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Intraguild Predation Model with Stage Structure and Cannibalism in Prey Population

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Abstract This paper concerns an intraguild predator model with stage structure and cannibalism in the prey population, where the prey population is divided into two stages, namely, a juvenile stage, and an adult stage. We investigated the dynamics of this model. We obtained the equilibrium points, the conditions of existence, and the condition for stability using the Jacobian matrix and Routh Hurwitz criterion. We use numerical simulation to illustrate the dynamics of the model over time. Furthermore, we illustrate the effects of the maturation rate on the adult prey population and the carrying capacity of the basal resources on the existence and stability region of the model. We also analysed the transcritical bifurcation with the maturation rate selected as the primary bifurcation parameter. We demonstrate herein that each equilibrium point undergoes stability switching of the equilibria with the change of maturation rate either from stable to unstable to stable, or unstable to stable to unstable. The obtained result indicates that a large maturation rate can lead to a system where the prey population can escape from predation yet negatively impacts the survival rate of basal resources in the ecosystem. Whereas, an intermediate maturation rate indicates that the species could co-exist whereby theoretically, it can ensure that the optimal predator population exists in the ecosystem. This is in parallel able to stabilise the effect of overpopulation of predators with competition factor. An ecosystem with all species coexists could ensure a healthy ecological interaction for a sustainable environment.

Keywords Intraguild predation; Population dynamics; Routh-Hurwitz criteria; Instability switching; Transcritical bifurcation

Mathematics Subject Classification 46N60, 92D25, 93C15

1 Introduction

The Biology Online Dictionary [1] defines ‘e*cosystem*’ as a system that includes all living organisms (biotic factors) in an area and the physical environment (abiotic factors) which function together as a unit. Deriving from this, an organism refers to an individual living thing that is able to react to stimuli, reproduce, grow, and maintain homeostasis. Nguyen [2] stated that organisms occupy space that is called as niche, a physical space that the organisms live in, with the resources that organisms use, and where the interaction among different organisms takes place. The interaction among organisms within or between overlapping of the niches can be divided into three types of interaction, namely, competition, predation, and symbiosis. According to Cronin and Carson [3], competition can be defined as an interaction that occurs between two or more organisms that require same and limited resources. It may occur within or between species. Competition reduces growth, reproduction, and survival whenever an organism is able to defend the other organism for the resource. Predation is cited as a relationship in which member of one species known as predator consumes or feeds on the member of another species, i.e., the prey. This predator–prey relationship accounts for the energy transfer in food chain and the food webs [4]. Predation is a relation that includes 'positive/ negative' interaction where one organism in the habitat consumes some or all other organisms [5].

Polis *et al.*, [6] remarked that intraguild predation (IGP) refers to the combination of competition and predation, i.e., killing and eating the species that feed on same and limited resource and thus potential competitors. Polis and Holt [7] suggested a general criterion for the coexistence of IGP systems, which states that the intermediate species (the prey in the IGP system) should be superior when competing for the shared resource, which also refers to the exploitative competition; meanwhile, the top species (the predator) should gain from its consumption of the intermediate species. Tuzin [8] mentioned that cannibalism is the exploitation of resources, i.e., the body, in which the resources had been assimilated and stored. Holt and Huxel [9] presented the summary of a typical three-species IGP theory and have described the results of theoretical consideration on how alternate prey can impact the preservation and stability of a principal IGP interaction. Bhattarcharya and Pal [10] analysed the effect of IGP of a two-dimensional single species that was divided into juvenile and adult stages of organisms with harvesting of the adult species. Namba *et al.* [11] studied the stabilizing effect of intra-specific competition on predator–prey dynamics with IGP which focuses on one-prey-two-predator model. Zhang *et al.* [12] developed a stage structured predator–prey model with cannibalism in the predator. The predator is divided into two categories, namely, adult predator and juvenile predator. The global stability of the model has been analysed to illustrate the effect of cannibalisation and benefits of cannibalism on the dynamic of the model. Ghosh *et al.* [13] developed a stage structured predator-prey model in continuous time, where both the prey and predator exhibit adult and juvenile species. The stability of the model with density-dependent effect has been studied in detail.

With reference to [12], this paper will incorporate the concept of IGP instead of predator-prey with addition of basal resources as a species. The stage structured concept is introduced among prey population and cannibalism effect is included in the model as Rudolf [14], who stated that previous studies on set of structured models showed that cannibalism can completely alter the dynamics and structure of three-species IGP systems, depending on the trophic position where cannibalism occurs, and it is frequent at the intermediate trophic levels. The concept of IGP highlights the competition among predator and adult prey and predation interaction between predator and juvenile prey.

The remainder of this paper is organised as follows. In Section 2, we first develop a nonlinear ordinary differential equation (ODE) system and further simplify the model by rescaling and introducing new parameters and then, obtain the equilibrium point. In Section 3, we discuss the conditions for existence and local stability of each equilibrium points. In Section 4, we discuss the stability region of the equilibrium points within two different parameters and illustrate the population density over time using numerical simulation. We have performed bifurcation analysis where we have analysed the effect of maturation rate and carrying capacity of basal resources towards the population density of adult prey.

2 Model Development

Figure 1 schematically depicts the IGP model with stage structure and cannibalism. The IGP relationship demonstrated with the predation occurs between IG predator and IG juvenile prey, while predator and adult prey compete with each other for basal resources. Stage structure and cannibalism interaction are introduced among prey population with adult prey cannibalising on the juvenile prey.

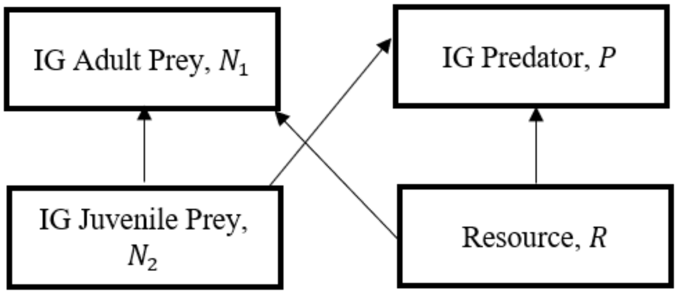


Figure 1: IGP Model with Stage Structure and Cannibalism in IG Prey

This paper proposes an asymmetric IGP model with stage structure and cannibalism in IG prey, which is mathematically described as follows:

$$\begin{aligned}\frac{dP}{dt} &= a_1PR + \alpha PN_2 - d_1P\\ \frac{dN_1}{dt} &= \mu N_2 + \rho N_1R + \gamma N_1N_2 - d_2N_1\\ \frac{dN_2}{dt} &= a_2N_1 - \mu N_2 - bN_2P - cN_1N_2\\ \frac{dR}{dt} &= R\left(a_3 - \frac{R}{K}\right) - m_1RP - m_2RN_1\end{aligned} \tag{1}$$

The biomass of the predator species is denoted by $P = P(t)$. Here, the prey species is divided into two classes: adult and juvenile, with their biomasses at time, t, denoted by $N_1 = N_1(t)$ and $N_2 = N_2(t)$ respectively. The basal resource defined at time, t is denoted by $R = R(t)$.

A few assumptions have been made to derive the system in Equation (1), namely, no immigration and emigration occur in the habitat. Also, the birth rate of both IG predator and IG juvenile prey are directly proportional to the consumption rate of basal resource, denoted by a_1 and a_2, respectively. The basal resource growth is indicated by a_3, and it grows logistically with carrying capacity, K. The parameters d_1, d_2 and d_3 are defined as the death rate of IG predator, IG adult prey, and IG juvenile prey, respectively. The effect of consumption rate of basal resource of IG predator and IG adult prey are denoted by m_1 and m_2. The IG juvenile prey matures into IG adult prey on growth rate indicated by μ with both positive interactions among IG adult prey population and negative interaction within IG juvenile prey population. The benefit of predation on IG juvenile prey by IG predator is represented by α, while the effect of predation on IG juvenile prey is denoted by b. The benefit of basal resource consumption by IG adult prey is denoted by ρ, and it is assumed that the IG juvenile prey population will grow when the IG adult prey gives birth. The benefit of cannibalism to IG adult prey is presented by γ, whereas the effect of cannibalism on IG juvenile prey is denoted by c.

In order to investigate the dynamical system behaviour of Equation (1), we have first non-dimensionalised the variable and then introduced new simplified parameters to better consider the effect of selected parameter on Equation (1).

