

Predicting Top Five Cryptocurrency Prices via Linear Structural Time Series (STS) Approach

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Abstract Predicting cryptocurrency prices are difficult due to dynamic data. At the same time, the hidden market behavior of trend and seasonal components in the history data is also critical as it provides an idea of what the price pattern will be in the future. Hence, this research proposes to identify and model the hidden pattern behavior in terms of component time series instead of removing it via the linear structural time series (STS) model approach. This study focuses on the top five cryptocurrencies relying on the highest market capitalization. From the results obtained, the top five cryptocurrencies have a different trend model, either deterministic or stochastic, which relies on the behavior of data. The five cryptocurrencies also show the crypto winter event, where the trend is downward after six months every year. The linear STS is the best model for predicting three cryptocurrencies' prices for nonstationary and volatility data behavior. It can also handle the hidden component behavior and is easy to interpret. Since the linear STS model can indirectly retain the information of data, it will assist investors and traders in accurately predicting cryptocurrency prices.

Keywords Cryptocurrency; Trend; Behavior; Nonstationary; Price prediction; Structural Time Series Optimisation

Mathematics Subject Classification 37M10, 62M10, 62R07

1 Introduction

Cryptocurrencies are decentralized virtual currency and new financial tools with innovative blockchain technology-based features as a medium of exchange that can transfer digitally and directly to a peer-to-peer network instead of a third party. It is not connected to any commodity or business,

and governments do not hold any regulatory authority over them [1]. Due to its fast data transfer and secure transactions, blockchain technology and the foundation of cryptocurrencies have captivated a vast interest and trust [2]. Other than electronically transferring funds, it was used for a variety of purposes, such as investment assets, an online transaction system, and the purchase of regular goods and services [3,4]. Since blockchain and cryptocurrency technologies are relatively novel, more research is needed to analyze cryptocurrency prices, including patterns and behavior [5], to obtain an accurate prediction. However, cryptocurrencies' price behavior is still predominantly unexplored, indirectly giving researchers new opportunities to highlight and compare the similarity and differences in the behavior of each cryptocurrency price. Even with the number of studies on the behavior and prediction of cryptocurrencies' price increases, the field is still limited and needs to be explored further.

Basically, the pattern or behavior of cryptocurrencies price is time-varying volatility [6], and it is hard to predict accurately. As a result, numerous studies have been undertaken in order to determine the best model for successful prediction. The techniques such as linear regression [7–9], logistic regression [10,11], Bayesian regression [12,13], artificial neural network [14–16], and deep learning [17] are used in this field of research. However, most prior literature focuses on and discusses the behavior of Bitcoin (BTC) in terms of volatility [18,19] and single or hybrid model prediction [20] to obtain the accuracy of the prediction of cryptocurrency price.

The state of the artwork agrees that deep learning is good for prediction. However, they lacked correct results because they avoided identifying hidden components of the market, and the absence of stationarity is the key cause of deep learning models' ineffectiveness [21,22]. Therefore, this study proposed the linear structural time series (STS) method that can model level, trend, and seasonal components and allow them to vary over time instead of removing it. This study will be carried out based on the advantage of the linear STS model, which can model the hidden component and predict cryptocurrency prices in time-varying very well. Moreover, this study intends to analyze and model the hidden pattern instead of removing it. This study also compares the performance of the linear STS model, autoregressive integrated moving average (ARIMA), and Seasonal autoregressive integrated moving average (SARIMA) model in predicting the cryptocurrencies' price in terms of time-varying volatility behavior. Apart from that, this study focuses on the top five cryptocurrencies relying on the highest market capitalization.

There are two contributions or novelties to this study. First, this study discusses the behavior and model of unobserved components of time series like level, trends, seasonal, and week-of-the-year effects for the top five cryptocurrencies that utilize the linear STS model. Secondly, the performance of the top five cryptocurrencies in predicting closing prices in time-varying volatility via linear STS, ARIMA, and SARIMA models will be compared. This paper is organized as follows. The second section provides a literature review of the various associated works that have already been carried out. The third section introduces the proposed workflow methodology, an explanation of the materials, and the methods of the experimental settings used. The fourth section provides detailed experimental settings, and the results will be obtained later. The last section includes the conclusion and future work, followed by the references for this paper's study purpose.

2 Literature Review

Bitcoin (BTC) is the most prominent cryptocurrency, and pseudonymous developer Satoshi Nakamoto was the first to introduce it [23]. Despite the system being introduced in 2009, growth only started in 2013. Since then, the number of available cryptocurrencies has exploded. There are more than 10,000 cryptocurrencies at present, as stated on the yahoo finance website. The complexity of price fluctuations, multiple market elements, and uncertainties, including the political and economic environments and investor behavior, might have been to the unpredictability of their price changes [24]. Additionally, cryptocurrency trading practices and hours differ from those of financial markets because, unlike financial markets, which have set trading hours, decentralized cryptocurrencies can be traded around the clock every day of the week. As a result, the price of cryptocurrencies can change instantly in response to news and events at any time [25]. Cryptocurrencies promise large returns and no correlation to traditional financial assets, despite being complex and risky [26].

Predicting cryptocurrency prices are difficult due to dynamic data, volatility, nonstationary, highly nonlinear [27,28], and heavily influenced by various seasons [29]. Consequently, numerous techniques are utilized in order to obtain an accurate prediction. That consist of econometric models, statistical methods, artificial intelligence, and hybrid models [30]. Other than that, the accurate prediction model can deepen investors' understanding of cryptocurrency market price fluctuations and provide a basis for optimal hedging, option pricing, and portfolio diversification. It can also help the government formulate regulatory policies [31,32]. The ARIMA model is considered the most common linear statistical model for time series analysis [33,34]. The models that are most widely used to investigate price fluctuation using econometrics are the generalized autoregressive conditional heteroscedasticity (GARCH) model [27,35], vector autoregressive (VAR) model [36,37], and multiple linear regression [38,39]. Note that these econometric models have specific assumptions needed to be fulfilled. A reasonable prediction can be made for facts that adhere to these assumptions. However, for time series data with nonstationary and nonlinear behavior, these models have low capabilities for prediction [40].

Besides, market behavior of trend and seasonal in the historical data is essential, which is part of the time series components. According to [41], the time series pattern is assumed to be the past price behavior that tends to repeat in the future. This gives an idea of what the price pattern or trend will be in the future, either increasing or decreasing. However, the presence of trend and seasonal rarely make the modeling process easier. Some of the studies lacked accurate results because they avoided identifying the critical features; one is a trend and seasonal component in time series. Consequently, most researchers isolate or remove the trend and seasonal component from the actual series, either linear or nonlinear time series model. Other than that, [42] discovered that deterministic seasonality is inadvisable to deseasonalize the time series. Deseasonalizing prevented the model from precisely estimating and affected forecasting accuracy.

According to [43–45], the structural time series (STS) model can handle nonstationary data very well without removing the information. The advantage of the linear STS model is that it can interpret and model the hidden component of time series. The linear STS model is one of the time series classes that model trend and seasonal components instead of removing that component. Therefore, in order to retain the important information from these critical features, this study utilizes linear STS or state-space modeling to take advantage of this model. Although many studies utilized the STS models in various forecasting fields (engineering, economics, meteorology, sociology, and many more), as far as we are concerned, limited studies were performed to analyze the STS model in predicting the top five

cryptocurrencies' prices. To the best of our knowledge, only [46] analyzed the BTC price dynamics using a Bayesian STS to investigate how macroeconomic, financial, and social factors influenced the price of BTC. Thus, to close this gap, this study uses the linear STS model to discover the behavior, model the hidden pattern of the top five cryptocurrencies, and predict prices.

3 Methodology

Based on their respective market capitalizations, the top five cryptocurrencies will be examined in this study's analysis of weekly historical data. Note that the amount of data that can be accessed through the yahoo finance website is restricted. Consequently, the data of the top five cryptocurrencies taken differ in the starting date, and all end on 24 January 2022. The weekly closing price for Bitcoin (BTC) data coverage was from 05 /01/2015 to 24/01/2022 (369 observations) generated from the website. Estimation from 05/01/2015 to 25/1/2021) (317 observations) was obtained while forecasting from 1/2/2021 to 24/1/2022 (52 observations). The details of other cryptocurrencies can see in Table 1. The structural time series (STS) analysis was analyzed utilizing Structural Time Series Analyser Modeller and Predictor (STAMP 8.2) software packaging.

This study utilizes the linear STS or state space modeling to take advantage of this model. It can easily handle critical features such as trend and seasonal for time-varying and direct interpretation. Consequently, we compare the benchmark model with the ARIMA model, in which model identification opens the analysis, followed by model estimation and forecasting for the top five cryptocurrencies' prices.

