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The Market Price of Risks in The Shipping Freight Market

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Abstract This paper aims to estimate the market price of risks from the relationship between the spot and forward price dynamics in the shipping freight market. We employ three different stochastic models, which are analytically tractable and enable for pricing of forwards. Subsequently, we examine the forward curve performance. By doing that in such a way, we offer a method for estimating the market price of risk and the market price of volatility risk by adjusting theoretical prices to today's observed forward curve. The findings of this study are essential for minimizing the difference of price gap between the theoretical and actual forward prices.

Keywords Freight markets; forward curve; spot and forward price; market price of risks; stochastic volatility; derivatives pricing.

Mathematics Subject Classification 60G51, 60H20, 62M10, 91B24, 91B70.

1 Introduction

According to Bernaschi *et al.* [1], the market price of risk is a measurement of the level of the market's risk aversion. The greater it is, the more compensation (measured by the excess expected rate of return) the market demands in exchange for accepting risks (measured in terms of the standard deviation of return). Hence, the ratio of expected return above the risk-free interest rate to the standard deviation of returns is used to measure a market price of risk for a stock. Interestingly, leverage does not affect this quantity. When someone borrows at the risk-free rate to invest in a risky asset, the expected return and risk both rise, but the market price of risk remains unchanged. This ratio is also known as the Sharpe ratio when suitably annualized (see Kolos and Ronn [2] as well Bailey and López de Prado [3]).

Hitherto, there have been plenty of trials in the literature to investigate the market price of risk, possibly because the market price of risk has a pivotal role in determining the pricing measure. Examples of this literature are Lintner [4], Schwartz [5], Lucia and Schwartz [6], Cartea and Figueroa [7], Bernaschi *et al.* [1], Rhee and Kim [8], Kolos and Ronn [2], Cartea and Williams [9], Benth *et al.* [10], Weron [11] and Bhar *et al.* [12]. To mention a few, Bernaschi *et al.* [1] describe a straightforward yet efficient technique for estimating the market price of risk.

They contribute an idea to compare the findings obtained using two alternative strategies to the Cox-Ingersoll-Ross (CIR) model calibration. Surprisingly, this CIR model can offer a reasonable approximation of the market price of risk, given these techniques enable the isolation of the market price of risk and the evaluation (under the Local Expectations Hypothesis) of the risk premium offered by the market for various maturities. Further, a study done by Kolos and Ronn [2] proposed that spot and futures prices be used to assess the market price of risk for energy commodities while accounting for the Samuelson effect. They found that the long-term market price of risk is typically positive, and the short-term market price of risk is generally negative, which is compatible with positive energy betas and hedging, respectively.

Since the introduction of the market price of risk, there have also been endeavours to bring about the market price of volatility risk. To the best of our knowledge, there is a vast body of literature on stochastic volatility models; nevertheless, only minimal information on the market price of volatility risk. Based on studies by Boswijk [13] and Negrea [14], Mohtar and Taib [15] defined the market price of volatility risk as a function of two variables: the price of the underlying asset and its volatility. Both reflect the risk preferences of agents in the market, which is fully used to determine the choice of relevant equivalent martingale measures for pricing derivatives. Therefore, the market price of volatility risk is crucial since it may be used to rate a market's appetite for risk. In the presence of stochastic volatility, Bakshi and Kapadia [16] found that a delta-hedged portfolio's underperformance was related to a negative market price of volatility risk. There has also been the endeavour to demonstrate the presence of a negative market price of volatility risk in commodities markets. Doran and Ronn [17] employed New York Mercantile Exchange options and futures data to illustrate that the divergence between the realized and implied volatility was directly linked to the presence of a negative market price of volatility risk in energy markets (i.e., heating oil, natural gas and crude oil).