For mathematical convenience, Equation (1) has been rescaled by letting

$$P = \frac{a_3 \overline{P}}{m_1}, N_1 = \frac{a_3 \overline{N_1}}{m_2}, N_2 = \frac{a_2 \overline{N_2}}{m_2}, R = \frac{a_3 \overline{R}}{\rho}, t = \frac{\bar{t}}{a_3} \tag{2}$$

and denoting

$$\begin{gathered} \psi = \frac{a_1}{\rho}, \quad \omega = \frac{\alpha a_2}{m_2 a_3}, \quad \tau = \frac{d_1}{a_3}, \quad \theta = \frac{\mu a_2}{a_3^2}, \quad \sigma = \frac{\gamma a_2}{m_2 a_3}, \quad \varepsilon = \frac{d_2}{a_3}, \quad \varphi = \frac{\mu}{a_3}, \\ \beta = \frac{b}{m_1}, \quad \delta = \frac{c}{m_2}, \quad \sigma = \frac{\gamma a_2}{m_2 a_3}, \quad \eta = \frac{1}{K\rho} \end{gathered} \tag{3}$$

Then, Equation (1) can be simplified as:

$$\begin{aligned} \frac{dP}{dt} &= \psi PR + \omega PN_2 - \tau P, \\ \frac{dN_1}{dt} &= \theta N_2 + N_1 R + \sigma N_1 N_2 - \varepsilon N_1, \\ \frac{dN_2}{dt} &= N_1 - \varphi N_2 - \beta PN_2 - \delta N_1 N_2, \\ \frac{dR}{dt} &= (1 - \eta) R - RP - RN_1 \end{aligned} \tag{4}$$

The Equation (4) can then be solved simultaneously by letting

$$\frac{dP}{dt} = \frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dR}{dt} = 0$$

to obtain the equilibrium points, and the results yield six equilibrium points that are biologically relevant. The biological relevance is determined by verifying that all equilibrium points are non-negative real numbers, satisfying $x \in \mathbb{R} \mid x \geq 0$. The equilibrium points are as follows:

(i) $E1\,(0, 0, 0, 0)$, the trivial solution,

(ii) $E2\left(1 - \eta, 0, 0, \frac{\tau}{\psi}\right)$, where only IG predator and basal resource survive,

(iii) $E3\left(0, \frac{\theta - \varphi\varepsilon}{\delta\varepsilon - \sigma}, \frac{\theta - \varphi\varepsilon}{\delta\theta - \sigma\varphi}, 0\right)$, only adult and juvenile IG prey coexist,

(iv) $E4\left(0, 1 - \eta, \frac{1 - \eta}{\delta(1 - \eta) + \varphi}, \frac{\varphi\varepsilon - \theta + (\delta\varepsilon - \sigma)(1 - \eta)}{\delta(1 - \eta) + \varphi}\right)$, denotes that all species exist except IG predator,

(v) $E5\left(\frac{\varphi(\omega\varepsilon - \sigma\tau) - \theta(\delta\tau + \omega)}{\beta(\omega\varepsilon - \sigma\tau)}, \frac{\tau\theta}{\omega\varepsilon - \sigma\tau}, \frac{\tau}{\omega}, 0\right)$, with all species, except basal resource exist and

(vi) $E6\left(P^*, N_1^*, N_2^*, R^*\right)$, the coexistence equilibrium point, where

$$P^* = 1 - \eta - N_1^*,$$

$$R^* = \tau\theta + (\sigma\tau - \omega)N_1^*,$$

$$N_2^* = \frac{N_1^*(\psi\varepsilon - \tau)}{\psi\theta + (\psi\sigma - \omega)N_1^*} \text{ and}$$

$$[(\psi\theta - \omega + (\tau - \psi\varepsilon)(\delta - \beta))N_1^* + (\psi\theta + (\tau - \psi\varepsilon)(\varphi + \beta - \beta\eta))]N_1^* = 0.$$

3 Existence and Stability Analysis of Equilibrium Points

In this section, we reconsider the system presented in Equation (4) and the existence conditions and local stability are analysed systematically. In order to define the stability of the equilibrium point, eigenvalue method and Routh-Hurwitz criterion method have been used, with consideration on the complexity of the eigenvalue. The existence of stable limit cycles is demonstrated using a numerical simulation approach.

The Jacobian matrix for the Equation (4) is defined as:

$$J = \begin{bmatrix} \omega N_2 + \psi R - \tau & 0 & \omega P & \psi P \\ 0 & \sigma N_2 + R - \varepsilon & \sigma N_1 + \theta & N_1 \\ -\beta N_2 & -\delta N_2 + 1 & -\delta N_1 - \beta P - \varphi & 0 \\ -R & -R & 0 & 1 - \eta - P - N_1 \end{bmatrix}$$

3.1 Equilibrium 1: $E1\,(0,0,0,0)$

The Jacobian matrix of Equation (4) at $E1\,(0,0,0,0)$ is:

$$J_{E1} = \begin{bmatrix} -\tau & 0 & 0 & 0 \\ 1 & -\varepsilon & \theta & 0 \\ 0 & 0 & -\varphi & 0 \\ 0 & 0 & 0 & 1 - \eta \end{bmatrix}$$

and further computation yields eigenvalues

$$\lambda_1 = -\tau, \lambda_2 = 1 - \eta, \lambda_{3,4} = -\frac{(\varepsilon + \varphi)}{2} \pm \frac{\sqrt{(\varepsilon - \varphi)^2 + 4\theta}}{2}.$$

Note that λ_1 is negative since τ is a positive real number. Therefore, $E1\,(0,0,0,0)$ is locally asymptotically stable if and only if $\eta > 1$ and $\theta < \varepsilon\varphi$.

Table 2 and Table 3 in Section 4 describe the assigned values for each parameter and the computed numerical simulation using Maple tool.

3.2 Equilibrium 2: $E2\left(1-\eta, 0, 0, \frac{\tau}{\psi}\right)$

Note that $E2$ exists if and only if $\eta < 1$.

The Jacobian matrix of Equation (4) at $E2\left(1-\eta, 0, 0, \frac{\tau}{\psi}\right)$ is:

$$J_{E2} = \begin{bmatrix} 0 & 0 & \omega(1-\eta) & \psi(1-\eta) \\ 0 & \frac{\tau}{\psi} - \varepsilon & \theta & 0 \\ 0 & 1 & -\beta(1-\eta) - \varphi & 0 \\ -\frac{\tau}{\psi} & -\frac{\tau}{\psi} & 0 & 0 \end{bmatrix}.$$

Therefore, the eigenvalues at $E2$ are defined as $\lambda_{1,2} = \pm\sqrt{\tau(\eta-1)}$, and

$$\lambda_{3,4} = \pm\frac{[\beta(\eta-1)-(\varphi+\varepsilon)]\,\psi+\tau}{2\psi} + \frac{\sqrt{[\beta(\eta-1)-(\varphi-\varepsilon)]\,\psi-\tau]^2+4\psi^2\theta}}{2\psi}.$$

Further analysis into the eigenvalues shows that the $E2\left(1-\eta, 0, 0, \frac{\tau}{\psi}\right)$ is unstable as not all real part of the eigenvalues is negative.

Proof

(i) $\lambda_{1,2} = \pm\sqrt{\tau(\eta-1)}$.

Both eigenvalues of λ_1 and λ_2 provide a pair of complex conjugates with zero real part, considering the existence condition of $\eta < 1$ is satisfied.

(ii) $\lambda_3 = \dfrac{[\beta(\eta-1)-(\varphi+\varepsilon)]\,\psi+\tau}{2\psi} + \dfrac{\sqrt{[\beta(\eta-1)-(\varphi-\varepsilon)]\,\psi-\tau]^2+4\psi^2\theta}}{2\psi}$.

As for λ_3, this will yield a negative real part when the following condition is satisfied:

$$\frac{[\beta(\eta-1)-(\varphi+\varepsilon)]\,\psi+\tau}{2\psi} < -\frac{\sqrt{[\beta(\eta-1)-(\varphi-\varepsilon)]\,\psi-\tau]^2+4\psi^2\theta}}{2\psi}$$

(iii) As for λ_4, this will yield a negative real part when the following condition is satisfied:

$$\frac{[\beta(\eta-1)-(\varphi+\varepsilon)]\,\psi+\tau}{2\psi} > \frac{\sqrt{[\beta(\eta-1)-(\varphi-\varepsilon)]\,\psi-\tau]^2+4\psi^2\theta}}{2\psi}$$

Notice that the conditions in (ii) and (iii) contradict each other. Hence, all eigenvalues do not have a negative real part. Thus, $E2$ is proven to be unstable. □

3.3 Equilibrium 3: $E3\,(0, N_{13}, N_{23}, 0)$

$E3\,(0, N_{13}, N_{23}, 0)$ indicates that only IG adult prey and juvenile prey exist with

$$N_{13} = \frac{\theta - \varphi\varepsilon}{\delta\varepsilon - \sigma},$$
$$N_{23} = \frac{\theta - \varphi\varepsilon}{\delta\theta - \sigma\varphi}.$$