Table 1: Sample Data

Cryptocurrency	Division	Period	Sample
Bitcoin (BTC)	Estimation	05/01/2015 to 25/1/2021	317 observations
	Forecast	1/2/2021 to 24/1/2022	52 observations
Ethereum (ETH)	Estimation	06/11/2017 to 25/1/2021	169 observations
	Forecast	1/2/2021 to 24/1/2022	52 observations
Tether (USDT)	Estimation	06/11/2017 to 25/1/2021	169 observations
	Forecast	1/2/2021 to 24/1/2022	52 observations
Binance Coin (BNB)	Estimation	06/11/2017 to 25/1/2021	169 observations
	Forecast	1/2/2021 to 24/1/2022	52 observations
U.S. Dollar Coin (USDC)	Estimation	05/11/2018 to 25/1/2021	117 observations
	Forecast	1/2/2021 to 24/1/2022	52 observations

Structural Time Series

On the basis of state space form, an STS is created, with the system's state serving to represent numerous unobserved elements, including trend and seasonality. As additional observations become available, a filtering technique can be used to update the estimate of the unobserved state. Smoothing

algorithms provide the most accurate estimate of the state at every point within the sample, and predictions are made by projecting these estimated components into the future. As per [47], an STS model can estimate the components and anticipate the data.

Conventional time series model as a sum of trend, μ_t , seasonal, γ_t , and irregular, ε_t components are given by

$$Y_t = \mu_t + \gamma_t + \varepsilon_t \quad (1)$$

Note that the basic STS is at a local level. The local level model is also known as the random walk, where the model consists of a stochastic level and noise or irregular component. The local level can be formulated as follows:

$$y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \quad t = 1, 2, \dots, T, \quad (2a)$$

where the stochastic level component measures the average of the series, which varies over time. A model can also be represented as follows:

$$\mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2) \quad t = 1, 2, \dots, T, \quad (2b)$$

where η_t is the level of disturbance and σ_ε^2 and σ_η^2 are normally and identically distributed with a zero mean and variance, which is constant. In the case that the level component does not change over time and is fixed for all time, the level component is equivalent to the intercept of the regression equation. When the state disturbances are all fixed on zero, models (2a) and (2b) reduce to a deterministic model(3). In other words, over time, the level does not change. On the other hand, a stochastic process is used when the level varies over time.

$$y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \quad t = 1, \dots, T. \quad (3)$$

The model that gave the local level model a slope is called the local linear trend model. The slope component in this model is given as an equation and may change over time (4).

$$\begin{aligned} \mu_t &= \mu_{t-1} + \nu_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2) \\ \nu_t &= \nu_{t-1} + \varsigma_t, \quad \varsigma_t \sim NID(0, \sigma_\varsigma^2), \end{aligned} \quad (4)$$

where ν_t is the slope component and ς_t is the slope disturbance. Letting $\sigma_\varsigma^2 = 0$, local linear trend model in (4) reduces to a random walk with constant drift, known as local level with drift. Sometimes, the time series is influenced by seasonal variation, such as calendar effects or policy changes. As a result, this study combines the trend specification with the seasonal component. The details of the combination of local with drift + deterministic seasonal can refer to the equation (10). Furthermore, in the smooth trend model, where $\sigma_\eta^2 = 0$ is the trend of integrated random walk, the resulting trend varies very smoothly over time. In other words, the smooth trend model treats a deterministic level and stochastic slope. If the local level has drift occurs when the level is stochastic, the slope is fixed. Therefore, trend specification is simplified, as depicted in Table 2.

Table 2. Trend Specification

Number	Trend Model	Specification
1.	Deterministic Level	‘Level – Fixed’ and ‘Slope’ not selected
2.	Local Level	‘Level – Stochastic’ and ‘Slope’ not selected
3.	Deterministic Trend	‘Level – Fixed’ and ‘Slope – Fixed’
4.	Smooth Trend	‘Level – Fixed’ and ‘Slope – Stochastic’
5.	Local Level with Drift	‘Level – Stochastic’ and ‘Slope -Fixed’
6.	Local Linear Trend	‘Level – Stochastic’ and ‘Slope -Stochastic’

*Note: Level – the actual value of the trend; Slope – this trend component may or may not be present

As mentioned before, time series data are influenced by seasonal effects. Modeling the seasonal effect involves including a seasonal component in equation (2a) and shown in equation (5)

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \quad t = 1, 2, \dots, T, \tag{5}$$

The seasonal component requires one state equation. The seasonal component generally requires $(s - 1)$ where s is the number of periodicity or seasonal. In this case, the seasonal period for weekly data is $s = 52$ (normally, a year contains 52 weeks). As a result, this study examines and investigates six combination trend and seasonal specifications of the STS model, as shown in equations (6)–(11).

Deterministic Level + Deterministic Seasonal

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) & (6) \\
 \mu_t &= \mu \\
 \gamma_t &= - \sum_{j=1}^{s-1} \gamma_{t-j}
 \end{aligned}$$

Local Level + Deterministic Seasonal

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) & (7) \\
 \mu_t &= \mu_{t-1} + \eta_t & \eta_t &\sim NID(0, \sigma_\eta^2) \\
 \mu_t &= \mu_{t-1} + \eta_t
 \end{aligned}$$

Deterministic Trend + Deterministic Seasonal

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) & (8) \\
 \mu_t &= \mu_{t-1} + \eta_t \\
 \nu_t &= \nu \\
 \gamma_t &= - \sum_{j=1}^{s-1} \gamma_{t-j}
 \end{aligned}$$

Smooth Trend + Deterministic Seasonal

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) \\
 \mu_t &= \mu_{t-1} + \nu_{t-1} & \varsigma &\sim NID(0, \sigma_\varsigma^2) \\
 \gamma_t &= - \sum_{j=1}^{s-1} \gamma_{t-j} \\
 \gamma_t &= - \sum_{j=1}^{s-1} \gamma_{t-j}
 \end{aligned} \tag{9}$$

Local Level with Drift + Deterministic Seasonal

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) \\
 \mu_t &= \mu_{t-1} + \nu_t + \eta_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) \\
 \nu_t &= \nu \\
 \gamma_t &= - \sum_{j=1}^{s-1} \gamma_{t-j}
 \end{aligned} \tag{10}$$

Local Linear Trend + Deterministic Seasonal

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) \\
 \mu_t &= \mu_{t-1} + \nu_{t-1} + \eta_t & \eta_t &\sim NID(0, \sigma_\eta^2) \\
 \nu_t &= \nu_{t-1} + \varsigma_t & \varepsilon_t &\sim NID(0, \sigma_\varepsilon^2) \\
 \gamma_t &= - \sum_{j=1}^{s-1} \gamma_{t-j}
 \end{aligned} \tag{11}$$

As mentioned above, the STS model summarises the dynamics of a system known as state space form. It enables the estimation of the STS parameters. On the basis of the state, space form consists of the observation and transition equations. The observation equation describes the relationship between the observed data y_t and the unobserved state α_t . The equation can be written as

$$y_t = Z_t \alpha_t + \varepsilon_t \quad \varepsilon_t \sim NID(0, H_t), \tag{12}$$

where y_t is a vector of observed data at time, t , α_t is a state vector, Z_t is a structural parameters, and ε_t is the vector of uncorrelated error and H_t variance of the matrix. Meanwhile, the transition equation can be represented as follows:

$$\alpha_t = T_t \alpha_{t-1} + R_t \tau_t \quad \tau_t \sim NID(0, Q_t), \tag{13}$$

where T_t and R_t are structural parameters, τ_t is a vector of uncorrelated error, and Q_t is the variance matrix. In addition, the specification state space form must complete the assumption mentioned in [48].

After the STS is expressed in a state space form, the Kalman filter technique’s iterative process will be utilized to estimate the unobserved state variable. Kalman filter estimation consists of two iterative

procedures that are predicting and updating. The aim of the filter is to update the state variable as new observation becomes available. Furthermore, for the hyperparameter or unknown parameter, such disturbance variance is estimated based on the maximum likelihood estimation function and predictive error decomposition. Information about the Kalman filter estimation procedure can be referred to [49,50]. Other than that, the diagnostic checking of estimated residuals for each model is conducted based on the Box-Ljung test of independence, both the homoscedasticity Goldfeld-Quandt (GQ) test and the normality Jarque-Bera (JB) test. Finally, the optimal model is chosen using both the Akaike Information Criteria (AIC) as well as the Schwartz Bayesian criterion (BIC).

Diagnostic Tests

In a linear Gaussian model, three analytical assumptions serve as the foundation for all significance tests of residual. These residuals should satisfy three properties, which are independence, homoscedasticity, and normality. It is possible to verify the assumption of the residuals' independence with the Box-Ljung statistic. Let

$$r_k = \frac{\sum_{t=1}^{n-k} (e_t - \bar{e})(e_{t+k} - \bar{e})}{\sum_{t=1}^m (e_t - \bar{e})^2}, \quad (14)$$

where r_k is the residual autocorrelation for lag k , while e is the mean of the n residuals. The Box-Ljung statistic is calculated using the following method:

$$Q(k) = n(n+2) = \sum_{l=1}^k \frac{r_l^2}{n-1} \quad l = 1, 2, \dots, k. \quad (15)$$

A chi-square distribution with $(k, w, \text{ and } 1)$ degrees of freedom should be used to test this, in which w denotes the number of expected disturbance variances. If $Q(k) < \chi^2_{0.05, k-w+1}$, it then illustrates that the independence null hypothesis is not rejected or that the residuals are serially uncorrelated.