Yet, since freight services are essentially non-storable, the market for freight rates is unlike the market for other commodities such as agriculture, crude oil or natural gas. Notwithstanding, the nonstorability feature makes the shipping freight market more analogous to energy markets, exemplifying the electricity and temperature market due to temperature cannot be traded, and the electricity must be used once generated. Thus, users in the freight market, including the market for temperature and electricity, typically execute a forward (or futures) contract associated with a particular delivery time to ensure that the commodity can be carried at such time. To this end, Taib [18] studies the pricing of the forward freight contracts under the spot-forward relationship structure by inferring forward prices from six distinct continuous stochastic models of spot freight rates proposed by Benth *et al.* [19]. The author also introduces a shift of measure from physical probability to risk-neutral measure utilizing the Esscher transform. As a result, the market price of risk is included in the formula of forward, and the market price of volatility risk is assimilated into models with stochastic volatility. Even so, the Esscher transform is nothing more than a traditional Girsanov transform if such a case is related to Brownian motion. As an extension of that study, a recent paper by Mohtar and Taib [15] then reviewed the pricing of the freight options rate under the spot-forward-option relationship structure.

We frequently seek to describe quantities under derivatives theory as stochastic, that is, random. The risk stems from such randomness, which compels us to examine how to value the risk, namely how much return we should expect through taking a risk. To price derivatives, one needs to consider the risk preferences of the investors, or in other words, the *risk premium*[1] that is discernible in the market. Traditionally, this is described by the spot-forward relation through a market price of risk and the market price of volatility risk charged for issuing the derivative, which is essentially the spec of a risk-neutral probability. By utilizing the Esscher transform to select the risk-neutral probabilities, we shall derive the equivalent martingale measures providing us with time-dependent parameters (i.e., the market price of risk and the market price of volatility risk). These parameters are also closely linked to the risk premium (refer to Benth *et al.* [20]). For example, a negative market price of risk (or market price of volatility risk) is equivalent to the negative risk premium in normal backwardation. Nonetheless, the sign of the market price of risk for power commodities may shift depending on the time duration in consideration (see Benth *et al.* [10]).

Motivated by the prior discussion, we calibrate the market price of risk and the market price of volatility risk from the relationship between the spot and forward (or futures) price dynamics in the shipping freight market. We shall look at how theoretical pricing may be fitted to today's observed forward curve to find the market price of risk and volatility risk. In many circumstances, this can be done perfectly; however, an approximation of the observed forward curve would be sought under the most realistic situations. We analyse data on the Panamax index, which is route averages for that shipping market segment. Here, we follow the procedure structured in Mohtar and Taib [15] for the stochastic dynamics of spot price modelling. The models are then analytically tractable, allowing for pricing forward.

Demand and supply, which differ seasonally in the forward pricing contracts, also affect the form of the forward curve under seasonally dependent commodities (the reader may refer to Borovkova and Geman [21] for a detailed explanation of seasonality behaviour in the forward curve). For example, the temperature market is seasonally dependent, while the electricity market is highly demanded all along the winter season for heating as opposed to summer. Even so, the inability of the supply side to act expeditiously to fulfil demand distinguishes the freight market from other seasonal-dependent markets. Based on the rejection of the existence of stochastic seasonality in freight rates (Kavussanov and Alizadeh [22]) and the absence of (deterministic) seasonality in the dry bulk freight time series (Benth *et al.* [19]), we agree with Taib [18] to disregard seasonality behaviour in our forward pricing since the discoveries of seasonality are mixed.

The discoveries of this paper are presented as follows. We start by introducing the stochastic dynamics of spot price that shall be employed in the pricing of forwards, simultaneously by investigating the empirical data that engage such dynamics in Section 2. Next, we put forward the analytic equations of the forward price in Section 3, where the forward curve's shapes for different stochastic models are also described. Section 4 discusses the forward curve performance of the non-stationary one-factor models. The market price of risk and market price of volatility risk is enumerated through the implementation of the Esscher transform in Section 5. Subsequently, we offer a method for adjusting those parameters to today's forward curve. Finally, Section 6 concludes the paper. All computations in this paper were carried out in Matlab version R2017b.