Note that E_3 exists if either

$$\frac{\sigma}{\delta} < \varepsilon < \frac{\theta}{\varphi} \text{ or } \frac{\theta}{\varphi} < \varepsilon < \frac{\sigma}{\delta}.$$

The Jacobian matrix evaluated at $E3\,(0, N_{13}, N_{23}, 0)$ is indicated by J_{E3}.

$$J_{E3} = \begin{bmatrix} \omega N_{23} - \tau & 0 & 0 & 0 \\ 0 & \sigma N_{23} - \varepsilon & \sigma N_{13} + \theta & N_{13} \\ -\beta N_{23} & -\delta N_{23} + 1 & -\delta N_{13} - \varphi & 0 \\ 0 & 0 & 0 & 1 - \eta - N_{13} \end{bmatrix}$$

Due to the complexity of the eigenvalues, Routh–Hurwitz criterion will be used. The characteristic polynomial is defined as:

$$P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4,$$

where

$$\begin{aligned}
a_1 &= -\omega N_{23} + \tau - \sigma N_{23} + \varepsilon + -\delta N_{13} + \varphi - 1 + \eta + N_{13}. \\
a_2 &= (\omega N_{23} - \tau)[1 - \eta - N_{13} + \sigma N_{23} - \varepsilon - \delta N_{13} - \varphi] + (\sigma N_{23} - \varepsilon) \\
&\quad [1 - \eta - N_{13} - \delta N_{13} - \varphi] - (\sigma N_{13} + \theta)(1 - \delta N_{23}) \\
&\quad + (-\delta N_{13} - \varphi)(1 - \eta - N_{13}). \\
a_3 &= (\omega N_{23} - \tau)[(1 - \eta - N_{13})(-\sigma N_{23} - \varepsilon + \delta N_{13} + \varphi) - (\sigma N_{23} - \varepsilon)(-\delta N_{13} - \varphi) \\
&\quad + (-\delta N_{23} + 1)(\sigma N_{13} + \theta)] - (1 - \eta - N_{13})[(\sigma N_{23} - \varepsilon)(-\delta N_{13} - \varphi) \\
&\quad - (\sigma N_{13} + \theta)(-\delta N_{23} + 1)]. \\
a_4 &= (\omega N_{23} - \tau)(1 - \eta - N_{13})[(\sigma N_{23} - \varepsilon)(-\delta N_{13} - \varphi) - (\sigma N_{13} + \theta)(-\delta N_{23} + 1)].
\end{aligned}$$

For numerical simulation, we have used the following corresponding parameter values:

$$\eta = 2,\ \varepsilon = 4,\ \varphi = 2,\ \omega = 0.5,\ \theta = 9,\ \sigma = 2,\ \beta = 0.5,\ \delta = 2,\ \tau = 0.1,\ \psi = 0.1.$$

Now, the Routh–Hurwitz criterion of $a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$, implies that $E3$ is locally stable.

Table 3 in Section 4 shows the obtained result of population density, the eigenvalues, and the Routh–Hurwitz criterion computed through the Maple tool, with the above assigned parameter values to support the above stability conditions.

3.4 Equilibrium 4: $E4\,(0, N_{14}, N_{24}, R_4)$

Equilibrium $E4\,(0, N_{14}, N_{24}, R_4)$ represents that IG adult prey, IG juvenile prey, and resource exist, but IG predator is absent, with

$$\begin{aligned}
N_{14} &= 1-\eta,\\
N_{24} &= \frac{1-\eta}{\delta\,(1-\eta)+\varphi},\\
R_4 &= \frac{\varphi\varepsilon-\theta+(\delta\varepsilon-\sigma)\,(1-\eta)}{\delta\,(1-\eta)+\varphi}.
\end{aligned}$$

$E4$ exists if it satisfies both conditions $\eta < 1$ and $(1-\eta)\,(\delta\varepsilon-\sigma) > \theta-\varphi\varepsilon$.

The Jacobian matrix evaluated at $E4$ is defined as:

$$J_{E4} = \begin{bmatrix} \omega N_{24}+\psi R_4-\tau & 0 & 0 & 0\\ 0 & \sigma N_{24}+R_4-\varepsilon & \sigma N_{14}+\theta & N_{14}\\ -\beta N_{24} & -\delta N_{24}+1 & -\delta N_{14}-\varphi & 0\\ -R_4 & -R_4 & 0 & 1-\eta-N_{14} \end{bmatrix}$$

From J_{E4}, the characteristic polynomial is derived as:

$$P\,(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4,$$

where

$$\begin{aligned}
a_1 = {} & -\omega N_{24}+\psi R_4+\tau-\sigma N_{24}-R_4+\varepsilon+\delta N_{14}+\varphi-1+\eta+N_{14}.\\
a_2 = {} & (\omega N_{24}+\psi R_4-\tau)\left[\sigma N_{24}+R_4-\varepsilon-\delta N_{14}-\varphi+1-\eta-N_{14}\right]+(\sigma N_{24}+R_4-\varepsilon)\\
& \left[-\delta N_{14}-\varphi+1-\eta-N_{14}\right]-(-\delta N_{24}+1)\,(\sigma N_{14}+\theta)+N_{14}R_4+\\
& (1-\eta-N_{14})(-\delta N_{14}-\varphi).\\
a_3 = {} & (\omega N_{24}+\psi R_4-\tau)\,[(-\delta N_{14}-\varphi)\,(-\sigma N_{24}+R_4+\varepsilon-1+\eta+N_{14})-\\
& (\sigma N_{24}+R_4-\varepsilon)\,(1-\eta-N_{14})+(-\delta N_{24}+1)\,(\sigma N_{14}+\theta)]+(-\delta N_{14}-\varphi)\\
& \left[(-\sigma N_{24}-R_4+\varepsilon)\,(1-\eta-N_{14})-N_{14}R_4\right]+(-\delta N_{24}+1)\,(\sigma N_{14}+\theta)\\
& (1-\eta-N_{14}).\\
a_4 = {} & (\omega N_{24}+\psi R_4-\tau)\,\{[[(\sigma N_{24}+R_4-\varepsilon)\,(-\delta N_{14}-\varphi)-(\sigma N_{14}+\theta)\,(-\delta N_{24}+1)\,]\\
& (1-\eta-N_{14})]-(-R_4)(N_{14})(-\delta N_{14}-\varphi)\}.
\end{aligned}$$

Further to this, we have used the following corresponding parameter values, $\eta = 0.8$, $\varepsilon = 4$, $\varphi = 2$, $\omega = 0.5$, $\theta = 9$, $\sigma = 2$, $\beta = 0.5$, $\delta = 2$, $\tau = 0.1$, $\psi = 0.1$ for numerical simulation using Maple tool. Hence, this shows that the Routh–Hurwitz criterion of $a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$, is satisfied. Thus, $E4$ is locally stable.

Table 3 in Section 4 displays the obtained result of population density, the eigenvalues and the Routh–Hurwitz criterion computed with above assigned parameter values.

3.5 Equilibrium 5: $E5\,(P_5, N_{15}, N_{25}, 0)$

The subsequent equilibrium point is $E5\,(P_5, N_{15}, N_{25}, 0)$, where:

$$\begin{aligned}
P_5 &= \frac{\theta\,(\omega-\delta\tau)-\varphi\,(\omega\varepsilon-\sigma\tau)}{\beta\,(\omega\varepsilon-\sigma\tau)},\\
N_{15} &= \frac{\tau\theta}{\omega\varepsilon-\sigma\tau},\\
N_{25} &= \frac{\tau}{\omega}.
\end{aligned}$$

which suggests that IG predator, IG adult prey and IG juvenile prey exist, while basal resource is absent.