The assumption of homoscedasticity of the residuals may be verified using the GQ test. Homoscedasticity is the null hypothesis for the GQ test. The larger the F-statistic, the more evidence you will have that the homoscedasticity assumption is false. Hence, the more likely you are to have heteroscedasticity (different variance for the two groups). If the variance of the residuals in the initial third of the series equals the variance of the residuals corresponding to the last third of the series, the statistic passes. Following is the GQ test statistic:

$$GQ(h) = \frac{\sum_{t=n-h+1}^n e_t^2}{\sum_{t=d+1}^{d+h} e_t^2}, \quad (16)$$

in which d denotes the number of estimated parameters in the model, and h represents the closest integer to $(n-d)/3$. In comparison to an F-distribution with (h, h) degrees of freedom, the GQ test statistic is evaluated.

The residuals are then assumed to be regularly distributed. Using the JB test, the residuals' normality can be verified as follows:

$$JB = n \left(\frac{S^2}{6} + \frac{(K-3)^2}{24} \right), \quad (17)$$

$$S = \frac{\frac{1}{n} \sum_{t=1}^n (e_t - \bar{e})^3}{\left(\sqrt{\frac{1}{n} \sum_{t=1}^n (e_t - \bar{e})^2}\right)^3} \text{ and } K = \frac{\frac{1}{n} \sum_{t=1}^n (e_t - \bar{e})^4}{\left(\frac{1}{n} \sum_{t=1}^n (e_t - \bar{e})^2\right)^2},$$

in which S denotes skewness and K denotes the residuals' kurtosis. It is recommended to compare the JB test statistic to a chi-square distribution having two degrees of freedom. If $JB < \chi^2_{\alpha,2}$, the residuals are normally distributed. Hence, the null hypothesis of normality is not rejected. Note that the possibility of outliers or structural breaks in the series is indicated by rejecting the null hypothesis, which is validated by examining the auxiliary residual.

4 Result and Discussion

Table 3: Descriptive Statistic

Cryptocurrency	Mean	Standard Deviation	Median	Min	Max	Mode	Skewness	Kurtosis	Augmented Dickey-Fuller Test statistic (p-value)
BTC	11608.63	16213.56	6482.35	210.34	65466.84	458.55	1.85	2.26	-0.8565 (0.8012)
ETH	970.02	1209.71	371.05	85.26	4626.35	N/A	1.57	1.23	-1.0348 (0.7409)
USDT	1.00	0.005	1.00	0.98	1.02	0.99	0.83	4.25	-8.3351 (0.000)**
BNB	111.26	181.37	19.49	1.52	662.23	12.12	1.65	1.25	-1.2158 (0.6679)
USDC	1.00	0.006	1.00	0.97	1.02	1.00	1.44	4.63	-3.2525 (0.0188)**

**significant at 5% level *significant at 10% level

Table 3 depicts the descriptive statistic of the top five cryptocurrencies' prices. Bitcoin (BTC) has the highest closing price of \$65466.84, with a standard deviation of \$16213.56. The average BTC price during the period between Jan 2015 to Jan 2022 is \$11608.63, which is the first ranking and highest value of cryptocurrency price compared to others. The frequent price of BTC in terms of that period is \$458.55. Meanwhile, the lowest closing price is USDT and USDC, which have the same value of \$1.02. The USDT and USDC also have low risk and low return since they have the lowest value standard deviation compared to other cryptocurrencies. Additionally, USDT and USDC are stable cryptocurrencies' prices by looking at the smallest standard deviation, whereby the price is more consistent and less spread out of data. It is supported by Augmented Dickey-Fuller Test, where both cryptocurrencies are more stationary than others. Subsequently, all cryptocurrency of closing price data is not normal and highly skewed with a value of skewness of more than 1, except USDT has moderate skewness.

Once checked, the behavior of nonstationary and stationary data was determined. Correspondingly, this research attempted to compare the linear structural time series model (STS), autoregressive integrated moving average (ARIMA), and SARIMA performance. In order to determine the superior model in predicting cryptocurrency prices in terms of the strength of each

model. The data has been transformed to log first before analyzing it in advance. Tables 4(a) until (e) demonstrate analysis model identification in the first step, and the STS model estimate comes next. The unobserved component trend and seasonality are used in the formulation of the linear STS models. Apart from that, this study attempts to combine trends with stochastic seasonal. However, the results obtained are the same with a combination of trend and deterministic seasonal. This shows that the seasonal component does not vary over time; instead, it is for trend. Therefore, this study finalizes six models of cryptocurrencies' prices of STS, combining trend and deterministic seasonal. The models are Deterministic Level + Deterministic Seasonal, Local Level + Deterministic Seasonal, Deterministic Trend + Deterministic Seasonal, Smooth Trend + Deterministic Seasonal, Local Level with Drift + Deterministic Seasonal, and Local Linear Trend + Deterministic Seasonal.

All models need to check the three assumptions of the residual model: independence, homoscedasticity, and normality. The best model was selected based on residual assumptions and had the lowest value of Bayesian Information Criteria (BIC) as well as Akaike's Information Criteria (AIC). From the outcomes, all the models do not meet all the residual assumptions, which indicates the existence of outliers or other explanatory variables not measured in the model. The Local Level with Drift + Deterministic Seasonal and Local Linear Trend + Deterministic Seasonal is the best estimation model of STS for BTC, Binance (BNB), and USDC. Both models have the same value of the variance of disturbance and residual diagnostic. Note that the model has the smallest value of AIC compared to others. Meaning that the level is varying to time while the slope changes to deterministic and stochastic conditions. The Local Level + Deterministic Seasonal is regarded as the best model for Ethereum (ETH) and USDT, where this model has the lowest value of both BIC and AIC. This means that the level is more varying to time, even though the slope is not present in the model.

Meanwhile, for the ARIMA and SARIMA model analysis, it is crucial to identify the suitable models to be fitted to the data series based on the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the stationary series. However, it needs to be reiterated that it is not easy to precisely specify the model class based on the evidence provided by the ACF and PACF. Therefore, in this predicament, several possible models will be selected, estimated, and then perform the validation or diagnostic tests. The model picked is the one that gives superior results based on the smallest AIC/BIC and the measurement error (MSE and MAPE) and meets all residual assumptions. Therefore, the simplest model and fewer parameters are the priority to be chosen basis on the concept of parsimony.

Table 4(a). Estimated Performance of STS Model (Specifications Trend + Seasonal) for Bitcoin

Parameter	Deterministic Level + Deterministic Seasonal	Local Level + Deterministic Seasonal	Deterministic Trend + Deterministic Seasonal	Smooth Trend + Deterministic Seasonal	Local Level with Drift + Deterministic Seasonal	Local Linear Trend + Deterministic Seasonal
Variance of disturbances	0.0000	0.0110	0.0000	0.0000	0.0108	0.0108
Slope			0.0000	0.0022	0.0000	0.0000
Seasonal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Irregular	2.5348	0.0000	0.3889	0.0037	0.0000	0.0000
Prediction error variance (p.e.v)	2.1430	0.0093	0.3277	0.0112	0.0091	0.0091
Residual	3446.8	8.9626	1622	35.581	9.0369	9.0369
Box-Ljung, Q-statistic (Independence)	[0.0000]**	[0.1106]	[0.0000]**	[0.0000]**	[0.0602]	[0.0602]
QQ	3.0464	1.3106	4.4033	1.4134	1.371	1.371
(Homoscedasticity)	[0.0000]**	[0.1033]	[0.0000]**	[0.0532]	[0.0704]	[0.0704]
JB	3.392	14.344	2.6303	22.384	16.639	16.639
(Normality)	[0.1834]	[0.0008]**	[0.2684]	[0.0000]**	[0.0002]**	[0.0002]**
AIC	1.0966	-4.3398	-0.7748	-4.147	-4.3607	-4.3607
BIC	1.725	-3.7113	-0.1345	-3.5067	-3.7204	-3.7204

**significant at 5% level *significant at 10% level

Table 4(b). Estimated Performance of STS Model (Specifications Trend + Seasonal) for Ethereum

