MATEMATIKA: MJIAM, 2021, Volume 37, Number 2, 77–90

An Optimal Control Analysis of the MERS-CoV Outbreak in South Korea

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Article history
Received: 3 June 2020
Received in revised form: 17 June 2021
Accepted: 25 July 2021
Published on line: 25 August 2021

Abstract MERS-CoV is a respiratory disease that originated in the Middle East. This disease is caused by a novel coronavirus which transmits from Dromedary camels to humans, then passed on to other humans through direct contact or droplets. Said to be the biggest to occur outside of Middle East with 186 cases, the outbreak in South Korea should be discussed further. By going through the chain of transmission, it was found that 44% of the infected individuals were exposed to the virus in the healthcare facilities' environment. To reduce the risk of exposure to the virus, several efforts have been made such as restricting healthcare facilities visitors and increasing the number of isolated individuals in hospitals. The previous study has developed a mathematical model for the outbreak in South Korea and estimated transmission rates from the data provided by the government. In this study, both efforts will be added as control variables to form an optimal control problem to minimize the number of infected individuals and the costs incurred. The optimal control will be obtained using the Pontryagin maximum principle, then simulated using the fourth-order Runge-Kutta forward-backward sweep method for two scenarios to see the effectiveness of both controls. The simulation showed that restricting health facilities visitors to limit the chance of exposure was the most effective strategy to control the MERS-CoV outbreak in South Korea.

Keywords MERS - CoV; optimal control; visitor restriction; isolation; Pontryagin maximum principle.

Mathematics Subject Classification 46N60, 92B99.

1 Introduction

In 2012, countries in the Middle East reported an infectious disease with the first case occurring in Saudi Arabia [1–3]. The disease, called Middle East respiratory syndrome, is caused by a novel coronavirus (MERS-CoV) found in Dromedary camels in several Middle Eastern, African and South Asian countries [4]. The virus then transmitted to other humans through direct contact or droplets [5, 6]. Some of the common symptoms of this disease are fever, cough, and shortness of breath [7]. In more severe cases, it is also found that infected individuals have respiratory complications such as pneumonia and respiratory failure. Since the first case report in 2012, this disease has spread to 27 countries with an estimated 80% of cases being in Saudi Arabia [4]. In Asia, the disease spread mostly to Western Asia countries. Countries outside Western Asia with known human cases are South Korea, China, Iran, Philippines, Thailand and Malaysia.

Said to be the biggest to occur outside of Middle East, the outbreak in South Korea should be discussed further. In South Korea itself, a total of 186 MERS-CoV cases were recorded including 39 deaths [8]. From the results of the transmission chain tracking, 44% of patients were infected by MERS-CoV in 16 hospitals [9]. where 44.1% of the infected individuals were patients, 32.8% were caregivers, and 13.4% were healthcare personnel [10]. In an effort to control the MERS-CoV outbreak, the South Korean government with the support of several Korean academic communities, created a guide for preventing and controlling MERS infections for health facilities and at the request of the government. Referring to the guidance and advice given by the experts, several ways that can be done to control the transmission of MERS-CoV include minimizing contact with patients with respiratory symptoms in the health facility environment, especially in the emergency department, socializing the community about how dangerous MERS-CoV and isolating infected patients in hospitals with adequate facilities [9].

Mathematical models has been widely used to study the dynamics of infectious diseases, including MERS-CoV. Malik *et al.* in [11] introduced a deterministic model for the MERS-CoV with vaccination and quarantine. Chowell *et al.* in [12] developed a stochastic SEIR-type compartmental transmission of the MERS-CoV outbreak during April-October 2013 by taking into account the zoonotic or index cases and secondary cases. Cachemez *et al.* in [13] evaluated the extent of human infection, the performance of case detection, and the transmission potential of MERS-CoV with and without control measures by estimating the incubation period and generation time. Aldila *et al.* in [14] analyze the efectiveness of medical mask usage and supportive care treatment to control the MERS disease, omitting the MERS-caused deaths and assuming that susceptible individuals who wear medical masks will not be infected. Xia *et al.* in [15] constructed two dynamical models to simulate the propagation processes in South Korea from the data provided by WHO and calculated the basic reproduction number. Kim *et al.* in [16] suggested a mathematical model for the MERS-CoV outbreak in South Korea based on the model developed by Chowell [12]. Using the data provided by the government, an estimation of the transmission rates of the infected and hospitalized individuals are obtained.

This article will discuss the effectiveness of several efforts stated in the guide to control the spread of MERS-CoV in South Korea provided by the government. Control measures that will be considered are efforts to increase the number of patients treated in hospitals (isolation) and limit the number of visitors to health facilities to reduce the chance of infection in health facilities. Both of these control efforts will be added to the mathematical model studied by Kim *et al.* [16] as a control variable, and the optimal value is then obtained using the Pontryagin Maximum Principle. Using parameters estimated by Kim *et al.* in [16], the effect of both efforts in controlling the outbreak in South Korea will be discussed.