$E5$ exists when both conditions of

$$\frac{\varphi}{\theta} < \frac{\omega-\delta\tau}{(\omega\varepsilon-\sigma\tau)} \text{ and } \frac{\varepsilon}{\sigma} > \frac{\tau}{\omega}$$

are met. The Jacobian matrix evaluated at $E5$ defined as J_{E5}.

$$J_{E5} = \begin{bmatrix} \omega N_{25}-\tau & 0 & \omega P_5 & \psi P_5 \\ 0 & \sigma N_{25}-\varepsilon & \sigma N_{15}+\theta & N_{15} \\ -\beta N_{25} & -\delta N_{25}+1 & -\delta N_{15}-\beta P_5-\varphi & 0 \\ 0 & 0 & 0 & 1-\eta-P_5-N_{15} \end{bmatrix}$$

The characteristic polynomial defined at this equilibrium point is

$$P\,(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$$

where

$$\begin{aligned}
a_1 = {} & -\omega N_{25}+\tau-\sigma N_{25}+\varepsilon+\delta N_{15}+\beta P_5+\varphi-1+\eta+P+N_{15}.\\
a_2 = {} & (\omega N_{25}-\tau)\,[\sigma N_{25}-\varepsilon-\delta N_{15}-\beta P_5-\varphi+1-\eta-P_5-N_{15}]+(\sigma N_{25}-\varepsilon)\\
& [-\delta\beta P_5-\varphi+1-\eta-P_5-N_{15}]+\beta N_{25}\omega P_5+(\sigma N_{15}+\theta)\,(\delta N_{25}+1)+\\
& (-\delta N_{15}-\beta P_5-\varphi)\,(1-\eta-P_5-N_{15}).\\
a_3 = {} & (1-\eta-P_5-N_{15})\,[(\omega N_{25}-\tau)\,(-\sigma N_{25}+\varepsilon+\delta N_{15}+\beta P_5+\varphi)-(\omega P_5\beta N_{25})\\
& -(\sigma N_{25}-\varepsilon)\,(-\delta N_{15}-\beta P_5-\varphi)+(-\delta N_{25}+1)\,(\sigma N_{15}+\theta)]\\
& (\omega N_{25}-\tau)\,[(-\sigma N_{25}+\varepsilon)\,(-\delta N_{15}-\beta P_5-\varphi)+(\sigma N_{15}+\theta)\,(\delta N_{25}+1)]+\\
& (\omega P_5)\,(\sigma N_{25}-\varepsilon)\,(-\beta N_{25}).\\
a_4 = {} & (1-\eta-P_5-N_{15})\,\{(\sigma N_{25}-\varepsilon)\,[(\omega N_{25}-\tau)\,(-\delta N_{15}-\beta P_5-\varphi)-(\omega P_5)\,(-\beta N_{25})]-\\
& (\omega N_{25}-\tau)(\sigma N_{15}+\theta)(-\delta N_{25}+1)\}.
\end{aligned}$$

The equilibrium point, $E5$ is further demonstrated using numerical simulation with parameter values of $\eta = 0.8$, $\varepsilon = 4$, $\varphi = 2$, $\omega = 0.5$, $\theta = 15$, $\sigma = 2$, $\beta = 0.5$, $\delta = 2$, $\tau = 0.1$, $\psi = 0.1$ The computation was carried out using Maple tool, and the results of population density, the eigenvalues, and the Routh–Hurwitz criterion are presented in Table 3, Section 4. Under this parameter set, the equilibrium is asymptotically stable as the Routh–Hurwitz criterion of $a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$ is satisfied.

3.6 Equilibrium 6: $E6\,(P^*, N_1^*, N_2^*, R^*)$

The final equilibrium examined is $E6\left(P^*, N_1^*, N_2^*, R^*\right)$, the co-existence where:

$$P^* = 1 - \eta - N_1^*,$$

$$R^* = \tau\theta + (\sigma\tau - \omega)\, N_1^*,$$

$$N_2^* = \frac{N_1^*\,(\psi\varepsilon - \tau)}{\psi\theta + (\psi\sigma - \omega)N_1^*},$$

$$[(\psi\theta - \omega + (\tau - \psi\varepsilon)(\delta - \beta))N_1^* + (\psi\theta + (\tau - \psi\varepsilon)(\varphi + \beta - \beta\eta))]N_1^* = 0.$$

Therefore, $E6$ exists if and only the condition of

$$\frac{\omega - 2\psi\theta}{\tau - \psi\varepsilon} > \varphi - \beta\eta + \delta$$

is satisfied. Subsequently, the Jacobian matrix evaluated at $E6$ is defined as J_{E6}

$$J_{E6} = \begin{bmatrix} \omega N_2^* + \psi R^* - \tau & 0 & \omega P^* & \psi P^* \\ 0 & \sigma N_2^* + R^* - \varepsilon & \sigma N_1^* + \theta & N_1^* \\ -\beta N_2^* & -\delta N_2^* + 1 & -\delta N_1^* - \beta P^* - \varphi & 0 \\ -R^* & -R^* & 0 & 1 - \eta - P^* - N_1^* \end{bmatrix}$$

From above, the characteristic polynomial is defined by

$$P\,(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4,$$

where

$$\begin{aligned}
a_1 = {} & -\omega N_2^* - \psi R^* + \tau - \sigma N_2^* - R^* + \varepsilon + \delta N_1^* + \beta P^* + \varphi - 1 + \eta + P^* + N_1^*. \\
a_2 = {} & (\omega N_2^* + \psi R^* - \tau)\,(1 - \eta - P^* - N_1^* + \sigma N_2^* + R^* - \varepsilon - \delta N_1^* - \beta P^* - \varphi) + \\
& (\sigma N_2^* + R^* - \varepsilon)\,(1 - \eta - P^* - N_1^* - \delta N_1^* - \beta P^* - \varphi) + \psi P^* R^* + N_1^* R^* + \\
& (-\delta N_1^* - \beta P^* - \varphi)\,(1 - \eta - P^* - N_1^*) + \omega P^* R^* - (\sigma N_1^* + \theta)\,(-\delta N_2^* + 1)\,. \\
a_3 = {} & (\omega N_2^* + \psi R^* - \tau)\,[(\delta N_1^* + \beta P^* + \varphi - 1 + \eta + P^* + N_1^*)\,(\sigma N_2^* + R^* - \varepsilon) - R^* N_1^* - \\
& (-\delta N_1^* - \beta P^* - \varphi)\,(1 - \eta - P^* - N_1^*) + (\sigma N_1^* + \theta)(-\delta N_2^* + 1)] + (\sigma N_2^* + R^* - \varepsilon) \\
& [(\omega P^*)\,(-\beta N_2^*) - \psi P^* R^* - (1 - \eta - P^* - N_1^*)\,(-\delta N_1^* - \beta P^* - \varphi)] + \\
& (-\delta N_1^* - \beta P^* - \varphi)\,[-R^* \psi P^* - R^* N_1^*] + (1 - \eta - P^* - N_1^*)\,[(-\beta N_2^* \omega P^*) + \\
& (\sigma N_1^* + \theta)(-\delta N_2^* +)]\,. \\
a_4 = {} & (\omega N_2^* + \psi R^* - \tau)\,[(\sigma N_2^* + R^* - \varepsilon)\,(-\delta N_1^* - \beta P^* - \varphi)\,(1 - \eta - P^* - N_1^*) - \\
& (\sigma N_1^* + \theta)\,(-\delta N_2^* + 1)\,(1 - \eta - P^* - N_1^*) - (N_1^*)(-\delta N_1^* - \beta P^* - \varphi)(-R^*)] + \omega P^* \\
& [(-\beta N_2^*)\,(\sigma N_2^* + R^* - \varepsilon)\,(-1 + \eta + P^* + N_1^*) - N_1^*[(\beta N_2^*)\,(-R^*) + \\
& (-\delta N_2^* + 1)(\,-\,R^*)]] - \psi P^*[(-R^*)(\sigma N_2^* + R^* - \varepsilon)(-\delta N_1^* - \beta P^* - \varphi) - (\sigma N_1^* + \theta) \\
& [-R^* \beta N_2^* + (-\delta N_2^* + 1)\,(-R^*)]].
\end{aligned}$$

With the approach same as that presented in Section 3.3, we have used the following corresponding parameter values, $\eta = 0.8$, $\varepsilon = 4$, $\varphi = 2$, $\omega = 0.5$, $\theta = 7.5$, $\sigma = 2$, $\beta = 0.5$, $\delta = 2$, $\tau = 0.1$, $\psi = 0.1$ for direct computation of Routh–Hurwitz criterion using Maple tool.

Hence, the results satisfied the intended criteria of $a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$, thus proving that $E6$ is locally stable. This is shown in Table 3.

The conditions of existence and stability for all equilibrium points are summarized in Table 1.

Table 1: Summary of Conditions of Existence and Stability

Equilibrium Point	**Condition for Existence**	**Condition for Stability**
$E1\,(0,0,0,0)$	-	$\eta > 1$ and $\theta < \varepsilon\varphi$
$E3\,(0, N_{13}, N_{23}, 0)$	$\frac{\sigma}{\delta} < \varepsilon < \frac{\theta}{\varphi}$ or $\frac{\theta}{\varphi} < \varepsilon < \frac{\sigma}{\delta}$.	Obtained using Routh Hurwitz criteria.
$E4\,(0, N_{14}, N_{24}, R_4)$	$\eta < 1$ and $(1-\eta)(\delta\varepsilon - \sigma) > \theta - \varphi\varepsilon$	$a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$.
$E5\,(P_5, N_{15}, N_{25}, 0)$	$\frac{\varphi}{\theta} < \frac{\omega - \delta\tau}{(\omega\varepsilon - \sigma\tau)}$ and $\frac{\varepsilon}{\sigma} > \frac{\tau}{\omega}$	
$E6\left(P^*, N_1^*, N_2^*, R^*\right)$	$\frac{\omega - 2\psi\theta}{\tau - \psi\varepsilon} > \varphi - \beta\eta + \delta$	Due to complexity, numerical simulation is used to illustrate the result in Table 3

4 Stability Region of Equilibrium Points

In this section, we will look into the (i) the stability region against two common parameters, in this case, chosen to be η and θ, denoted by the carrying capacity of the basal resource and the maturation rate from IG juvenile prey into IG adult prey respectively, (ii) the population density against time approaching the stable limit cycle. It is derived hypothetically that the higher the maturation rate, IG juvenile prey will be able to escape from the predation and cannibalism, thus leading to higher growth of the prey population, subject to the rate of carrying capacity of the basal resource.