Parameter	Deterministic Level + Deterministic Seasonal	Local Level + Deterministic Seasonal	Deterministic Trend + Deterministic Seasonal	Smooth Trend + Deterministic Seasonal	Local Level with Drift + Deterministic Seasonal	Local Linear Trend + Deterministic Seasonal
Variance of disturbances	0.0000	0.0212	0.0000	0.0000	0.0213	0.0205
Slope			0.0000	0.0054	0.0000	2.6297e-005
Seasonal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Irregular	0.5663	0.0000	0.5543	0.0063	0.0000	0.0000
Prediction error variance (p.e.v)	0.3967	0.0148	0.3852	0.0179	0.0148	0.0147
Residual	562.1	4.6496	485.05	19.29	4.0148	3.7589
Box-Ljung, Q-statistic (Independence)	[0.0000]**	[0.4601]	[0.0000]**	[0.0007]**	[0.4040]	[0.4396]
QQ	0.4875	0.3627 [0.9990]	6.55	0.3454 [0.9993]	0.3521 [0.9991]	0.3096
(Homoscedasticity)	[0.9863]	[0.0000]**	[0.0000]**	[0.0000]**	[0.9998]	[0.9998]
JB	3.5995	11.973	0.1628	2.4476	10.418	5.3534
(Normality)	[0.1653]	[0.0025]**	[0.9218]	[0.2941]	[0.0055]**	[0.0688]**
AIC	-0.2974	-3.5837	-0.3148	-3.3854	-3.575	-3.5776
BIC	0.6842	-2.6022	0.6853	-2.3853	-2.5749	-2.5776

**significant at 5% level *significant at 10% level

Table 4 (c). Estimated Performance of STS Model (Specifications Trend + Seasonal) for USD1

Parameter	Deterministic Level + Deterministic Seasonal	Local Level + Deterministic Seasonal	Deterministic Trend + Deterministic Seasonal	Smooth Trend + Deterministic Seasonal	Local Level with Drift + Deterministic Seasonal	Local Linear Trend + Deterministic Seasonal
Variance of disturbances	0.0000	5.8807e-006	0.0000	0.0000	6.3235e-006	6.3235e-006
Slope			0.0000	2.9131e-007	0.0000	0.0000
Seasonal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Irregular	3.9372e-005	1.6012e-005	3.9441e-005	2.1198e-005	1.5701e-005	1.5701e-005
Prediction error variance (p.e.v)	2.7581e-005	2.0375e-005	2.7412e-005	2.3944e-005	2.0362e-005	2.0362e-005
Residual						
Box-Ljung, Q-statistic (Independence)	67.964 [0.0000]**	5.5125 [0.3566]	97.301 [0.0000]**	11.01 [0.0265]	4.7212 [0.3171]	4.7213 [0.3171]
Diagnosics	0.4222	0.3256 [0.9997]	0.43732 [0.9938]	0.4049 [0.9968]	0.3213 [0.9997]	0.3213 [0.9997]
(Homoscedasticity)	[0.9958]	6.9806	2.6962	35.22	8.1905	8.1905
JB	4.346	[0.0305]**	[0.2597]	[0.0000]**	[0.0167]**	[0.0167]**
(Normality)	[0.1138]	-9.8712	-9.8655	-10.001	-10.163	-10.163
AIC	-9.8712	-9.1924	-8.8654	-9.0007	-9.1627	-9.1627
BIC	-8.8896					

**significant at 5% level *significant at 10% level

Table 4(d). Estimated Performance of STS Model (Specifications Trend + Seasonal) for BNB

Parameter	Deterministic Level + Deterministic Seasonal	Local Level + Deterministic Seasonal	Deterministic Trend + Deterministic Seasonal	Smooth Trend + Deterministic Seasonal	Local Level with Drift + Deterministic Seasonal	Local Linear Trend + Deterministic Seasonal
Variance of disturbances	0.0000	0.0279	0.0000	0.0000	0.0277	0.0277
Slope			0.0000	0.0053	0.0000	0.0000
Seasonal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Irregular	0.4514	0.0000	0.2226	0.0085	0.0000	0.0000
Prediction error variance (p.e.v)	0.3162	0.0196	0.1547	0.0216	0.0192	0.0192
Residual						
Box-Ljung, Q-statistic (Independence)	339.44 [0.0000]**	6.0097 [0.3053]	418.97 [0.0000]**	8.0966 [0.0881]*	6.1853 [0.1857]	6.1853 [0.1857]
Diagnosics	2.1471	0.2670	1.212	0.2918 [0.9999]	0.2543 [1.0000]	0.2543 [1.0000]
(Homoscedasticity)	[0.0096]**	[1.0000]	[0.2782]			
JB	0.3969	25.085	5.3946	16.136	23.671	23.671
(Normality)	[0.8200]	[0.0000]**	[0.0674]*	[0.0003]**	[0.0000]**	[0.0000]**
AIC	-0.5241	-3.3063	-1.2274	-3.1974	-3.3124	-3.3124
BIC	0.4575	-2.3247	-0.2273	-2.1973	-2.3123	-2.3123

**significant at 5% level *significant at 10% level

Table 4(e). Estimated Performance of STS Model (Specifications Trend + Seasonal) for USDC

Parameter	Deterministic Level + Deterministic Seasonal	Local Level + Deterministic Seasonal	Deterministic Trend + Deterministic Seasonal	Smooth Trend + Deterministic Seasonal	Local Level with Drift + Deterministic Seasonal	Local Linear Trend + Deterministic Seasonal
Variance of disturbances	0.0000	1.7113e-006	0.0000	0.0000	1.4501e-006	1.4501e-006
Slope			0.0000	2.1309e-009	0.0000	0.0000
Seasonal	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Irregular	4.64550e-005	1.8210e-005	2.6332e-005	2.2343e-005	1.8541e-005	1.8541e-005
Prediction error variance (p.e.v)	2.6119e-005	1.3892e-005	1.4588e-005	1.4222e-005	1.3564e-005	1.3564e-005
Residual	23.277	6.4294	12.428	7.765	6.6545	6.6545
Diagnosics	[0.0003] **	[0.2667]	[0.0144] *	[0.1006]	[0.1553]	[0.1553]
Box-Ljung, Q-statistic (Independence)	0.7304	0.3637	0.3781	0.3576	0.3312 [0.9927]	0.3312
GG (Homoscedasticity)	[0.7611]	[0.9876]	[0.9887]	[0.9887]	[0.9927]	[0.9927]
JB	0.3251	1.4675	0.3382	2.3114	1.2905	1.2905
(Normality)	[0.8500]	[0.4801]	[0.8444]	[0.3148]	[0.5245]	[0.5245]
AIC	-9.6469	-10.278	-10.212	-10.238	-10.285	-10.285
BIC	-8.3956	-9.027	-8.9374	-8.9628	-9.0101	-9.0101

**significant at 5% level *significant at 10% level

The top image of Figure 1 depicts that the closing price of BTC exhibits a stochastic trend, while the bottom image of the figure shows the deterministic seasonal pattern. Meanwhile, Tables 5(a)-(e) present the estimated unobserved component where a combination of trend and seasonal specifications of the STS model is mentioned in equations (6) - (11). For Table 5(a), the final estimate of the level of BTC closing price is ($e^{10.43693}$) \$34095.82 per week, and the estimated deterministic slope coefficient indicates that the level of the closing price of BTC is rising at a pace of 1.6% per week. It is evident from the predictable seasonal pattern in Figure 1 and Table 5(a) that during the first week of each year (2015-2021), the closing price of BTC increases. It might be the cause of the January effect phenomenon. In comparison, the lowest closing price is on the week of thirteen every year. Consequently, Table 5(a) demonstrated that 4 out of 52 weeks are significant seasonal effects, which are the first, eleven, twelve, and thirteen weeks. The BTC closing price decreasing rate can be seen in the eleventh week until the thirteenth week every year (2015-2021), around 14%, 17%, and 18%, respectively.

For ETH, the final estimate of the level closing price is significantly per week by ($e^{7.0789}$) \$1186.66, as stated in Table 5(b). The level is varying to time, whereby the middle of the year 2018 shows a downward trend until January 2019, as shown in Figure 2. Afterwards, the trend demonstrated increases until January 2021. The deterministic seasonal shows at the seventh and eighth weeks are increasing trend and significantly affect every year by 30% and 29%, respectively. Note that both weeks are the highest cryptocurrencies price every year. Meanwhile, every 49th-week closing price of ETH decreases significantly by 29% each year. Starting from 32nd until the 52nd week, the cryptocurrency price trend dropped significantly, or it is called crypto winter, where the cryptocurrency price occurred downturn in that period.

Meanwhile, the trend of the cryptocurrency price of USDT also shows the level is varying to time, where the level estimate is \$1.0001, as shown in Table 5(c). Every first and second week of January of each year, the cryptocurrency price of USDT is bulling by 0.5% to 0.7%. The pattern is demonstrated in Figure 3. Subsequently, three weeks are seen significantly increasing closing prices in 49th, 51st, and 52nd weeks. However, when comparing the plot of the actual closing price and the level at the top of Figure 3, it is seen the plot is slightly different. It might be the behavior of USDT has gone stationary. As a result, the STS model is not the most accurate for predicting. However, this research will prove this by comparing the results obtained by the ARIMA model.