2 Mathematical Model

The MERS-CoV transmission model used to represent the MERS-CoV transmission chain in South Korea is a SEIR model that was developed into six compartments, namely individuals who are prone to be infected or Susceptible (S), individuals who were exposed to the virus or Exposed (E), individuals who are infected but does not show symptoms or Asymptomatic (A), individuals who are infected and show symptoms or Infected (I), individuals who get treatment in hospital or Hospitalized (H), and individuals who have recovered (R). In this model, it is assumed that the total population is constant, the number of births and deaths is ignored and transmission occurs only between humans. In addition, only infected individuals who shows symptoms and hospitalized individuals can transmit the virus to susceptible individuals.

Based on the description above, the transmission model of MERS-CoV in South Korea is expressed by system of ODEs as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= -S\frac{\beta\left(I+lH\right)}{N},\\
\frac{dE}{dt} &= S\frac{\beta\left(I+lH\right)}{N}-\kappa E,\\
\frac{dI}{dt} &= \kappa\rho E-\left(\gamma_a+\gamma_I\right)I,\\
\frac{dA}{dt} &= \kappa\left(1-\rho\right)E,\\
\frac{dH}{dt} &= \gamma_a I-\gamma_r H,\\
\frac{dR}{dt} &= \gamma_I I+\gamma_r H,
\end{aligned}
\tag{1}
$$

where β is the rate of transmission between individuals per unit of time (days), l is the relative transmission of hospitalized individuals, κ is the rate of change in the number of individuals exposed and become infected, ρ is the proportion of exposed individuals who turn into infected individuals with symptoms, γ_a is the average rate of infected individuals with symptoms being treated in hospital, γ_I is the rate of recovery without treatment in hospital, and γ_r is the rate of recovery of individuals who are hospitalized [16]. The compartment diagram for the model (1) is presented in Figure 1.

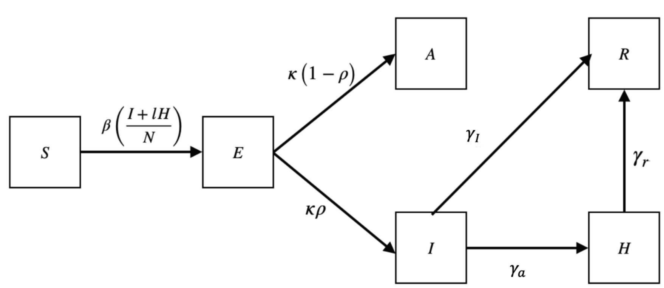


Figure 1: Compartment diagram for the transmission todel of MERS-CoV in South Korea

It is easy to verify that the model (1) has only one equilibrium point $(S^*, E^*, I^*, A^*, H^*, R^*) = (S^*, 0, 0, A^*, 0, R^*)$ where S^*, A^*, and R^* are of positive values. Since the number of infected population is zero, this equilibrium point leads to disease free population. Furthermore, one can predict whether there will be an outbreak if one infectious individual is introduced to a population by looking at the basic reproduction number. Basic reproduction number can be interpreted simply as the average number of infection case produced, that is the average number of new infection caused by one infectious individual [17]. By using the next generation matrix approach as outlined in [18], the basic reproduction number of model (1) is attained as:

$$\mathcal{R}_0 = \frac{\rho\beta}{\gamma_a + \gamma_I}\left(1 + \frac{\gamma_a l}{\gamma_r}\right), \tag{2}$$

where ρ is the spectral radius of the next generation matrix. The first term represent the average number of individual infected by an individual in the I population and the second term represent the average number of individual infected by an individual in the H population [16].

While it can be tell directly that efforts for isolating infected individuals in hospital with adequate facilities will increase the number of hospitalized individuals, visitors restriction will help to minimize the chance of people being exposed since most infected individuals were exposed to the disease in health facilities. By adding controls u_1 and u_2 which each represent restriction on visitors to health facilities and effort to increase the number of isolated individuals in hospital, the MERS-CoV disease spread model in South Korea given by model (1) becomes:

$$\begin{aligned}
\frac{dS}{dt} &= -S\frac{\beta\left(1 - u_1(t)\right)\left(I + lH\right)}{N}, \\
\frac{dE}{dt} &= S\frac{\beta\left(1 - u_1(t)\right)\left(I + lH\right)}{N} - \kappa E, \\
\frac{dI}{dt} &= \kappa\rho E - \left(\gamma_a\left(1 + u_2(t)\right) + \gamma_I\right)I, \\
\frac{dA}{dt} &= \kappa\left(1 - \rho\right)E, \\
\frac{dH}{dt} &= \gamma_a\left(1 + u_2(t)\right)I - \gamma_r H, \\
\frac{dR}{dt} &= \gamma_I I + \gamma_r H,
\end{aligned} \tag{3}$$

with total population $N = S + E + I + A + H + R$ and initial conditions

$$S(0) \geq 0, \quad E(0) \geq 0, \quad I(0) \geq 0, \quad A(0) \geq 0, \quad H(0) \geq 0, \quad R(0) \geq 0. \tag{4}$$

3 Optimal Control Analysis

Our aim is to minimize the number of infected individuals, both with or without symptoms, and the costs incurred for control efforts. Mathematically, the goal can be written in the form of an objective function as follows

$$J(u_1, u_2) = \int_0^{T_f}\left(A_1 I + A_2 H + \frac{B_1}{2}u_1^2 + \frac{B_2}{2}u_2^2\right)dt, \tag{5}$$

where A_1 and A_2 are the weights of costs for I and H populations, and B_1 and B_2 are the weights of costs incurred to limit health facilities visitors and increase the number of patients isolated in hospitals. The objective function in equation (5) along with a mathematical model of the spread of MERS-CoV disease in system (3) forms an optimal control problem of obtaining the optimal controls u_1^* and u_2^* such that

$$J\left(u_1^*, u_2^*\right) = \min\left\{J\left(u_1, u_2\right) | u_1, u_2 \in \mathcal{U}\right\}.$$

The optimal controls u_1^* and u_2^* must satisfy the necessary conditions obtained through the Pontryagin maximum principle [19,20] so that the optimal control of the system (3) to minimize the objective function (5) can be obtained using the Pontryagin Maximum Principle [19].

