Input data							
4	Demand parameters	$a =$	100		$b =$	-20		
5	Cost parameters	$c_1 =$	100		$c_2 =$	7.5		
6	Time horizon	$H =$	5					
7	Number of replenishment, n	6						
8	Batch time	t_0	t_1	t_2	t_3	t_4	t_5	t_6
9		0	0.5411	1.1198	1.7496	2.4562	3.3041	5.0000
10	Inventory per unit time per unit time		13.5848	13.6398	13.7252	13.8768	14.2250	16.2572
11								
12								
13	Total inventory cost	639.8156						
14								
15	Total cost	1239.8156						
16								
17	Batch time constraints	0.5411	<=	1.1198				
18		1.1198	<=	1.7496				
19		1.7496	<=	2.4562				
20		2.4562	<=	3.3041				
21		3.3041	<=	5.0000				

The assignment of fitness to each individual is achieved using the linear ranking methods whereby individual(s) with the lowest fitness is assigned a value of 2 and individual(s) with the lowest objective value is assigned a value of 0. The values of other individuals are interpolated linearly between these two values.

Individuals are selected for breeding using the stochastic universal sampling that has been shown to have zero bias [2]. Pairs of individuals are then recombined using the intermediate recombination operator to produce offspring and mutation is achieved using the breeder genetic algorithm proposed by Mühlenbein and Schlierkamp-Voosen [9].

The population size, crossover rate and mutation rate are problem dependent. We note that for each problem the program were run for five times and we observed that the algorithm converges to the same value over the five runs for all the problems.

To demonstrate the effectiveness of these methods, we present three numerical examples with different demand function.

Example 1

In this example, the demand rate is linearly decreasing with:

$$a = 100, b = -20, c_1 = 100, c_2 = 7.5 \,\&\, t_n(= H) = 5.$$

For this problem, both GAs and Microsoft Excel Solver found the optimal solutions cited in the literature [3] whereby we replenish at times 0, 0.5410, 1.1197, 1.7494, 2.4560 and 3.3040, with the minimum total cost of 1239.8156.

Example 2

In this example, we consider a linearly increasing demand rate with:

$$a = 6, b = 1, c_1 = 90, c_2 = 1 \,\&\, t_n(= H) = 11.$$

The GA and Microsoft Excel Solver give the optimal number of replenishment orders as 3 with the corresponding replenishment times at 0, 4.2099 and 7.7915. Both algorithms found the best optimal total relevant cost of 510.8392 as given by Donaldson [4].

Example 3

This example considers a problem with an exponentially declining demand rate where:

$$a = 500, b = 0.5, c_1 = 30, c_2 = 0.2 \,\&\, t_n(= H) = 10.$$

The optimal results obtained by both procedures are comparable with those found by Omar et al [11]. The replenishment times are at 0, 0.9165, 2.1424 and 4.0408 with the minimum total cost of 259.0128.

These numerical results suggest that the genetic algorithm and Microsoft Excel Solver are able to produce competitive results. The advantage of using the genetic algorithms and Microsoft Excel Solver is that they are very straight forward to use and can easily be adapted to accomodate different demand patterns as compared to the existing heuristic methods.

6 Conclusions

We have shown that GAs and Microsoft Excel Solver can easily be adapted to solve deterministic inventory problems and in all the examples considered in this paper, both procedures obtained the optimal solutions reported in the literature.

Since most of the deterministic time-varying demand process can be expressed as a function of t_i (see for example [12]), the above procedures can offer a very attractive alternative solution methods for other deterministic inventory problems.

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