Consequently, for BNB, the level varies to the time when the closing price is \$47.46 per week. While the slope change to stochastic and deterministic conditions by increasing by 1.9% per week, as depicted in Table 5(d). BNB also has a crypto winter which starts on the 34th and until the 52nd week. The closing price pattern is the same as ETH, where the trend has a negative pattern after six months. There are four weeks that have a significant seasonal effect, which is weeks 46th, 47th, 48th, and 49th, where the trend is downward. In other words, the cryptocurrency price is bearish from every November to December each year. It also can be seen from the bottom of Figure 4.

Next, for USDC, the best model identified is the Local Level with Drift + Deterministic Seasonal and Local Linear Trend + Deterministic Seasonal. From the top of Figure 5, when comparing the level and actual closing price plot, the level shows the points are more around the mean compared to the other plots. It is seen the linear STS model is not suited for predicting the closing price of USDC. It also might be that the USDC closing price is stationary, like USDT. Table 5(e) illustrates that USDC showed that only one cryptocurrency price has a negative level and slope among the top five. This means that the USDC closing price level decreases by \$1.005 per week.

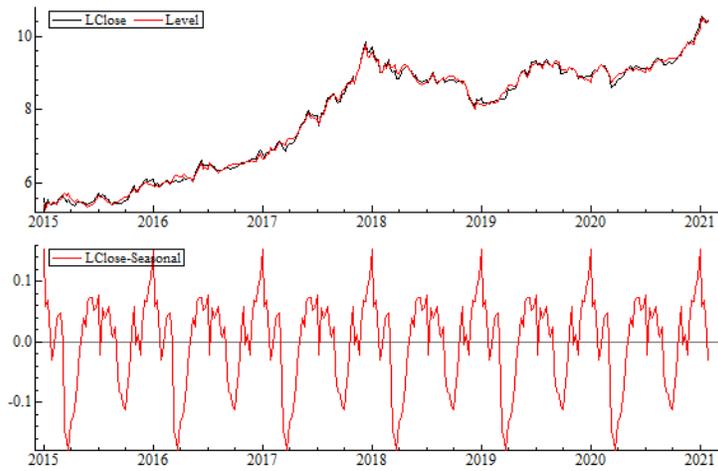


Figure 1: Local Level with Drift + Deterministic Seasonal for Bitcoin

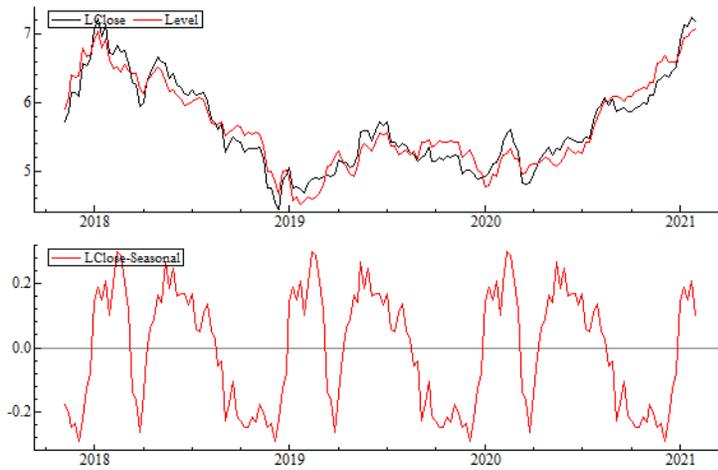


Figure 2: Local Level + Deterministic Seasonal for ETH

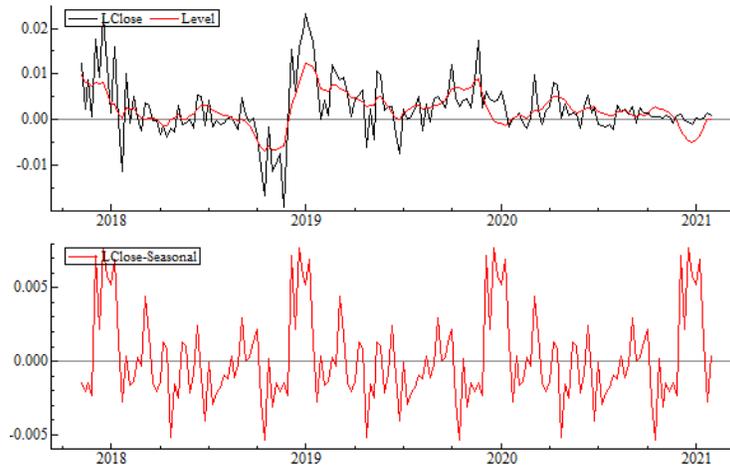


Figure 3: Local Level + Deterministic Seasonal for USDT

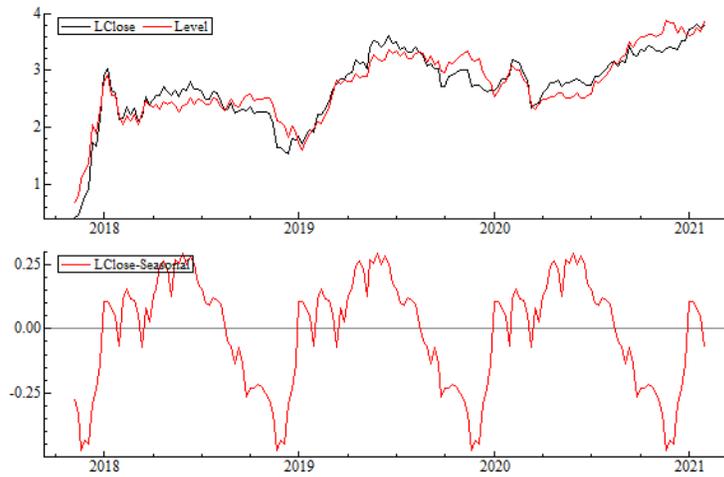


Figure 4: Local Level + Deterministic Seasonal for BNB

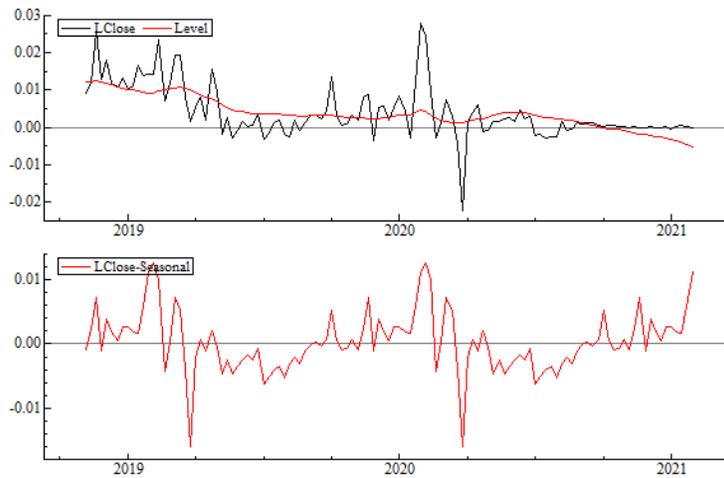


Figure 5: Local Level with Drill + Deterministic Seasonal for USDC

Table 5(a). The Best Model Final Estimation Performance for the Bitcoin

Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.
μ_1	10.43693**	[0.00000]												
V_1	0.01585**	[0.00715]												
γ_1	0.15366*	[0.07846]	γ_{12}	-0.16502*	[0.06072]	γ_{23}	0.07286	[0.40926]	γ_{34}	0.00801	[0.92771]	γ_{45}	-0.00496	[0.95490]
γ_2	0.05854	[0.50118]	γ_{13}	0.17948**	[0.04165]	γ_{24}	0.07305	[0.40821]	γ_{35}	0.02505	[0.77651]	γ_{46}	0.01246	[0.88701]
γ_3	0.07005	[0.42082]	γ_{14}	-0.13251	[0.13219]	γ_{25}	0.05231	[0.55361]	γ_{36}	-0.04992	[0.57160]	γ_{47}	-0.02147	[0.80642]
γ_4	0.00989	[0.90947]	γ_{15}	-0.12484	[0.15629]	γ_{26}	0.05544	[0.53020]	γ_{37}	-0.08297	[0.34715]	γ_{48}	0.0241	[0.78305]
γ_5	-0.02921	[0.73731]	γ_{16}	-0.10895	[0.21611]	γ_{27}	0.07838	[0.37510]	γ_{38}	-0.08562	[0.33177]	γ_{49}	0.06832	[0.43494]
γ_6	-0.00757	[0.93077]	γ_{17}	-0.07233	[0.41147]	γ_{28}	-0.02076	[0.81412]	γ_{39}	-0.10585	[0.23021]	γ_{50}	0.0673	[0.44135]
γ_7	0.03746	[0.66780]	γ_{18}	-0.01352	[0.87794]	γ_{29}	0.05745	[0.51553]	γ_{40}	-0.11104	[0.20799]	γ_{51}	0.0883	[0.31208]
γ_8	0.04484	[0.60785]	γ_{19}	-0.00013	[0.99880]	γ_{30}	0.04021	[0.64895]	γ_{41}	-0.06648	[0.45028]	γ_{52}	0.10711	[0.21981]
γ_9	0.04773	[0.58535]	γ_{20}	0.03973	[0.65226]	γ_{31}	0.04669	[0.59708]	γ_{42}	-0.05101	[0.56203]			
γ_{10}	0.00021	[0.99807]	γ_{21}	0.02487	[0.77795]	γ_{32}	0.05853	[0.50760]	γ_{43}	0.01716	[0.84522]			
γ_{11}	-0.14449*	[0.09999]	γ_{22}	0.06525	[0.45973]	γ_{33}	0.01546	[0.86096]	γ_{44}	0.05771	[0.51129]			