Based on the objective function (5) and systems (3) as the governing equation, the Hamiltonian function can be formed as:

$$\begin{aligned}\mathcal{H}\left(I, H, u_1, u_2, \lambda\right) &= A_1 I + A_2 H + \frac{B_1}{2}u_1^2 + \frac{B_2}{2}u_2^2 \\ &+ \lambda_1\left[-S\frac{\beta\left(1-u_1\right)\left(I+lH\right)}{N}\right] \\ &+ \lambda_2\left[S\frac{\beta\left(1-u_1\right)\left(I+lH\right)}{N} - \kappa E\right] \\ &+ \lambda_3\left[\kappa\rho E - \left(\gamma_a\left(1+u_2\right)+\gamma_I\right)I\right] \\ &+ \lambda_4\left[\kappa\left(1-\rho\right)E\right] + \lambda_5\left[\gamma_a\left(1+u_2\right)I - \gamma_r H\right] \\ &+ \lambda_6\left[\gamma_I I + \gamma_r H\right],\end{aligned} \tag{6}$$

where $\Lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$ being the adjoint vector related to the state variables $\backsim = (S, E, I, A, H, R)$.

Stationary conditions are attained by deriving equation (6) with respect to the control vectors u_1 and u_2:

$$\frac{\partial\mathcal{H}}{\partial u_1} = 0, \qquad \frac{\partial\mathcal{H}}{\partial u_2} = 0,$$
$$\Rightarrow u_1^* = \frac{S^*\beta\left(I^*+lH^*\right)\left(\lambda_2^*-\lambda_1^*\right)}{NB_1}, \quad \Rightarrow u_2^* = \frac{\gamma_I I^*\left(\lambda_3^*-\lambda_5^*\right)}{B_2}.$$

Assuming that both controls have an effectiveness level of 60%, the following solutions are obtained:

$$u_1^* = \begin{cases} 0, & \text{if} \quad \frac{S^*\beta(I^*+lH^*)\left(\lambda_2^*-\lambda_1^*\right)}{NB_1} \leq 0, \\ \frac{S^*\beta(I^*+lH^*)\left(\lambda_2^*-\lambda_1^*\right)}{NB_1}, & \text{if} \quad 0 < \frac{S^*\beta(I^*+lH^*)\left(\lambda_2^*-\lambda_1^*\right)}{NB_1} < 0.6, \\ 0.6, & \text{if} \quad \frac{S^*\beta(I^*+lH^*)\left(\lambda_2^*-\lambda_1^*\right)}{NB_1} \geq 0.6, \end{cases}$$

$$u_2^* = \begin{cases} 0, & \text{if} \quad \frac{\gamma_I I^*\left(\lambda_3^*-\lambda_5^*\right)}{B_2} \leq 0, \\ \frac{\gamma_I I^*\left(\lambda_3^*-\lambda_5^*\right)}{B_2}, & \text{if} \quad 0 < \frac{\gamma_I I^*\left(\lambda_3^*-\lambda_5^*\right)}{B_2} < 0.6, \\ 0.6, & \text{if} \quad \frac{\gamma_I I^*\left(\lambda_3^*-\lambda_5^*\right)}{B_2} \geq 0.6. \end{cases}$$

Both correspond to

$$u_1^* = \min\left\{0.6, \max\left\{0, \frac{S^*\beta\left(I^* + lH^*\right)\left(\lambda_2^* - \lambda_1^*\right)}{NB_1}\right\}\right\}, \tag{7}$$

$$u_2^* = \min\left\{0.6, \max\left\{0, \frac{\gamma_I I^*\left(\lambda_3^* - \lambda_5^*\right)}{B_2}\right\}\right\}. \tag{8}$$

Furthermore, from equation (6) one can derive the state equations:

$$\begin{aligned}
\dot{S}^*(t) &= \frac{\partial \mathcal{H}^*}{\partial \lambda_1^*} = -S^*\frac{\beta\left(1-u_1^*\right)\left(I^*+lH^*\right)}{N},\\
\dot{E}^*(t) &= \frac{\partial \mathcal{H}^*}{\partial \lambda_2^*} = S^*\frac{\beta\left(1-u_1^*\right)\left(I^*+lH^*\right)}{N} - \kappa E^*,\\
\dot{I}^*(t) &= \frac{\partial \mathcal{H}^*}{\partial \lambda_3^*} = \kappa\rho E^* - \left(\gamma_a\left(1+u_2^*\right)+\gamma_I\right) I^*,\\
\dot{A}^*(t) &= \frac{\partial \mathcal{H}^*}{\partial \lambda_4^*} = \kappa\left(1-\rho\right) E^*,\\
\dot{H}^*(t) &= \frac{\partial \mathcal{H}^*}{\partial \lambda_5^*} = \gamma_a\left(1+u_2^*\right) I^* - \gamma_r H^*,\\
\dot{R}^*(t) &= \frac{\partial \mathcal{H}^*}{\partial \lambda_6^*} = \gamma_I I^* + \gamma_r H^*,
\end{aligned} \tag{9}$$