4.1 Stability Region

The stability region is plotted between two selected parameters that have been varied across the equilibrium points, while other parameters have been kept constant. The values of other parameters are chosen randomly, instead of any real published data from past literature. This can illustrate the dynamics of the equilibrium points when two critical parameters increase, which are η, the carrying capacity of the basal resource, and θ, the positive effect of maturation rate on IG adult prey, N_1.

The regions are constructed with θ against η, as such both the existence and the stability conditions of each equilibrium point as previously described in Section 3 are satisfied in Figure 2.

4.2 Population Density Against Time

In order to understand more clearly, Table 2 derives the corresponding parameter values assigned for each equilibrium points of $E1$, $E3$, $E4$, $E5$, $E6$. As previously described in Section 4.1, all parameters are kept constant with the values selected theoretically for numerical simulation purpose solely and varying only η and θ. The values for η and θ are then chosen within the range based on the stability region in Figure 2.

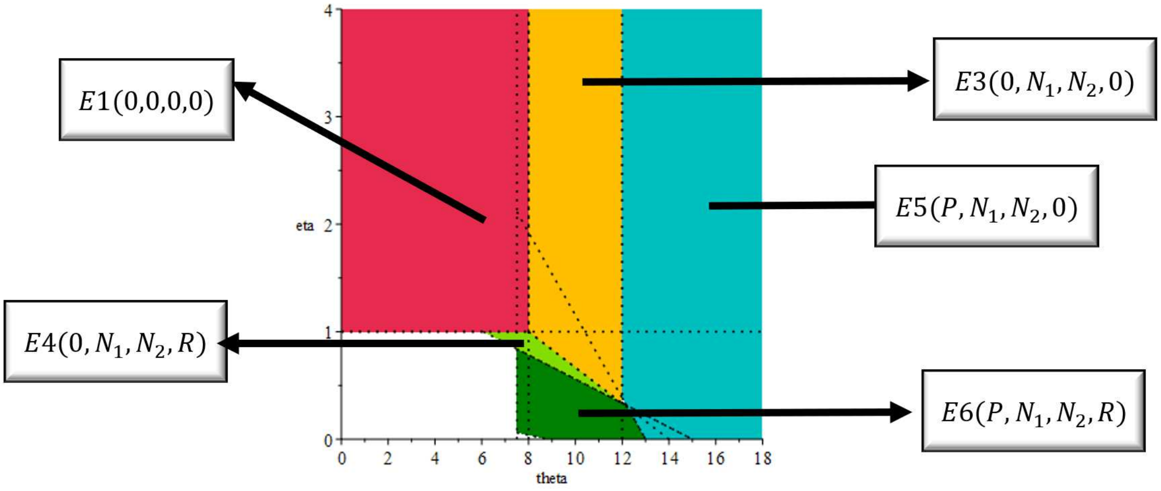


Figure 2: Stability Region of the System, θ against η

Table 2: Parameter Values Assigned for Each Stable Equilibrium Point

Parameter	**Description**	**Values**				
		$E1$	$E3$	$E4$	$E5$	$E6$
η	Carrying capacity of R	2	2	0.8	0.8	0.8
ε	Death rate of N_1	4	4	4	4	4
φ	Effect of maturation on N_2	2	2	2	2	2
ω	Benefit of predation to P	0.5	0.5	0.5	0.5	0.5
θ	Benefit of maturation to N_1	5	11	9	15	7.5
σ	Benefit of cannibalism to N_1	2	2	2	2	2
β	Effect of predation on N_2	0.5	0.5	0.5	0.5	0.5
δ	Effect of cannibalism to N_2	2	2	2	2	2
τ	Death rate of P	0.1	0.1	0.1	0.1	0.1
ψ	Birth rate of P	0.1	0.1	0.1	0.1	0.1

Table 3 computes the eigenvalues and the Routh–Hurwitz criterion using the above parameter values, using Maple software, hence proving that the equilibrium points $E1$, $E3$, $E4$, $E5$ and $E6$ exist and are locally stable. In addition to that, we illustrate that the population of the system approaches the respective equilibrium points by letting $\eta = 2$ and $\theta = 5$ (Figure 3), $\eta = 2$ and $\theta = 11$ (Figure 4), $\eta = 0.8$ and $\theta = 9$ (Figure 5), $\eta = 0.8$ and $\theta = 15$ (Figure 6), $\eta = 0.8$ and $\theta = 7.5$ (Figure 7). Figure 8 displays the growth of population of adult prey, N_1, with combination of $E1$, $E3$, $E4$, $E5$ and $E6$ in a single plot. The significance of this combination plot will be further discussed in Section 5.

Table 3: Stability Conditions of Each Stable Equilibrium Point

Numerical Simulation	**Eigenvalues**	**Routh Hurwitz Criteria**
$E1(0,0,0,0)$	$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} -0.1000 \\ -5.4490 \\ -0.5505 \\ -1.0000 \end{bmatrix}$	$a_1 = 8.1833$ $a_3 = 4.7167$ $a_4 = 0.0750$ $a_1a_2a_3 - a_1^2a_4 - a_3^2 = 479.7589$
$E3\,(0, 0.50, 0.17, 0)$	$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} -0.0167 \\ -0.4853 \\ -6.1813 \\ -1.5000 \end{bmatrix}$	$a_1 = 8.1833$ $a_3 = 4.7167$ $a_4 = 0.0750$ $a_1a_2a_3 - a_1^2a_4 - a_3^2 = 479.7589$
$E4\,(0, 0.20, 0.08, 0.08)$	$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} -5.9523 \\ -0.0436 \\ -0.0500 \\ -0.1541 \end{bmatrix}$	$a_1 = 6.2000$ $a_3 = 0.0992$ $a_4 = 0.0020$ $a_1a_2a_3 - a_1^2a_4 - a_3^2 = 0.8299$
$E5\,(1, 0.83, 0.20, 0)$	$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} -7.0544 \\ -0.0378 \\ -0.6744 \\ -1.6333 \end{bmatrix}$	$a_1 = 9.4000$ $a_3 = 8.4283$ $a_4 = 0.2940$ $a_1a_2a_3 - a_1^2a_4 - a_3^2 = 1308.10$
$E6\,(0.040, 0.16, 0.07, 0.66)$	$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} -5.3951 \\ -0.0688 + 0.2055i \\ -0.0688 - 0.2055i \\ -0.0125 \end{bmatrix}$	$a_1 = 5.5451$ $a_3 = 0.2632$ $a_4 = 0.0032$ $a_1a_2a_3 - a_1^2a_4 - a_3^2 = 1.0858$