Table 5(b). The Best Model Final Estimation Performance for the Ethereum

Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.
H_4	7.0789**	[0.00000]												
γ_1	0.14605	[0.37144]	γ_{12}	-0.16583	[0.33283]	γ_{23}	0.15898	[0.36415]	γ_{34}	-0.05786	[0.73816]	γ_{45}	-0.17485	[0.29090]
γ_2	0.1894	[0.24789]	γ_{13}	-0.2641	[0.12549]	γ_{24}	0.16722	[0.34011]	γ_{35}	-0.04112	[0.81168]	γ_{46}	-0.19873	[0.22836]
γ_3	0.15052	[0.35925]	γ_{14}	-0.13929	[0.41886]	γ_{25}	0.1672	[0.34024]	γ_{36}	-0.22847	[0.18589]	γ_{47}	-0.24751	[0.13286]
γ_4	0.20721	[0.20923]	γ_{15}	-0.02686	[0.87631]	γ_{26}	0.13249	[0.44942]	γ_{37}	-0.17169	[0.31781]	γ_{48}	-0.2342	[0.15368]
γ_5	0.10268	[0.53446]	γ_{16}	0.06204	[0.71999]	γ_{27}	0.17005	[0.33184]	γ_{38}	-0.10401	[0.54310]	γ_{49}	-0.2923*	[0.07512]
γ_6	0.2009	[0.22804]	γ_{17}	0.09029	[0.60286]	γ_{28}	0.05679	[0.74527]	γ_{39}	-0.21194	[0.21466]	γ_{50}	-0.22221	[0.17434]
γ_7	0.30081*	[0.07374]	γ_{18}	0.16443	[0.34502]	γ_{29}	0.04853	[0.78107]	γ_{40}	-0.22884	[0.17872]	γ_{51}	-0.12976	[0.42624]
γ_8	0.28634*	[0.09017]	γ_{19}	0.13934	[0.42420]	γ_{30}	0.11103	[0.52467]	γ_{41}	-0.2469	[0.14529]	γ_{52}	-0.07576	[0.64211]
γ_9	0.20394	[0.22831]	γ_{20}	0.26568	[0.12949]	γ_{31}	0.13838	[0.42736]	γ_{42}	-0.24822	[0.14124]			
γ_{10}	0.11827	[0.48586]	γ_{21}	0.18349	[0.29442]	γ_{32}	0.05528	[0.75048]	γ_{43}	-0.21544	[0.19881]			
γ_{11}	-0.13697	[0.42170]	γ_{22}	0.24952	[0.15514]	γ_{33}	0.02703	[0.87618]	γ_{44}	-0.23104	[0.16609]			

Table 5(c). The Best Model Final Estimation Performance for the USDT

Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.
\mathcal{H}_i	0.00013	[0.97315]												
γ_1	0.00517	[0.12978]	γ_{23}	-0.00145	[0.69360]	γ_{34}	-0.00083	[0.82376]	γ_{45}	-0.00117	[0.75229]	γ_{46}	-0.0015	[0.66132]
γ_2	0.00698	[0.04172]	γ_{24}	-0.00206	[0.57578]	γ_{35}	0.00247	[0.50677]	γ_{46}	-0.00028	[0.94034]	γ_{47}	-0.00203	[0.55258]
γ_3	0.00133	[0.69703]	γ_{14}	-0.00135	[0.71355]	γ_{25}	-0.00104	[0.77943]	γ_{36}	0.00294	[0.42538]	γ_{47}	-0.00149	[0.66227]
γ_4	-0.00274	[0.42289]	γ_{15}	0.00132	[0.72122]	γ_{26}	-0.00409	[0.27157]	γ_{37}	0.00001	[0.99828]	γ_{48}	-0.00235	[0.49044]
γ_5	0.00034	[0.92062]	γ_{16}	0.00085	[0.81712]	γ_{27}	-0.0001	[0.97744]	γ_{38}	0.00031	[0.93169]	γ_{49}	0.00724**	[0.03470]
γ_6	-0.00163	[0.65192]	γ_{17}	-0.0052	[0.16099]	γ_{28}	-0.00291	[0.43308]	γ_{39}	0.00128	[0.72693]	γ_{50}	0.00217	[0.52353]
γ_7	-0.00139	[0.70162]	γ_{18}	-0.00153	[0.67974]	γ_{29}	-0.00219	[0.55507]	γ_{40}	0.00216	[0.55443]	γ_{51}	0.00772**	[0.02431]
γ_8	0.0003	[0.93376]	γ_{19}	-0.00249	[0.50223]	γ_{30}	-0.00178	[0.63133]	γ_{41}	-0.00271	[0.45771]	γ_{52}	0.00577*	[0.09076]
γ_9	-0.00033	[0.92822]	γ_{20}	0.00136	[0.71450]	γ_{31}	-0.00097	[0.79398]	γ_{42}	-0.00534	[0.14408]			
γ_{10}	0.00443	[0.22690]	γ_{21}	0.00102	[0.78231]	γ_{32}	-0.00122	[0.74109]	γ_{43}	0.00015	[0.96787]			
γ_{11}	0.00179	[0.62593]	γ_{22}	-0.0022	[0.55400]	γ_{33}	0.00035	[0.92532]	γ_{44}	-0.0031	[0.39288]			

Table 5(d). The Best Model Final Estimation Performance for the BNB

Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.
μ_t	3.85998**	[0.00000]												
ν_t	0.01886	[0.14744]												
γ_1	0.10543	[0.57230]	γ_{12}	0.0825	[0.67342]	γ_{23}	0.24856	[0.21536]	γ_{34}	-0.04597	[0.81637]	γ_{45}	-0.27302	[0.15146]
γ_2	0.10709	[0.56722]	γ_{13}	0.02613	[0.89412]	γ_{24}	0.28586	[0.15477]	γ_{35}	-0.06835	[0.72930]	γ_{46}	-0.32854*	[0.08334]
γ_3	0.07869	[0.67512]	γ_{14}	0.12955	[0.51102]	γ_{25}	0.25512	[0.20734]	γ_{36}	-0.13615	[0.48977]	γ_{47}	-0.47177**	[0.01313]
γ_4	0.05163	[0.78418]	γ_{15}	0.15757	[0.42554]	γ_{26}	0.17579	[0.38029]	γ_{37}	-0.07316	[0.70948]	γ_{48}	-0.43541**	[0.02137]
γ_5	-0.06945	[0.71407]	γ_{16}	0.24458	[0.21815]	γ_{27}	0.15672	[0.43378]	γ_{38}	-0.13625	[0.48665]	γ_{49}	-0.44914**	[0.01741]
γ_6	0.11703	[0.53943]	γ_{17}	0.2645	[0.18415]	γ_{28}	0.10295	[0.60660]	γ_{39}	-0.26385	[0.17762]	γ_{50}	-0.29169	[0.11936]
γ_7	0.15579	[0.41670]	γ_{18}	0.23655	[0.23550]	γ_{29}	0.09225	[0.64420]	γ_{40}	-0.23246	[0.23271]	γ_{51}	-0.23702	[0.20465]
γ_8	0.11347	[0.55381]	γ_{19}	0.1265	[0.52562]	γ_{30}	0.11906	[0.55078]	γ_{41}	-0.23094	[0.23378]	γ_{52}	-0.13498	[0.46925]
γ_9	0.11067	[0.56737]	γ_{20}	0.26841	[0.17998]	γ_{31}	0.11059	[0.57890]	γ_{42}	-0.21547	[0.26424]			
γ_{10}	0.05594	[0.77331]	γ_{21}	0.25644	[0.20056]	γ_{32}	0.09481	[0.63356]	γ_{43}	-0.22671	[0.23799]			
γ_{11}	-0.0705	[0.71769]	γ_{22}	0.29585	[0.14058]	γ_{33}	0.01807	[0.92744]	γ_{44}	-0.25124	[0.18898]			