and adjoint equations:

$$\begin{aligned}
\dot{\lambda}_1^*(t) &= -\frac{\partial \mathcal{H}^*}{\partial S^*} = \frac{\beta\left(1-u_1^*\right)\left(I^*+lH^*\right)\left(\lambda_1^*-\lambda_2^*\right)}{N},\\
\dot{\lambda}_2^*(t) &= -\frac{\partial \mathcal{H}^*}{\partial E^*} = \kappa\left(\lambda_2^* - \lambda_3^*\rho - \lambda_4^*\left(1-\rho\right)\right),\\
\dot{\lambda}_3^*(t) &= -\frac{\partial \mathcal{H}^*}{\partial I^*} = -A_1 + \frac{S^*\beta\left(1-u_1^*\right)\left(\lambda_1^*-\lambda_2^*\right)}{N} + \gamma_a\left(1+u_2^*\right)\left(\lambda_3^*-\lambda_5^*\right) + \gamma_I\left(\lambda_3^*-\lambda_6^*\right),\\
\dot{\lambda}_4^*(t) &= -\frac{\partial \mathcal{H}^*}{\partial A^*} = 0,\\
\dot{\lambda}_5^*(t) &= -\frac{\partial \mathcal{H}^*}{\partial H^*} = -A_2 + \frac{S^*\beta I^*\left(1-u_1^*\right)\left(\lambda_1^*-\lambda_2^*\right)}{N} + \gamma_r\left(\lambda_5^*-\lambda_6^*\right),\\
\dot{\lambda}_6^*(t) &= -\frac{\partial \mathcal{H}^*}{\partial R^*} = 0.
\end{aligned} \tag{10}$$

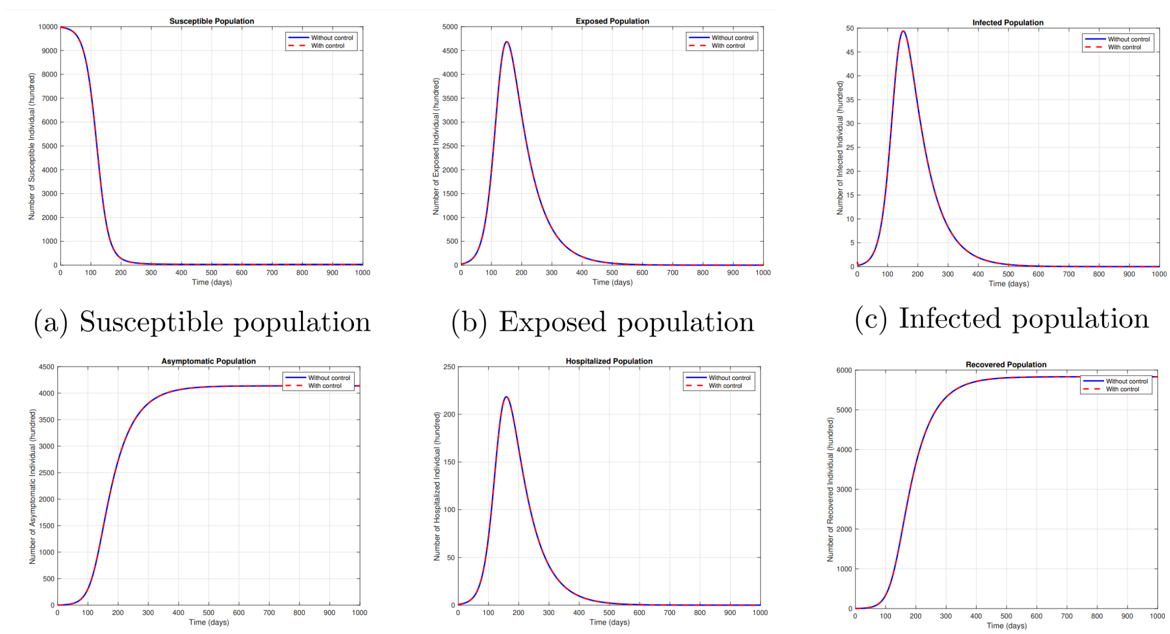
4 Numerical Simulations

In this section, the optimal controls will be obtained by solving the state and adjoint equations (9-10) numerically using the fourth-order Runge-Kutta forward-backward sweep method [20]. Afterward, the optimal controls will be simulated for the two periods described in [16] to see which control is most influential to overcome the MERS-CoV outbreak in Korea. Hence, the simulation will be carried out for three cases, where the first case is increasing the number of infected individuals being isolated in hospitals without restricting healthcare facilities visitor ($u_1 = 0, u_2 \neq 0$), the second is restricting healthcare facilities visitor without increasing the number of infected individuals being isolated in hospital ($u_1 \neq 0, u_2 = 0$) and the third is combining both the restriction on healthcare facilities visitors and effort to increase the number of infected individuals being isolated in hospitals ($u_1, u_2 \neq 0$). To simulate the outbreak, it is assumed that the population consists of 10,000 individuals with 26 individuals being exposed, 1 infected and 1 asymptomatic. Referring to Kim *et al.* in [16], the parameter values used are:

$$\begin{aligned} &\kappa = \frac{1}{66}, \quad \rho = 0.585, \quad \gamma_a = 0.6403, \\ &\gamma_I = \frac{1}{5}, \quad \gamma_r = \frac{1}{7}. \end{aligned} \tag{11}$$