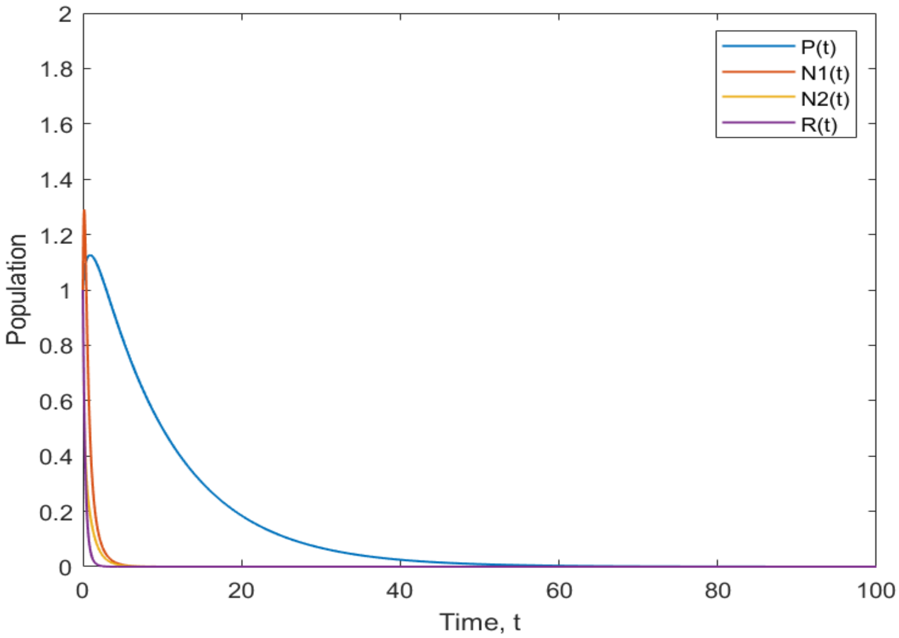


Figure 3: System Approaches $E1\,(0,0,0,0)$ with Initial Values $(1,1,1,1)$

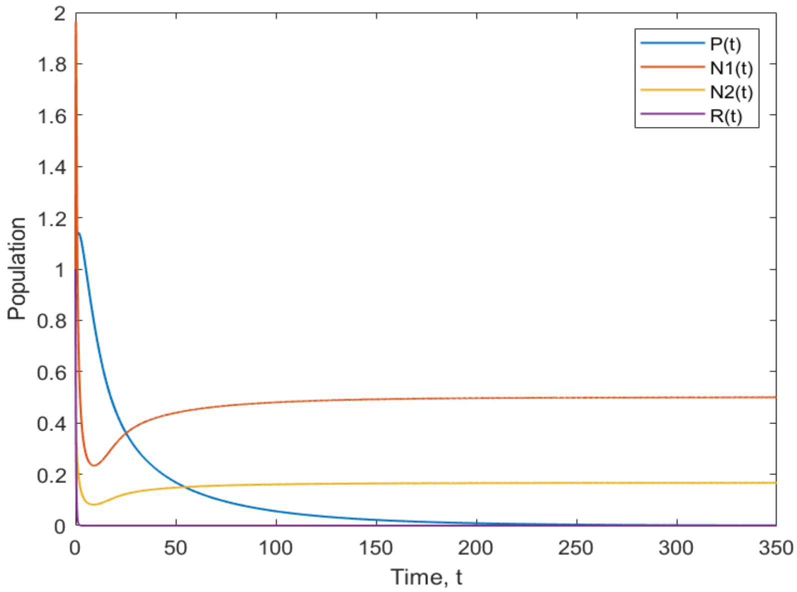


Figure 4: System Approaches $E3\,(0, 0.50, 0.17, 0)$ with Initial Values $(1, 1, 1, 1)$

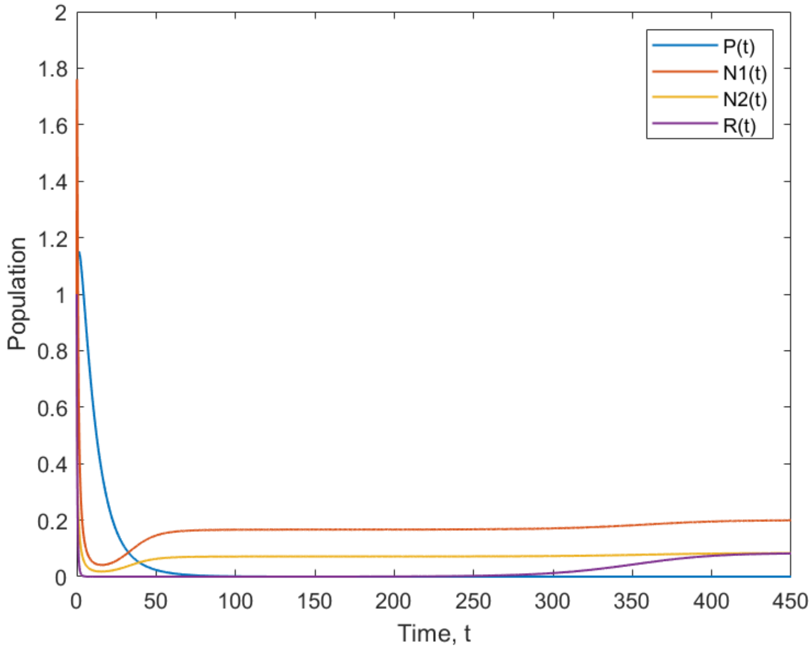


Figure 5: System Approaches $E4\,(0, 0.20, 0.08, 0.08)$ with Initial Values $(1, 1, 1, 1)$

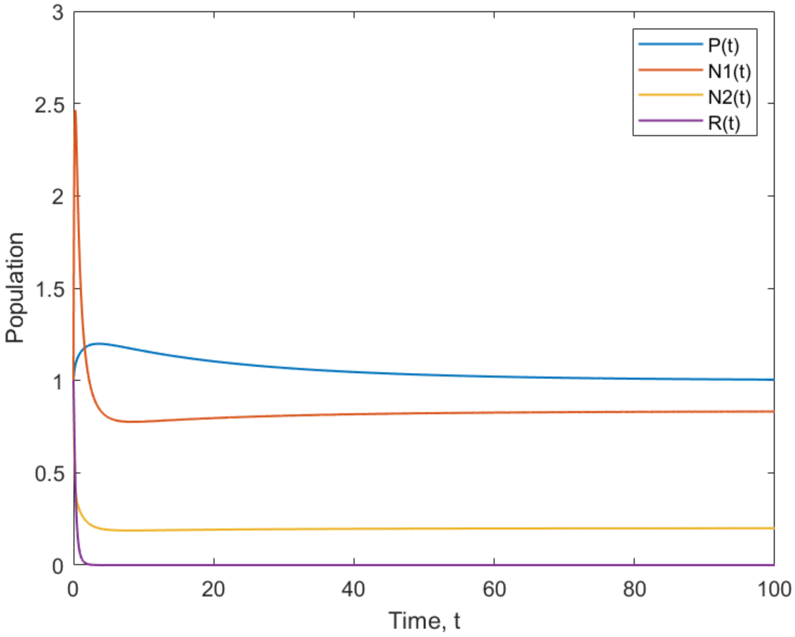


Figure 6: System Approaches $E5\,(1, 0.83, 0.20, 0)$ with Initial Values $(1, 1, 1, 1)$

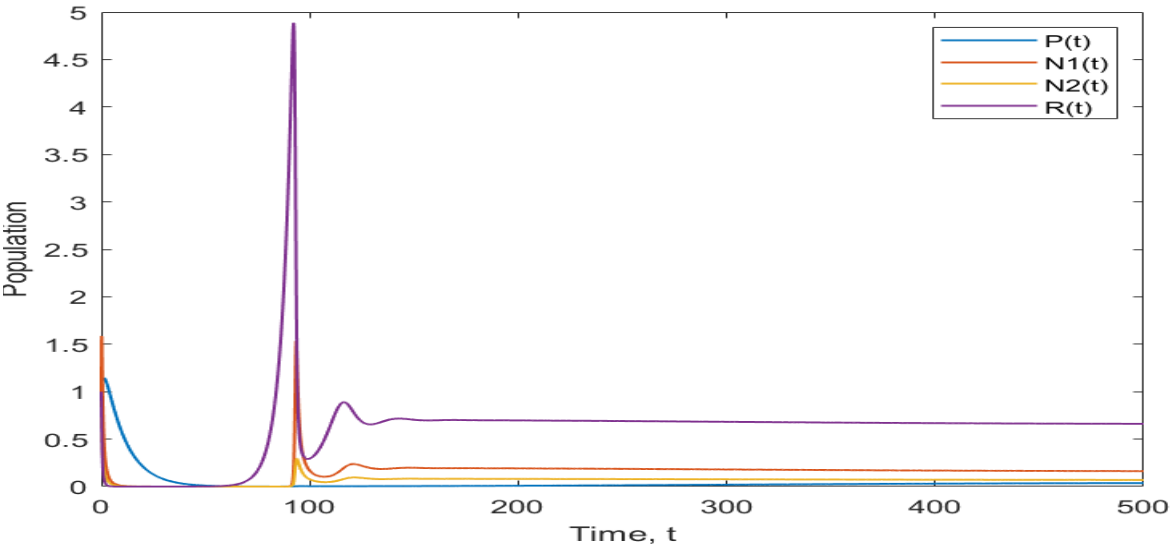


Figure 7: System Approaches $E6\,(0.040, 0.16, 0.07, 0.66)$ with Initial Values $(1, 1, 1, 1)$