Table 5(e). The Best Model Final Estimation Performance for the USDC

Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.	Coeff.	Value	Prob.
μ_1	-0.00525*	[0.06059]	γ_{23}	-0.00248	[0.48139]	γ_{34}	-0.00317	[0.36641]	γ_{45}	-0.00094	[0.75530]
ν_1	-0.00015	[0.19921]	γ_{24}	-0.00161	[0.64610]	γ_{35}	-0.0011	[0.75315]	γ_{46}	0.00225	[0.45491]
γ_1	0.00267	[0.37468]	γ_{25}	-0.00251	[0.47536]	γ_{36}	-0.00005	[0.98829]	γ_{47}	0.00717**	[0.01932]
γ_2	0.00188	[0.53044]	γ_{26}	-0.00064	[0.85545]	γ_{37}	0.00031	[0.92904]	γ_{48}	-0.0011	[0.71349]
γ_3	0.00165	[0.58316]	γ_{27}	-0.00627*	[0.07745]	γ_{38}	-0.00023	[0.94701]	γ_{49}	0.00392	[0.19358]
γ_4	0.00574*	[0.06006]	γ_{28}	-0.00498	[0.15923]	γ_{39}	0.00075	[0.83097]	γ_{50}	0.00195	[0.51581]
γ_5	0.01116**	[0.00043]	γ_{29}	-0.00398	[0.25921]	γ_{40}	0.00533	[0.13071]	γ_{51}	0.0005	[0.86741]
γ_6	0.01259**	[0.00057]	γ_{30}	-0.00337	[0.33814]	γ_{41}	0.00052	[0.88212]	γ_{52}	0.00257	[0.39106]
γ_7	0.01**	[0.00538]	γ_{31}	-0.00522	[0.14029]	γ_{42}	-0.00082	[0.81475]			
γ_8	-0.00417	[0.23450]	γ_{32}	-0.00314	[0.37132]	γ_{43}	-0.00063	[0.85708]			
γ_9	0.00031	[0.93014]	γ_{33}	-0.00211	[0.54722]	γ_{44}	0.00077	[0.82458]			
γ_{10}	0.00727**	[0.04051]									
γ_{11}	0.00526	[0.13561]									

**significant at 5% level *significant at 10% level

Table 6(a). Estimated Performance of ARIMA Model for BTC

Model	ARIMA	Test Of		Homoscedasticity	AIC	BIC	MSE	MAPE
		Normality	Autocorrelation					
Model 1	ARIMA (0,1,0)	/	/	x	-528.648	-521.1365	107.4983	1.0364
Model 2	ARIMA (1,1,1)	/	/	/	-526.7017	-511.6914	107.7232	1.0374
Model 3	ARIMA (1,1,2)	/	/	/	-524.7899	-506.027	107.6949	1.0373
Model 4	ARIMA (2,1,1)	/	/	/	-525.2153	-506.4683	107.6062	1.0369
Model 5	ARIMA (2,1,2)	/	/	x	-533.1596	-510.6632	107.5465	1.0366
Model 6	SARIMA (1,1,1) (0,1,1) ₃₂	/	/	x	-521.7443	-506.734	115.8899	1.0763
Model 7	SARIMA (1,1,2) (0,1,1) ₃₂	/	/	/	-524.2325	-501.717	107.7164	1.0374
Model 8	SARIMA (1,1,2) (1,1,1) ₃₂	/	/	/	-550.0311	-528.5982	115.8899	1.0763
Model 9	SARIMA (2,1,2) (1,1,1) ₃₂	/	/	x	-557.1173	-532.1389	115.8899	1.0763
Model 10	SARIMA (2,1,3) (1,1,1) ₃₂	/	/	/	-555.8756	-527.3288	115.8899	1.0763

Table 6(b). Estimated Performance of ARIMA Model for ETH

Model	ARIMA	Test Of		Homoscedasticity	AIC	BIC	MSE	MAPE
		Normality	Autocorrelation					
Model 1	ARIMA (0,1,0)	/	/	/	-163.4831	-157.2352	59.5716	0.7715
Model 2	ARIMA (1,1,1)	/	/	/	-163.6468	-151.1748	62.2521	0.7884
Model 3	ARIMA (1,1,2)	/	/	/	-162.7202	-147.1302	59.5424	0.7713
Model 4	ARIMA (2,1,1)	/	/	/	-161.4928	-145.9329	59.5787	0.7715
Model 5	ARIMA (2,1,2)	/	/	/	-167.5775	-148.9056	59.0375	0.7680
Model 6	SARIMA (1,1,1) (0,1,1) ₃₂	/	/	/	-163.8361	-151.3642	63.1190	0.7938
Model 7	SARIMA (1,1,2) (0,1,1) ₃₂	/	/	/	-163.0837	-147.4937	63.1190	0.7938
Model 8	SARIMA (1,1,2) (1,1,1) ₃₂	/	/	/	-212.3626	-195.893	63.1190	0.7938
Model 9	SARIMA (2,1,2) (1,1,1) ₃₂	/	/	/	-210.6243	-191.471	63.1190	0.7938
Model 10	SARIMA (2,1,3) (1,1,1) ₃₂	/	/	/	-210.5211	-188.6315	63.1190	0.7938

Table 6(c). Estimated Performance of ARIMA Model for BNB

Model	Test Of Normality	Test Of Autocorrelation	Test Of Homoscedasticity	AIC	BIC	MSE	MAPE
Model 1	/	/	/	-133.2964	-127.0604	29.7914	0.5448
Model 2	/	/	/	-140.6141	-128.1662	34.5886	0.5869
Model 3	/	/	/	-132.4938	-116.9339	29.8038	0.5449
Model 4	/	/	/	-131.9564	-116.4267	30.0930	0.5476
Model 5	/	/	/	-130.9111	-112.2754	29.6105	0.5431
Model 6	/	/	/	-139.6745	-127.2266	35.7062	0.5962
Model 7	/	/	/	-131.1777	-115.6178	35.7062	0.5962
Model 8	/	/	/	-188.696	-172.2789	35.7062	0.5962
Model 9	/	/	/	-186.7118	-167.6201	35.7062	0.5962
Model 10	/	/	/	-185.1326	-163.3135	35.7062	0.5962

Table 6(d). Estimated Performance of ARIMA Model for USDC

Model	Test Of Normality	Test Of Autocorrelation	Test Of Homoscedasticity	AIC	BIC	MSE	MAPE
Model 1	/	x	x	-1055.5458	-1050.0386	1.0395e-06	9.0587e-05
Model 2	/	/	x	-1083.895	-1072.9153	1.2331e-06	9.8197e-05
Model 3	/	/	x	-1082.5956	-1068.8709	1.1633e-06	9.5618e-05
Model 4	/	/	x	-1083.2483	-1069.5673	2.1022e-06	1.2945e-04
Model 5	/	/	x	-1081.4185	-1065.0013	2.1208e-06	1.3006e-04
Model 6	/	/	x	-1081.8503	-1070.8706	9.7442e-08	1.9598e-05
Model 7	/	/	x	-1103.6051	-1092.8894	9.7442e-08	1.9598e-05
Model 8	/	/	x	-1111.5999	-1098.7411	9.7442e-08	1.9598e-05
Model 9	/	/	x	-1110.9432	-1093.9261	4.4894e-07	5.7815e-05
Model 10	/	/	x	-1149.6494	-1130.5052	9.9437e-08	1.9731e-05

Table 6(e). Estimated Performance of ARIMA Model for USDT

Model	Test Of Normality	Test Of Autocorrelation	Test Of Homoscedasticity	AIC	BIC	MSE	MAPE
Model 1	/	x	x	-1245.2929	-1239.0449	6.6769e-06	2.3439e-04
Model 2	/	/	x	-1281.3184	-1268.8464	7.0413e-06	2.4071e-04
Model 3	/	/	x	-1279.3667	-1263.7767	7.0376e-06	2.4067e-04
Model 4	/	/	x	-1280.7245	-1265.1646	1.0262e-06	7.9167e-05
Model 5	/	/	x	-1277.3667	-1258.6948	7.0376e-06	2.4067e-04
Model 6	/	/	x	-1279.3832	-1263.7932	6.7780e-06	2.3620e-04
Model 7	/	/	x	-1293.951	-1275.243	1.7438e-06	1.1339e-04
Model 8	/	/	x	-1293.139	-1271.3551	1.0244e-05	2.9159e-04
Model 9	/	/	x	-1273.3608	-1251.4712	7.0571e-06	2.4101e-04
Model 10	/	/	x	-1325.5072	-1300.8814	5.1151e-06	1.9759e-04

Tables 6 (a) until (e) show ten possible estimated benchmark models, divided into five models from ARIMA and another five for SARIMA models. All the models were identified through the Econometric Application Modeller in MATLAB, and then predictive values were generated using ARIMA coding in MATLAB version 2022 software packaging. All the models met the requirements for the residual assumption. For BTC, four models out of ten violated the assumption of homoscedasticity. Meanwhile, in ETH and BNB, all the ARIMA and SARIMA models meet the residual assumptions. Again, for USDT and USDC, none of the models met all residual assumptions. In other words, all the ARIMA and SARIMA models violated the assumption of homoscedasticity. It might be the cause of the volatile behavior, which still exists in error. The result of the model in detail can be seen in Tables 6 (a) until (e).