[1]The risk premium is the compensation for keeping a risky investment rather than a risk-free one, according to Weron [11]. More precisely, risk premium represents the additional return above the risk-free rate that an investor needs to compensate for a particular investment's risk. Put another way, the higher the return required by the investor, the riskier the investment.

2 Empirical Analysis of The Spot Freight Data

This study analyses a freight rates series from the dry bulk market segment. The Panamax index is four daily Panamax time charter rates on average. This index is formally denoted as PM4TC and quoted in USD/day[2]. We shall call this the Panamax index for the rest of the study. We also have obtained data for this index from the *Shipping Intelligence Network*[3]. The data for the Panamax index is included over the period ranging from May 6, 1998, to September 3, 2020. There about 5,587 records covering 22 years of daily observations data are observed. Nonetheless, weekends and holidays are not included in the data set. Figure 1 shows the time series plot for these spot freight rates and the corresponding logreturns.

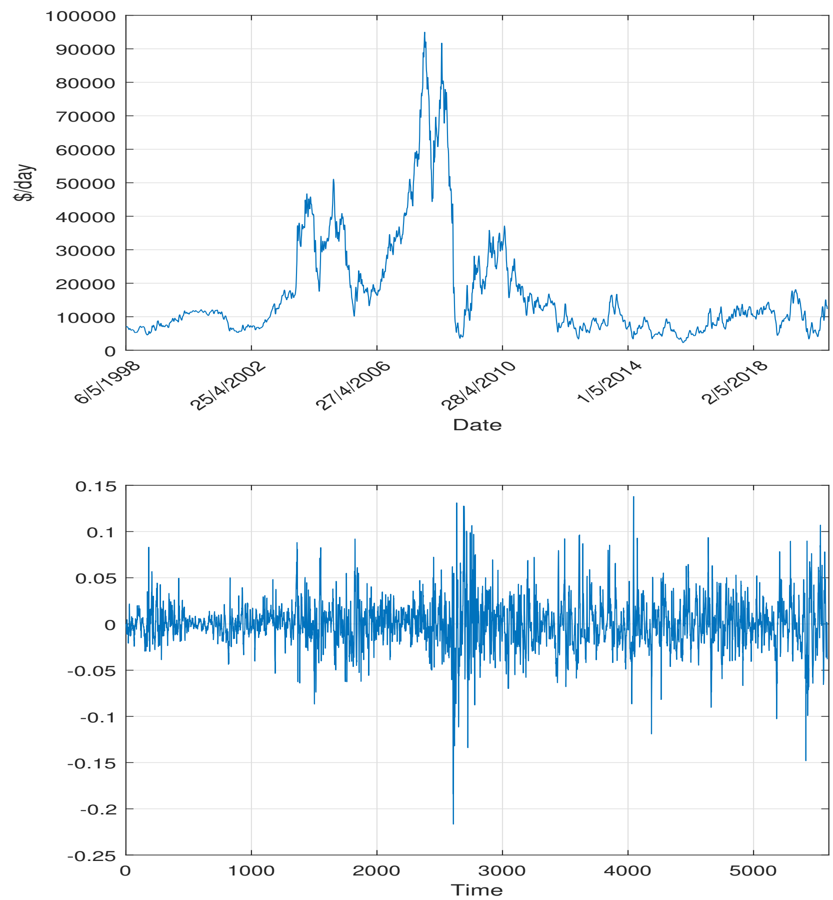

Figure 1: Daily spot freight rates (top) and time series of the logreturns (bottom) for the Panamax index.

[2]A detailed index description is provided at www.balticexchange.com on the different routes and an overview of panellists reporting daily quotes.
[3]The report was retrieved from https://sin.clarksons.net.