4.1 Simulation for Period 1

Period 1 is the period in which the number of newly infected individuals continues to increase. Kim, *et al.* in [16] estimated the rate of transmission between individuals and the relative transmission of individuals treated at the hospital by $\beta = 0.0835$ and $l = 22$, respectively, so that the basic reproductive value for this period is $\mathcal{R}_0 = 5.3973$. In other words, the first period leads to the MERS-CoV spreading in the population, therefore control is needed to reduce the spread of disease. Figure 2-4 shows the effect of the optimal controls in the first period for the first, second and third case, respectively.

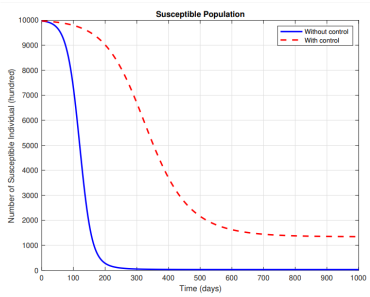


(a) Susceptible population (b) Exposed population (c) Infected population

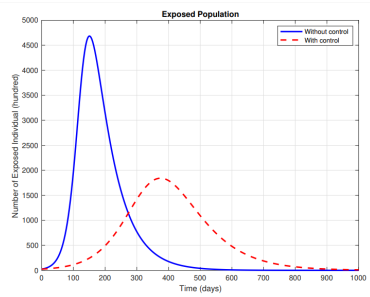
(d) Asymptomatic population (e) Hospitalized population (f) Recovered population

Figure 2: Simulation for period 1 with isolation only ($u_1 = 0$)

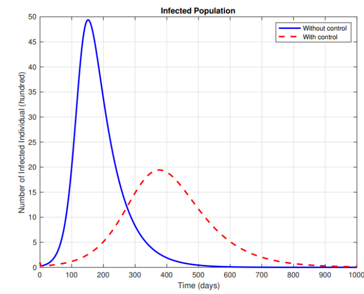
Figure 2 shows the effect of increasing the number of infected individuals being isolated in hospitals on the first period. It can be seen that for period 1 the graphs for the population with control coincides with the graph for the population without control. This shows that increasing the number of individuals isolated in hospitals has no effect in suppressing MERS-CoV transmission.



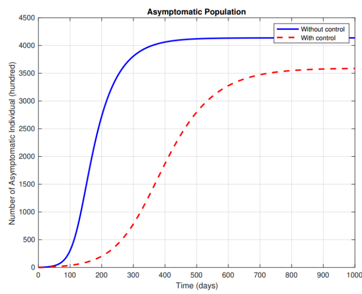
(a) Susceptible population



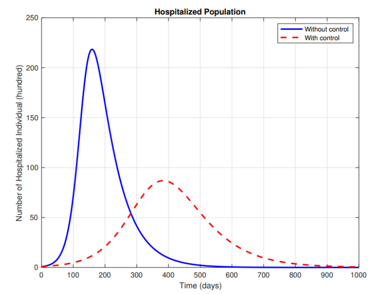
(b) Exposed population



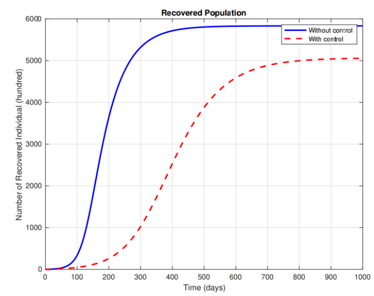
(c) Infected population



(d) Asymptomatic population



(e) Hospitalized population



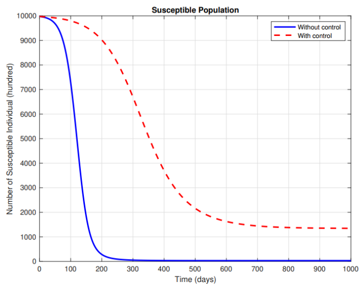
(f) Recovered population

Figure 3: Simulation for period 1 with restriction on healthcare facilities visitors only ($u_2 = 0$)

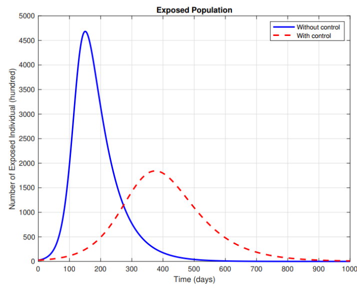
From Figure 3 it can be seen that for period 1, restricting healthcare facilities visitors results in reduced number of infected and asymptomatic individuals (Figure 3b-3d), hence the drop in the hospitalized and recovered population (Figures 3e-3f) and the rise in the susceptible population (Figure 3a). This indicates that limiting health facilities visitors to reduce the chance of contact with infected individuals is efficacious.