Table 7: Best Model of Top Five Cryptocurrencies

Cryptocurrency	Model	MSE	RMSE	MAPE
BTC	Local Level with Drift + Deterministic Seasonal	0.1336	0.3655	2.8225
	ARIMA (1,1,1)	107.7233	10.3790	103.7435
ETH	Local Level + Deterministic Seasonal	0.9345	0.9667	10.6447
	ARIMA (2,1,2)	59.0375	7.6836	76.80
USDT	Local Level + Deterministic Seasonal	9.4176e-06	0.0031	2200.0738
	ARIMA (2,1,1)	1.0263e-06	0.0010	195.883
BNB	Local Level with Drift + Deterministic Seasonal	2.7106	0.0521	26.5955
	ARIMA (2,1,2)	29.6105	5.4416	54.3135
USDC	Local Level with Drift + Deterministic Seasonal	7.9632e-05	0.0089	204.9413
	ARIMA (1,1,1)	1.2331e-06	0.00275	9.8197e-05

Subsequently, once the best STS and ARIMA models are determined, these models will be utilized to forecast the top five cryptocurrencies' prices. Table 7 demonstrates the best ARIMA and linear STS model for each cryptocurrency. These two models were utilized to forecast out of the sample in determining the best model performance. The comparison of the model in determining the best model prediction of the top five cryptocurrencies' prices is based on the minimum measurement of error. For BTC and BNB, the best model is local level with drift deterministic seasonal where the minimum mean square error is 0.1336 and 2.7106, respectively. Correspondingly, the best model for ETH is local-level deterministic seasonal with a mean square error (MSE) is 0.9345 compared to ARIMA (2,1,2) with 59.0375. Subsequently, USDT indicated the ARIMA (2,1,1) as a performing model in predicting closing price with a little bit different on the decimal of MSEs of the STS model. Other than that, for USDC, the ARIMA (1,1,1) is the best model with the lowest error compared to the Local Level with Drift Deterministic Seasonal. From that outcome, there are significantly different errors (MSE, MAE, and MAPE) between the structural time series and the ARIMA models. The three cryptocurrencies (BTC, ETH, and BNB) have the smallest error for the structural time series model compared to the ARIMA model. Therefore, it can be concluded that the structural time series is the

best model for nonstationary data. Meanwhile, the ARIMA model is the best option for stationary data, such as the USDT and USDC data series.

The top images of Figures 6 (a) to (e) depict a plot of the estimation part, while the bottom image is estimation + forecast horizon using the best model, respectively. From that figure, BTC has the smallest error difference between actual and forecast values. This indicates that the most suitable model for predicting BTC cryptocurrency price is the linear STS or Local Level with Drift + Deterministic Seasonal. Meanwhile, USDT and USDC also have the smallest gap between the real closing price and the forecast closing price. In other words, the ARIMA model is the best model when data behavior is stationary. However, these models are misleading because they violate the assumption of homoscedasticity; it can cause the prediction to be unreliable.

5 Conclusion and Future Work

Numerous studies have been developed to find the best-predicting cryptocurrency price model. Since cryptocurrency price has nonstationary data, it is hard to predict accurately. Traditionally, the estimation model eliminates the trend and seasonal patterns. In order to estimate the pattern of trend and seasonal variation, this study presented a linear STS model. Apart from that, when composing this component, the behavior of cryptocurrency price can be detected, indirectly having an exceptional impact on the prediction system.

From the result obtained, the top five cryptocurrencies have a different model of the trend, either deterministic or stochastic, which relies on the behavior of data. Most of the top five cryptocurrencies are affected by the crypto winter, where the trend is downward after six months every year. Apart from that, the linear STS is the best model for predicting three cryptocurrencies' prices which are Bitcoin (BTC), Ethereum (ETH), and Binance (BNB). The other two cryptocurrencies can be handled by the autoregressive integrated moving average (ARIMA) model. When the data is stationary, the ARIMA model is the most accurate prediction algorithm for cryptocurrency values. Though, the best final estimated ARIMA model for USDT and USDC does not meet the assumption of homoscedasticity. Even though the behavior of data is stationary, it still has volatility. This indicates that the ARIMA model is inadequate for the data with volatility and nonstationary behaviors.

Meanwhile, a linear STS model can be considered the best to handle nonstationary and easily interpret the hidden component. Hence, it can retain the information of data. Note that the result is different from the ARIMA model. However, the best model of the univariate model of linear STS does not meet the assumptions of normality. This means that the linear STS model can handle volatility easily but not for asymmetric data. It might be the cause or effect of the outliers or structural breaks that do not account for in the model. If the study does not handle this problem properly, it causes the model to be misspecified.

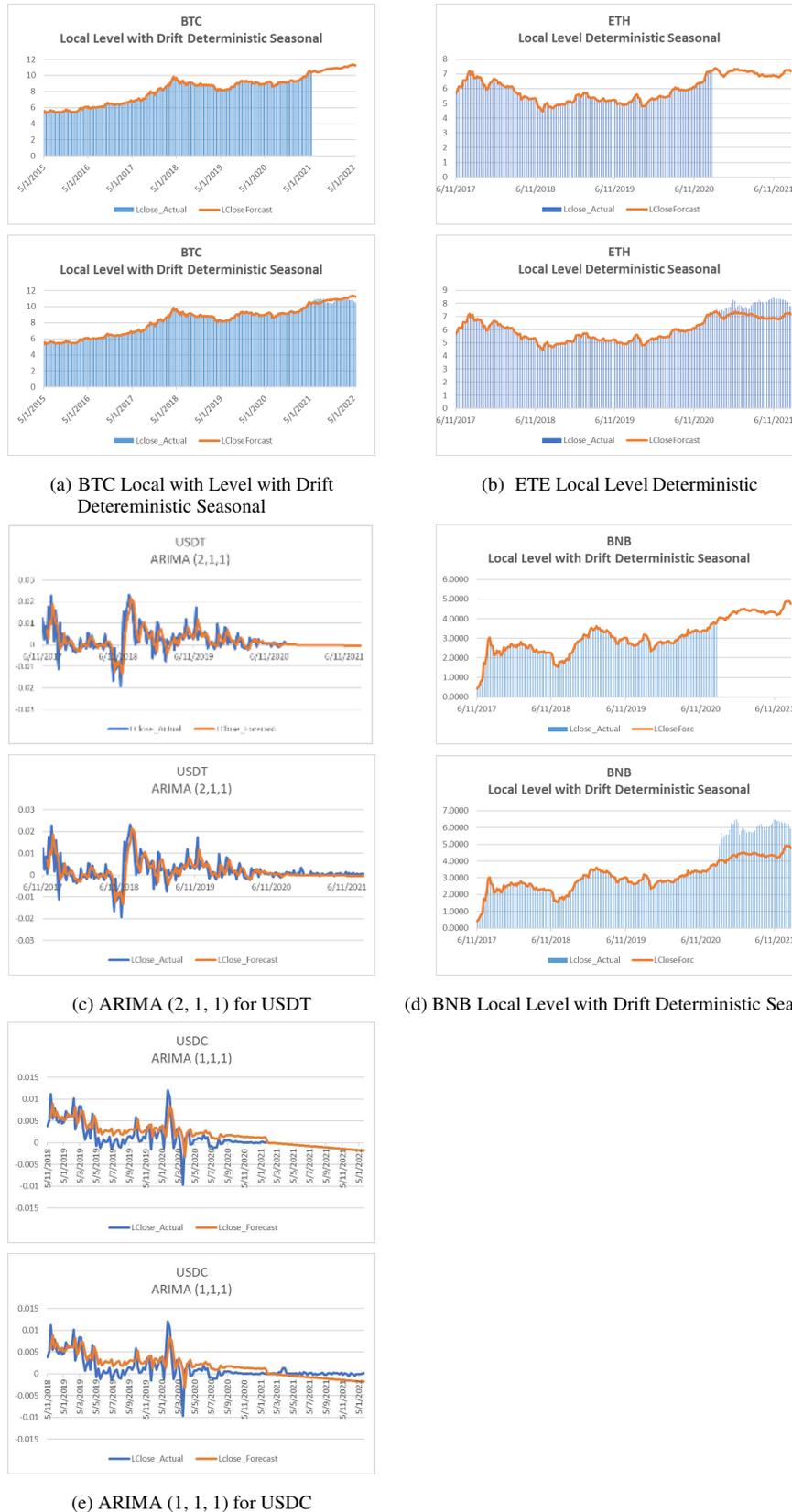


Figure 6: The Best Model (Actual vs Forecast)

Therefore, this study will investigate the structural break and outlier in the linear STS model. Consequently, we attempt to account for the explanatory variables contributing to cryptocurrency price instead of analyzing the univariate itself. Other than that, this study will propose a novel hybrid linear STS with machine learning and a nonlinear model that might assist in encountering the linearity problem and capture multiple characteristics of data. It will indirectly assist the analysis in giving information and accurately predicting cryptocurrency prices.

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