2.1 Geometric Brownian motion

Let $B(t)$ be a Brownian motion defined on a complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, P)$ under the real-world probability measure P and suppose $S(t)$ represents the spot price at time $t \geqslant 0$. The geometric Brownian motion (GBM for short) model describes the spot using the dynamics

$$\mathrm{d}S(t) = \mu S(t)\mathrm{d}t + \sigma S(t)\mathrm{d}B(t), \qquad \forall \mu, \sigma > 0, \tag{1}$$

where the constant μ and σ are the drift and volatility, respectively. Solving equation (1) for $0 \leqslant t \leqslant T$ gives

$$S(T) = S(t)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma\int_t^T \mathrm{d}B(u)\right). \tag{2}$$

Since we fit our freight rate data to the GBM, Figure 1 also exhibits the graph plot of corresponding logreturns, defined as the logarithm of the ratio between two consecutive prices. In Table 1, we present the descriptive statistics of such an index. The mean daily value for the Panamax spot rates is 17,066.60, and the standard deviation is 15,894.42. The average value for the logreturns is 0.0001, corresponding to 2.52% in annual logreturn[4]. In annual terms, the Panamax logreturns have a volatility of 38.89% based on the value of the standard deviation of 0.0245. Since the positive skewness indicates that the Panamax spot rates skewed to the right, unfortunately, the Panamax logreturns show the opposite, whereby it is slightly left-skewed.

Table 1: Descriptive statistics for the Panamax index.

	Min	Max	Mean	Standard Deviation	Skewness	Kurtosis
Spot rates	2,260.00	94,977.00	17,066.60	15,894.42	2.2280	8.2562
Logreturns	-0.2166	0.1379	0.0001	0.0245	-0.0711	8.3071

The logreturns, in this model, are assumed to be independent and normally distributed. Figure 2 exhibits the empirical distribution of freight rate logreturns, illustrating the mass concentration at the centre of the distribution. Furthermore, the tails are heavier compared to normal. Then, based on normality tests of the logreturns, the Kolmogorov-Smirnov test formally rejects the hypothesis, with a statistic of 0.4652 for the Panamax Index (the series are significant at the 1% level) and eventually, the GBM is not suitable spot model.

2.2 Lévy-based dynamics

Rather than utilizing GBM for modelling the spot price, we might use an exponential Lévy model, which is a straightforward generalization of a Brownian motion to the Lévy process. The normal inverse Gaussian (NIG) Lévy process is a possible nomination for explaining the spot. For $t \geqslant 0$, the spot price $S(t)$ is defined by

$$S(t) = S(0)\exp(L(t)) \tag{3}$$

as an exponential Lévy process. We define $L(t)$ as a stochastic process with NIG distributed increments, the so-called NIG Lévy process, and because of that, equation (3) represents NIG

[4]We use the convention of 252 trading days in a financial year.

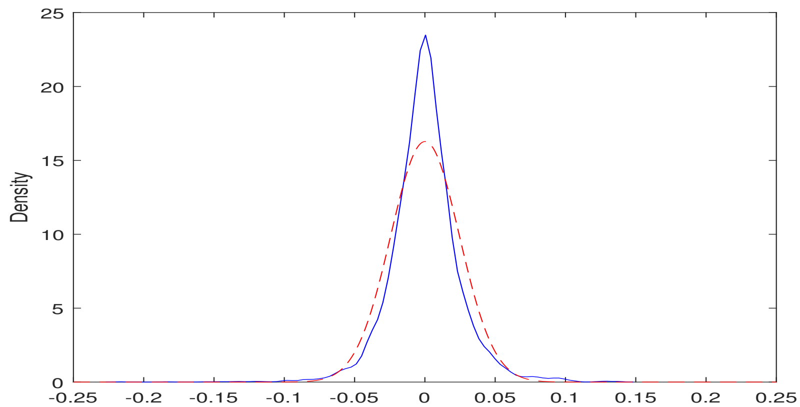

Figure 2: Density plot of the empirical (complete line) with fitted normal distributions (dashed line) for the logreturns of the Panamax spot freight rates.