From Figure 4 it can be seen that for period 1, the combination between restricting healthcare facilities visitors and increasing the number of isolated infected individuals in hospitals results in reduced number of exposed, infected, asymptomatic, hospitalized and recovered individuals (Figure 4b-4f) and increased number of susceptible individuals (Figure 4a). This outcome is identical with the result for the practice of restricting healthcare facilities visitors alone as seen in Figure 3, meaning that the combination of both controls is no more effective than only restricting healthcare facilities visitors.

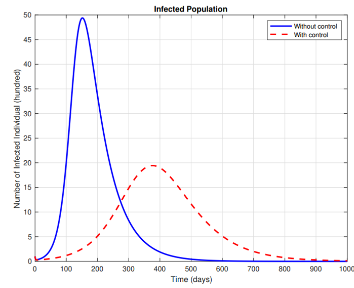
Figure 5 shows the optimal control profiles for the three cases in the first period. For the first case where there is only increasing the number of isolated infected individuals in hospitals, the control is at the lower bound throughout the period, where at the end, that is at the day 1000, the control risen (Figure 5a). For the second case, the control is at the maximum for more than 900 days, then dropped down until it reach the lower bound at day 1000 (Figure 5b). As seen in (Figure 5c), the control profile for the third case is one and the same with the profile for the second case, that is the control u_2 which indicates effort in increasing the number of isolated infected individuals is no help in suppressing the outbreak.



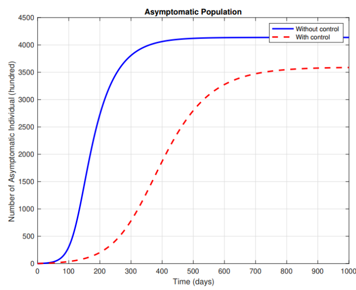
(a) Susceptible population



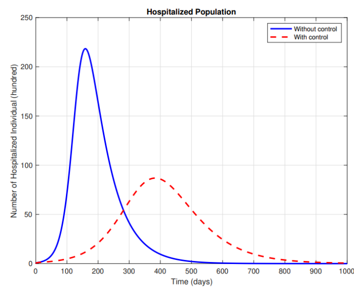
(b) Exposed population



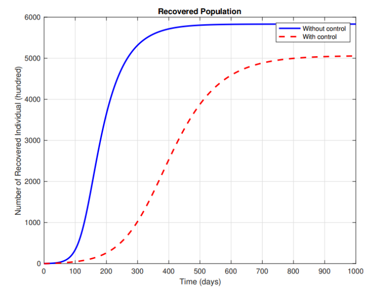
(c) Infected population



(d) Asymptomatic population

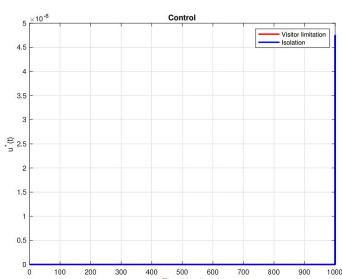


(e) Hospitalized population

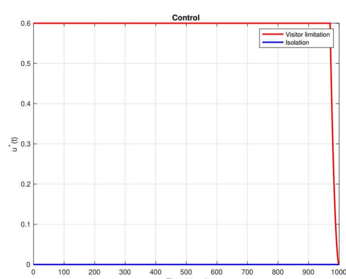


(f) Recovered population

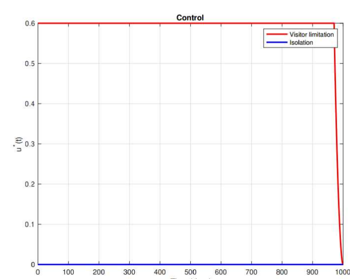
Figure 4: Simulation for period 1 with visitor restriction and isolation ($u_1, u_2 \neq 0$)



(a) Infected individuals isolation only ($u_1 = 0$)



(b) Restriction on healthcare facility only ($u_2 = 0$)