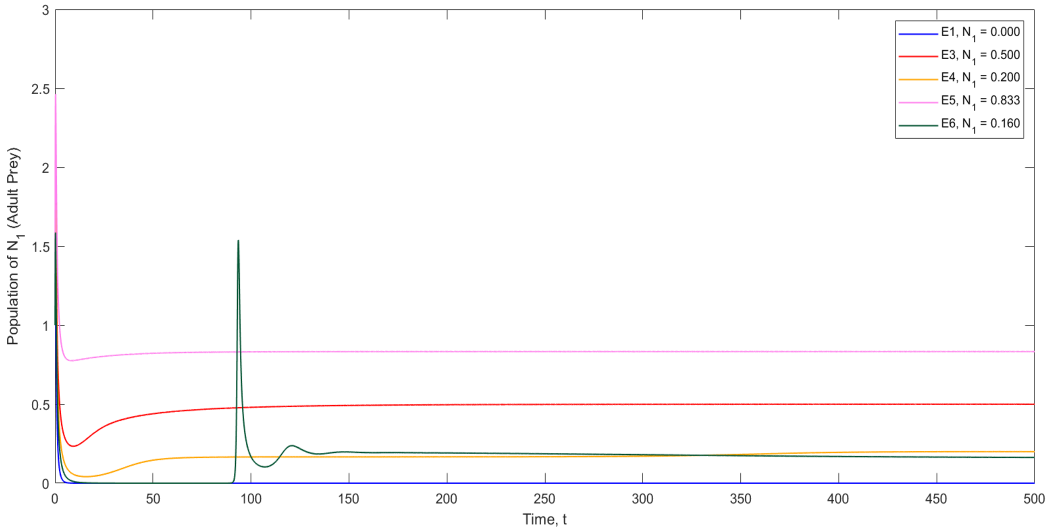


Figure 8: System Approaches Multiple Steady State for Population of N_1 over Time, t

5 Bifurcation Analysis

In general, bifurcation theory refers to the study of qualitative changes to the state of the system when the parameter varies. There are two types of bifurcation theories, namely, local bifurcation and global bifurcation. The former can be explained in terms of changes to the stability of the simple steady states, while the latter often causes catastrophic changes to the attractor of the changes. Classic example of local bifurcation is the Hopf bifurcation, which leads to the onset of oscillation and the saddle-node bifurcation, where a stable steady state is either created or destroyed, which often lead to bi-stability [15].

This section delves into bifurcation analysis of the system in Equation (4), analyzing further the effect of changing a single parameter on the system's dynamics. XPPAUT, a numerical tool for simulating, animating, and analyzing dynamical system, has been widely used for studying of ODEs, specifically in analyzing the effect of changing the selected parameter on the stability of the equilibrium of the system.

Using the obtained stability region from Figure 2 as framework, we have explored the effect of bifurcating one parameter, namely, θ, which, as stated previously, denotes the positive effect of maturation rate in IG adult prey. This gives us two different perspectives on the impact of varying a selected parameter, while keeping other parameters intact changes the stability of the concerned population N_1 – the population of IG adult prey as follows:

(i) Region 1: Parameter θ against the population, N_1, with η set constant at $\eta = 2$

(ii) Region 2: Parameter θ against the population, N_1, with η set constant at $\eta = 0.6$

Following that, we have studied the effect of bifurcating another critical parameter based on Figure 2, which is η defined by the carrying capacity of the basal resource.

(iii) Region 3: Parameter η against the population, θ, with θ set constant at $\theta = 8.5$

Figure 9 illustrates the focused area within the stability region used to perform both horizontal and vertical bifurcation analyses.

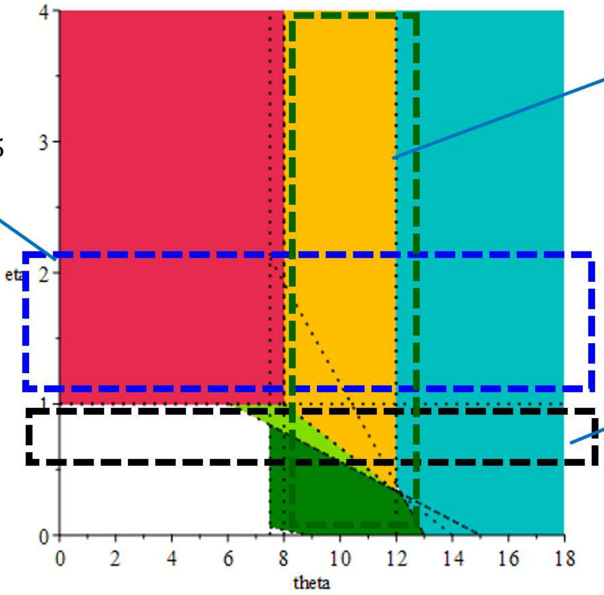
We have derived an initial hypothesis based on Figure 8, which could further explain the growth of N_1 population over time until it reaches the steady state when regions specified in Figure 9 are taken into consideration.

(i) Region 1: Population size of N_1 increases from $N_1 = 0$ and grows more to reach $N_1 = 0.500$. The population is larger until reaches $N_1 = 0.833$.

(ii) Region 2: Population size of N_1 increases from $N_1 = 0.160$ and grows slightly to reach $N_1 = 0.200$. The population grows further until reaches $N_1 = 0.500$ and reaches the larger population size at $N_1 = 0.833$.

(iii) Region 3: Population size of N_1 increases from $N_1 \sim 0.150$ and grows slightly to reach $N_1 \sim 0.280$. The population then declines until it reaches $N_1 \sim 0.080$ and reaches a constant population size at $N_1 \sim 0.080$.

However, it is important to note that the population size during plotting bifurcation diagrams might differ from the steady state value in Figure 8 due to steady state of the population are explained through the population growth against time, t, while bifurcation focuses on the population size when a specific parameter varies, where time was not part of the consideration. The objective of this hypothesis is to provide an overview of the expectation of the bifurcation analysis. In this model, stability switching phenomenon can be observed when each equilibrium point switches over with maturation rate increased gradually, depicting the transcritical bifurcation, as described similarly in a spatially coupled models of predator–prey by Barman and Ghosh [16].



(a) Region 1:
Vary θ, at η = 1.5

(b) Region 3:
Vary η, at θ = 8.5

(b) Region 2:
Vary θ, at η = 0.6

Figure 9: Focused Area within Stability Region to Three Different Bifurcation Diagrams

5.1 Transcritical Bifurcation within Region 1

Figure 10 shows that when varying θ with η set at 2, the stability of the population N_1 shifts from E1→E3→E5, which indicates that the population of IG adult prey, N_1 increases when θ increases.

(i) $E1$, the trivial solution, is stable when $0 < \theta < 8$, which denotes that N_1 does not exist in the ecosystem;

(ii) $E3$, where both IG adult and juvenile prey exist, is stable when $8 < \theta < 12$ which denotes that N_1 grows slightly linear;

(iii) $E5$, which IG predator, IG adult, and juvenile prey exist, is stable when $\theta > 12$, which denotes that N_1 grows significantly linear.

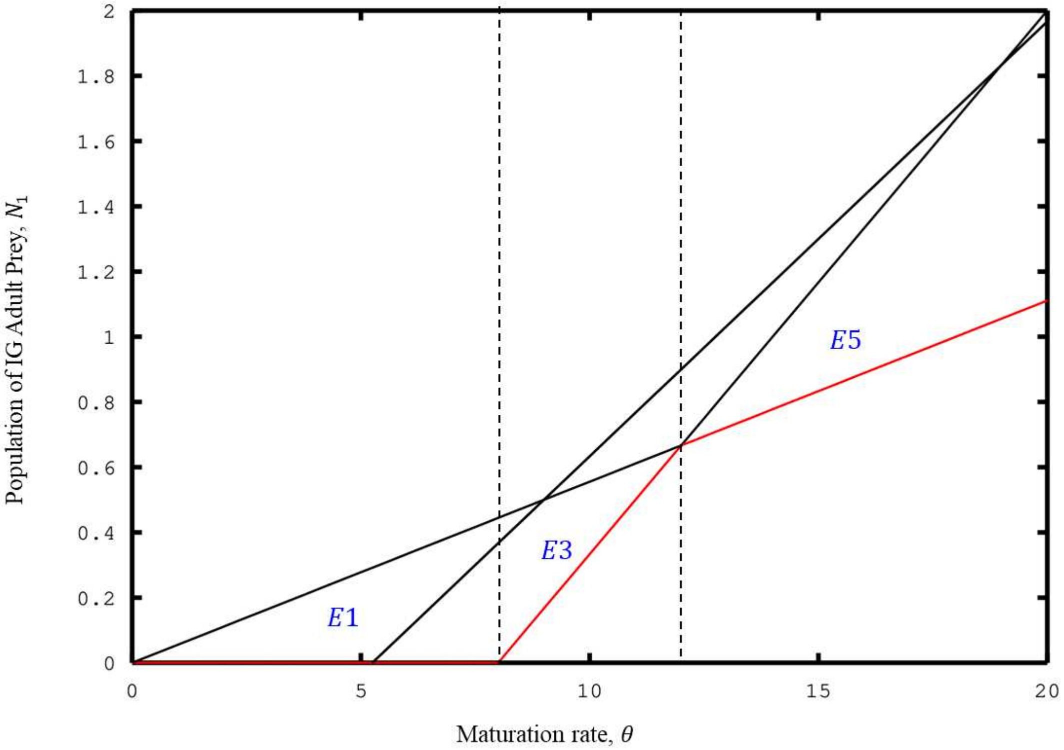


Figure 10: Parameter θ against the Population of IG Adult Prey, N_1 with Carrying Capacity Set Constant at η= 2

5.2 Transcritical Bifurcation within Region 2

Figure 11 shows that when varying θ with η set at 0.6 instead, the stability of the population N_1 shifts from $E6 \rightarrow E4 \rightarrow E3 \rightarrow E5$, indicating that the population of IG adult prey, N_1, increases when θ increases.