Lévy model. We describe the NIG distribution briefly below, and further information may be found in Benth *et al.* [20].

Let $\mathrm{NIG}(\alpha, \beta, \delta, \mu)$ denotes the NIG with four parameters is a generalized hyperbolic distributions class with density functions,

$$f(x|\alpha, \beta, \delta, \mu) = c \exp(\beta(x - \mu)) \frac{K_1(\alpha\sqrt{\delta^2 + (x - \mu)^2})}{\sqrt{\delta^2 + (x - \mu)^2}}, \tag{4}$$

in which $c = \delta\alpha \exp(\delta\sqrt{\alpha^2 - \beta^2})/\pi$ and $K_1(x)$ is the modified Bessel function of the third kind of order 1 evaluated at x. Then, we may label $L(t)$ to be NIG Lévy process when $L(1)$ is distributed based on the NIG distribution with $(\alpha, \beta, \delta, \mu)$ parameters. Here, $\alpha \geqslant |\beta|$ measures the tail heaviness of such distribution, $\beta \in \mathbb{R}$ is the skewness parameter, $\delta > 0$ is the scale parameter, and lastly, $\mu \in \mathbb{R}$ is the density location. The Lévy measure of $L(t)$ can be stated as (refer to Barndorff-Nielsen and Shephard [23])

$$\ell(\mathrm{d}z) = \frac{\alpha\delta}{\pi|z|} e^{\beta z} K_1(\alpha|z|)\mathrm{d}z, \tag{5}$$

and the cumulant function as shown by (refer to Benth and Šaltytė Benth [24])

$$\psi(\lambda) = i\lambda\mu + \delta\left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + i\lambda)^2}\right). \tag{6}$$

Figure 3 and the study by Mohtar and Taib [15] indicate that normal inverse Gaussian fitted the logreturns of the freight rates effectively and achieved the high peak in the centre. We also put forward the density plot with a log-scale on the frequency axis, highlighting how the tails are captured by NIG.

Table 2 reports the parameters estimated using the maximum likelihood for the normal inverse Gaussian distribution. The logreturns of the Panamax index are nearly 0.0004, while the negative value of $\hat{\beta}$ reflects that the logreturns left-skewness. The third parameter,

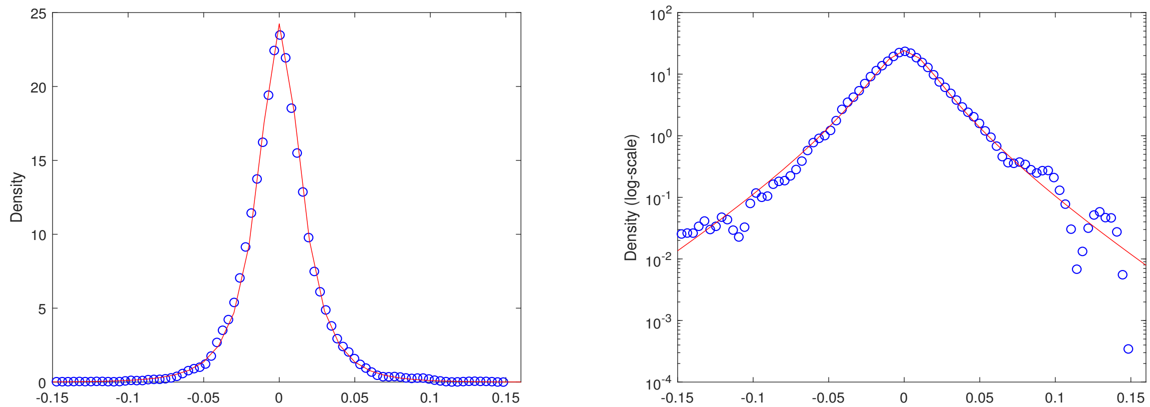

Figure 3: Density plot (left) and density plot with logarithmic frequency axis (right) of the empirical (bullet marker) together with the NIG distributions (complete line) for the logreturns of the Panamax spot freight rates.