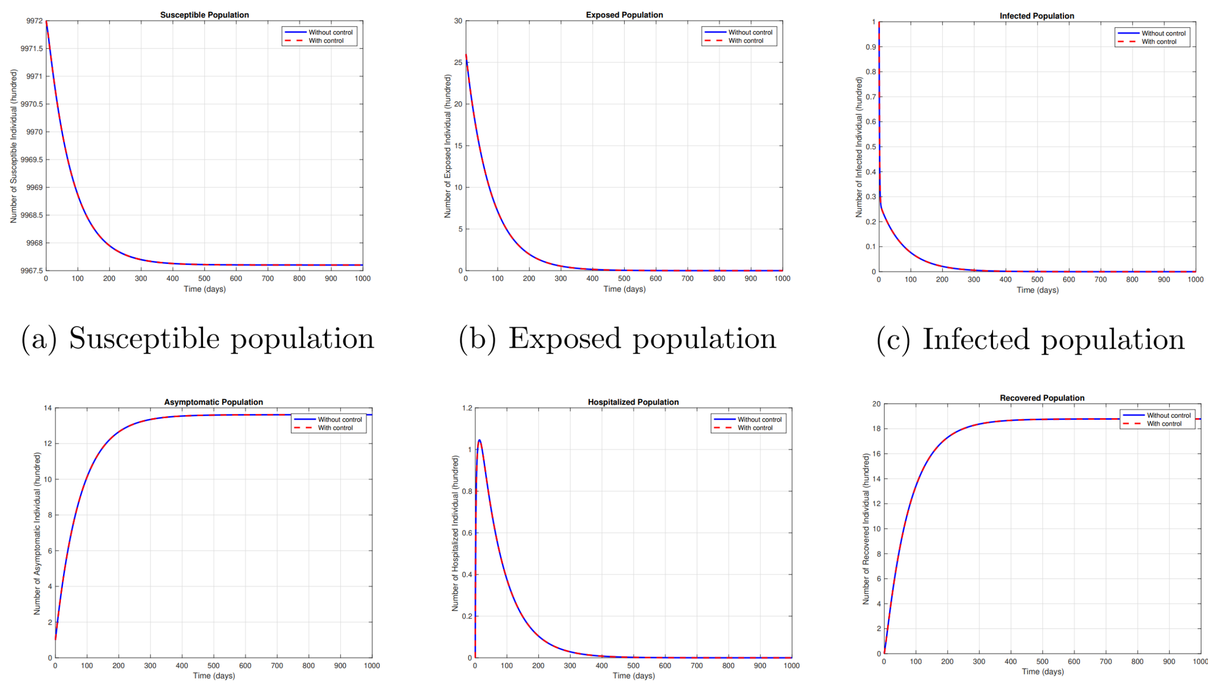


(c) With visitor restriction and isolation ($u_1, u_2 \neq 0$)

Figure 5: Optimal control profile for period 1

4.2 Simulation for Period 2

In contrary to period 1, period 2 is the period in where the trend in the number of infected individuals decreases. The rates of transmission between individuals and the relative transmission of individuals hospitalized for this period is estimated in [16] by $\beta = 0.036$ and $l = 1$, hence the basic reproductive value for this period is $\mathcal{R}_0 = 0.1351$ or namely the population will tend to be disease free. Accordingly, control is needed to accelerate the disappearance of disease in the population. Figure 6-8 shows the effect of the optimal controls in the second period for the first, second and third case, respectively.



(a) Susceptible population (b) Exposed population (c) Infected population

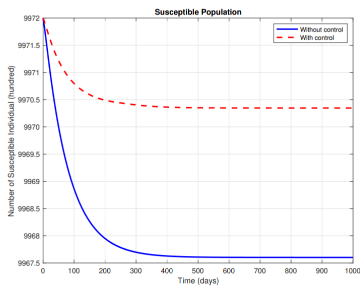
(d) Asymptomatic population (e) Hospitalized population (f) Recovered population

Figure 6: Simulation for period 2 with isolation only ($u_1 = 0$)

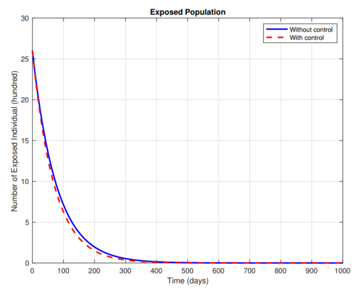
From Figure 6 it can be seen that the susceptible, exposed, infected, hospitalized and recovered populations are decreasing, while the rising of the asymptomatic population is meaningless since it is assumed that they are unable to infect another individual. Thus, the disease will be gone eventually. Furthermore, there is no difference between populations without and with controls, meaning that the attempt to increase the number of isolated infected individuals in hospitals has no effect to get rid of the disease faster.

Figure 7 shows that restriction on healthcare facilities visitors caused the susceptible population to rise greatly (Figure 7a) and the other to drop (Figure 7b-7f). Moreover, the asymptomatic and recovered populations experienced sizeable drop compared to the exposed, infected and hospitalized populations. This implies that for period 2, restriction on healthcare facilities visitors were beneficial.

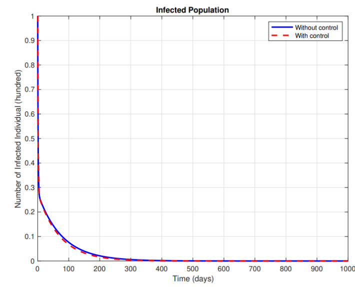
Combining both visitors restriction and isolation (Figure 8) seems to produce the same outcome as restricting healthcare facilities visitors alone (Figure 7), meaning that the additional effort of increasing the number of isolated infected individuals is futile.