(i) $E6$, the coexistence point, is stable when $7.03 < \theta < 9.6$, denoting that N_1 displays a linear growth;

(ii) $E4$, where both IG adult, juvenile prey, and resource exist, is stable when $9.6 < \theta < 10.4$ denoting that N_1 remains constant at $N_1 = 0.40$;

(iii) $E3$, where only both IG adult and juvenile prey exist, is stable and grows significantly when $10.4 < \theta < 12$;

(iv) $E5$, where IG predator, IG adult, and juvenile prey exist, is stable when $\theta > 12$, which denotes that N_1 grows significantly.

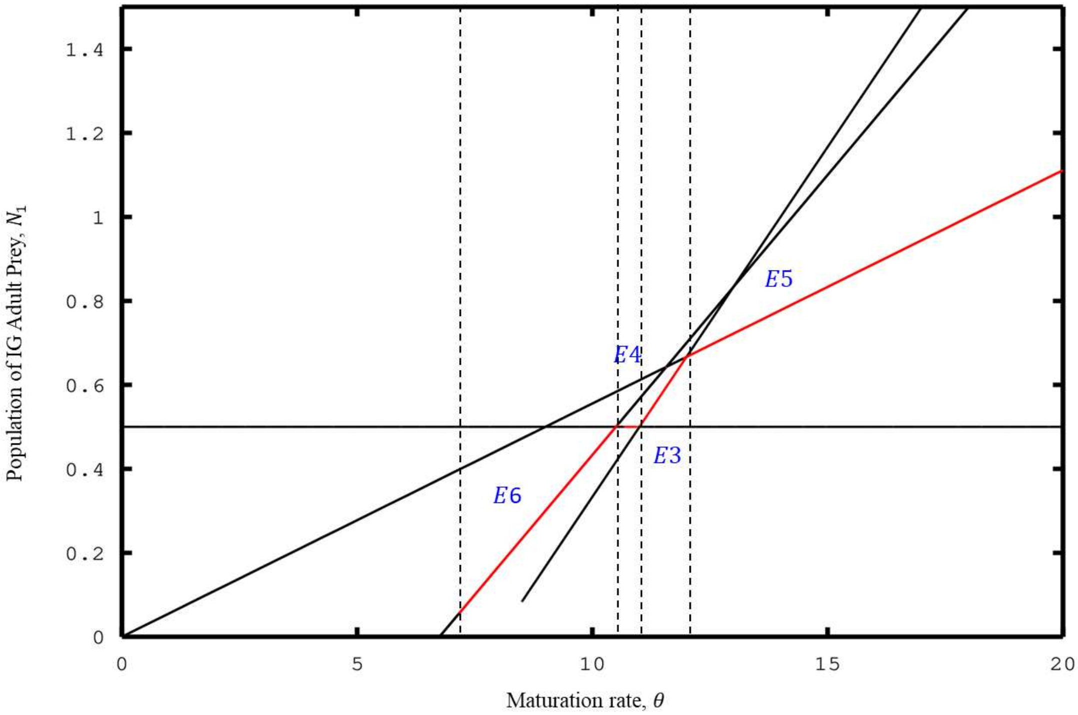


Figure 11: Parameter θ against the Population of IG Adult Prey, N_1 with Carrying Capacity Set constant at η= 0.6

5.3 Transcritical Bifurcation within Region 3

Figure 12 shows that when varying η with θ set at 8.5 instead, the stability of the population N_1 shifts from $E6 \rightarrow E4 \rightarrow E3$, which indicates that the population of IG adult prey, N_1, increases when η increases.

(i) $E6$, the coexistence point, is stable with the population of N_1 displaying slight growth until reaches $N_1 \sim 0.08$;

(ii) $E4$, where IG adult, juvenile prey and resource exist, is stable, however, we could see that the population of N_1 continues to reduce until reaches $N_1 \sim 0.08$;

(iii) $E3$, where only both IG adult and juvenile prey exist, is stable, and N_1 grows linearly at $N_1 \sim 0.08$.

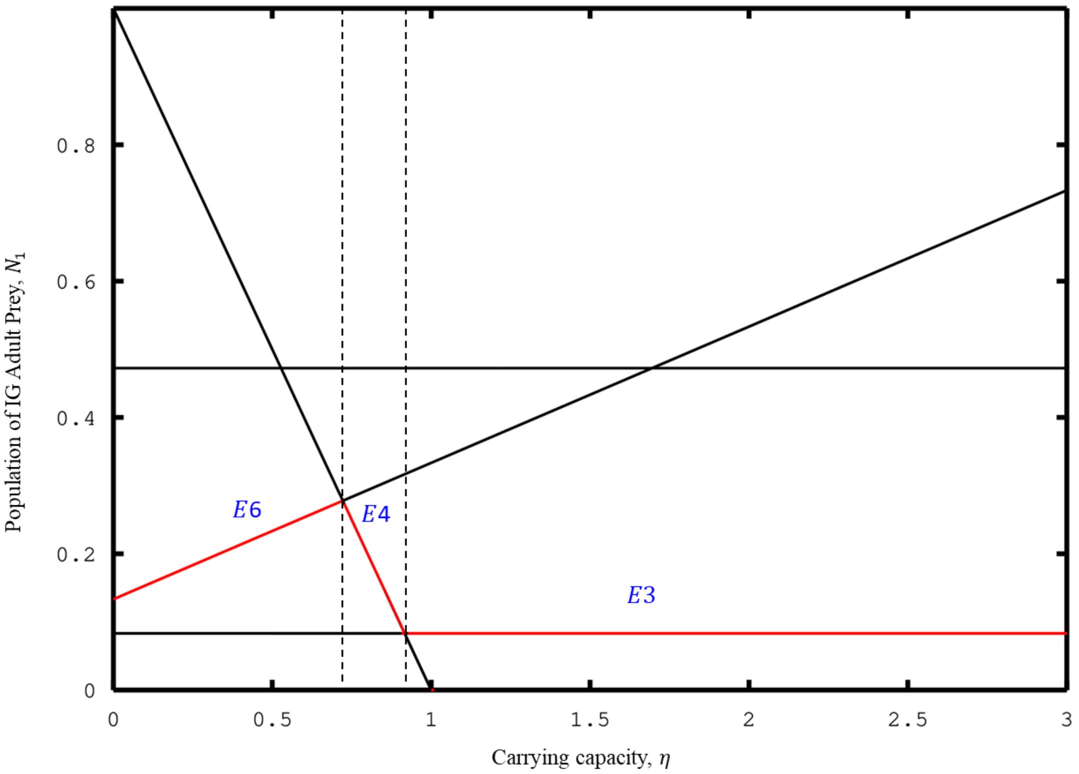


Figure 12: Parameter η against the Population of IG Adult Prey, N_1, with Maturation Rate Set Constant at θ= 8.5

6 Conclusion and Discussion

In this paper, we have considered an IGP model with stage structure in prey population, namely, juvenile prey and adult prey, which exhibit cannibalism interactions. We have assumed that the adult prey compete with predator for basal resources and give birth to juvenile prey. In this case, we have concentrated on the benefits of maturation rate from juvenile to adult prey population with hypotheses that (i) higher maturation rate leads to lower predation rate as predator only feeds on juvenile prey, (ii) higher maturation rate can encourage optimal competition rate between predator and adult prey. Stability region and bifurcation analysis performed have shown the dynamics of the model, which can lead to physical interpretation of the model.

The conclusions of this study are enumerated as follows:

(i) A higher maturation rate leads to an increase in prey population, whereby the population density increases, thus enabling the prey to escape predation and sustaining the ecosystem;

(ii) A higher maturation rate leads to a significant increase in competition rate between predator and adult prey to compete for basal resources, which is important for maintaining an optimal ecosystem;

(iii) The negative impact of a higher maturation rate is due to an increased level of competition, the basal resource reaches the carrying capacity faster and leads to the extinction of the resources;

(iv) The maturation rate is expected to be at a medium level to enable a 'coexist' ecosystem that could guarantee a healthy environmental interaction.

Providing an example of real-life species, dragonflies belong to the intermediate predator group, which also means they can prey on something and can be preyed on. More to that, dragonflies are known to exhibit IGP interaction, where it competes with other species to feed on smaller insects as a basal resource and cannibalise their young as well. The model described in this paper could be applied in ecosystem control of species, especially more in sustaining environments with species exhibiting IGP such as wolf spiders, and cannibalism such as dragon flies.

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