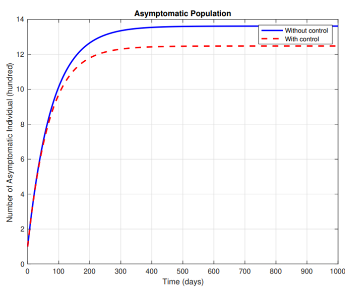
(a) Susceptible population



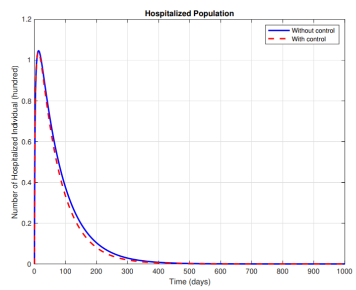
(b) Exposed population



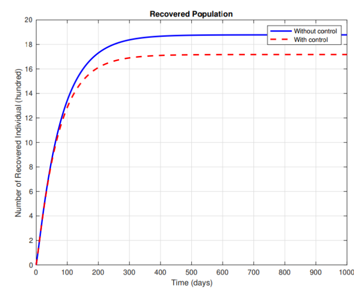
(c) Infected population



(d) Asymptomatic population

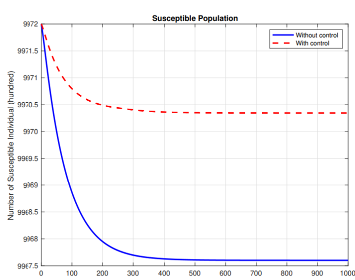


(e) Hospitalized population

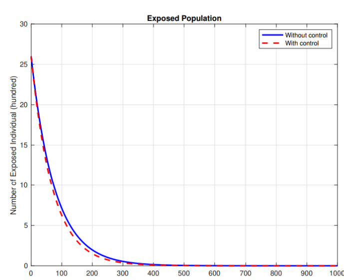


(f) Recovered population

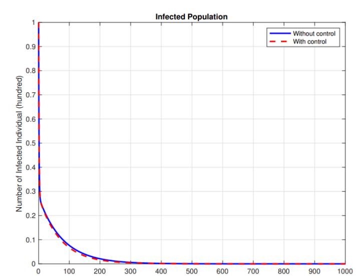
Figure 7: Simulation for period 2 with restriction on healthcare facilities visitors only ($u_2 = 0$)



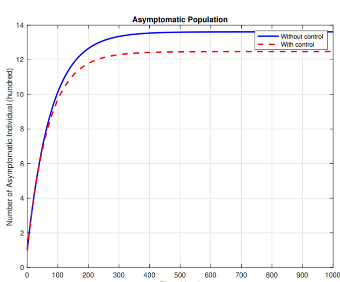
(a) Susceptible population



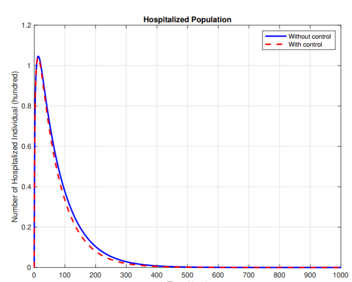
(b) Exposed population



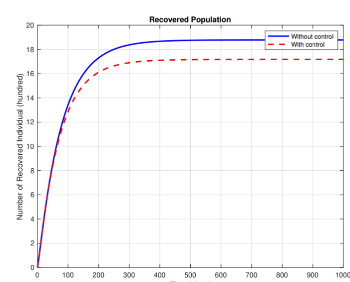
(c) Infected population



(d) Asymptomatic population

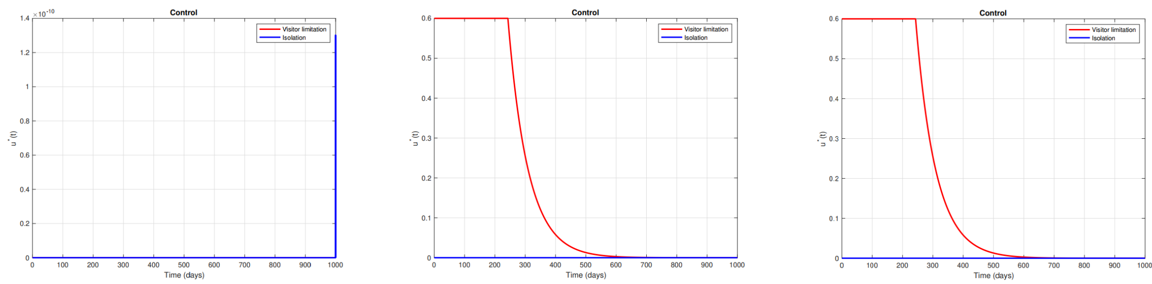


(e) Hospitalized population



(f) Recovered population

Figure 8: Simulation for period 1 with visitor restriction and isolation($u_1, u_2 \neq 0$)



(a) Infected individuals isolation only ($u_1 = 0$)

(b) Restriction on healthcare facility only ($u_2 = 0$)

(c) With visitor restriction and isolation ($u_1, u_2 \neq 0$)

Figure 9: Optimal control profile for period 1

Figure 9 shows the optimal control profiles for the three cases in the second period. For the first case, the profile is identical to the profile for the first case in the first period, that is the control is at the lower bound throughout the period and the control risen at the end (Figure 9a). For the second case, the control is at the maximum for more than 2 days, then slowly dropping to zero at day 600 (Figure 9b). For the third case, the control profile is identical to that of second case, as seen in (Figure 9c). It means that just like in the first period, the control u_2 does not help to speed up the MERS-CoV disapperance.

5 Conclusion

In order to overcome the MERS-CoV outbreak in South Korea in 2015, the South Korean government made several efforts such as limiting visitors to health facilities and isolating infected individuals in hospitals. To see which efforts are more effective in overcoming the MERS-CoV outbreak in South Korea, an analysis of optimal control using Pontryagin's maximum principle is carried out. From the simulation results, it can be concluded that reducing the chance of being exposed to the virus by restricting health facilities visitors is the most effective strategy to be implemented as compared to isolating infected individuals or the combination of both.

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