$\hat{\delta}$, corresponds to the normal inverse Gaussian distribution scale. Small values narrow the distribution, making the larger ones wider. Parameter $\hat{\delta}$ plays the same role as the standard deviation, σ (or volatility), in the normal distribution. The value of the tail heaviness parameter $\hat{\alpha}$ is 30.7049.

Table 2: Parameter estimates of the logreturns for the NIG distribution.

$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\mu}$
30.7049	-0.5472	0.0184	0.0004

According to Mohtar and Taib [15], the empirical logreturns' time series of freight rates indicate volatility clustering. This factuality, along with the presence of heavy-tailed logreturns, reveals the existence of stochastic volatility in the dynamics. Besides, empirical studies by Benth *et al.* [20] and Benth [25] demonstrate that using a model of stochastic volatility might capture various stylized facts of empirical logreturns data, and Benth [25] proposed to employ the famous stochastic volatility model, namely Barndorff-Nielsen and Shephard (BNS) model. Following those studies, we represent the time-varying volatility process using the BNS model.

2.3 Barndoff-Nielsen and Shephard stochastic volatility model

Suppose $X(t) = \ln S(t)$ is the stochastic differential equation solution,

$$\mathrm{d}X(t) = (\mu + \beta\sigma^2(t))\mathrm{d}t + \sigma(t)\mathrm{d}B(t), \tag{7}$$

whereby $B(t)$ is the standard Brownian motion. Let $\sigma^2(t)$ be a stationary process that depends on a weighted sum of the processes $Y_i(t)$ given by

$$\sigma^2(t) = \sum_{i=1}^{n} \omega_i Y_i(t), \qquad \omega_i \in [0, 1]. \tag{8}$$

The OrnsteinUhlenbeck process $Y_i(t)$ takes the form

$$\mathrm{d}Y_i(t) = -\lambda_i Y_i(t)\mathrm{d}t + \mathrm{d}U_i(\lambda_i t), \qquad i = 1, 2, \ldots, n, \tag{9}$$

whereby λ_i is strictly positive constant, the so-called speed of mean reversion of the volatility process. The $U(\lambda t)$ process, also known as subordinator, has only positive increments and no drift to ensure the positivity of variable $Y(t)$. We make $Y(t)$ obey the inverse Gaussian law; thus, the increments of equation (7) would more or less be NIG distributed. For $0 \leqslant t \leqslant T$, we may rewrite equation (7) to be

$$S(T) = S(t) \exp\left(\mu(T-t) + \beta \int_t^T \sigma^2(u)\mathrm{d}u + \int_t^T \sigma(u)\mathrm{d}B(u)\right). \tag{10}$$

As in Mohtar and Taib [15], fitting the empirical autocorrelation function (ACF) for squared logreturns with a single exponential function is challenging. Because of the combination of rapidly dwindling for small-order lags and, afterwards, gradually diminishing ACF for bigger-order lags, they suggest leastwise two exponential functions employed for calibrating the BNS model over this empirical ACF. The fitted ACF for the squared logreturns of the Panamax index with one and two exponential functions is shown in Figure 4. Table 3 reports the estimated λs, where λ_i is a positive constant in the Ornstein-Uhlenbeck process dynamics. The weights $\omega_i \in [0, 1]$ for $i = 1, 2, \ldots, n$ summed to 1. Here, the non-linear least squares method is employed to calibrate the exponential functions for both cases. Notice that the speed of mean reversion is confined to be positive for the stationary volatility process, $\sigma^2(t)$. Fitting two exponential functions procedure over Panamax index data was oversensitive to lag selection. We attained positive results for the fitted λs by selecting the number of lags until 50, which seems to be a sensible calibration of the data. Readers may refer to Mohtar and Taib [15] for a detailed explanation.

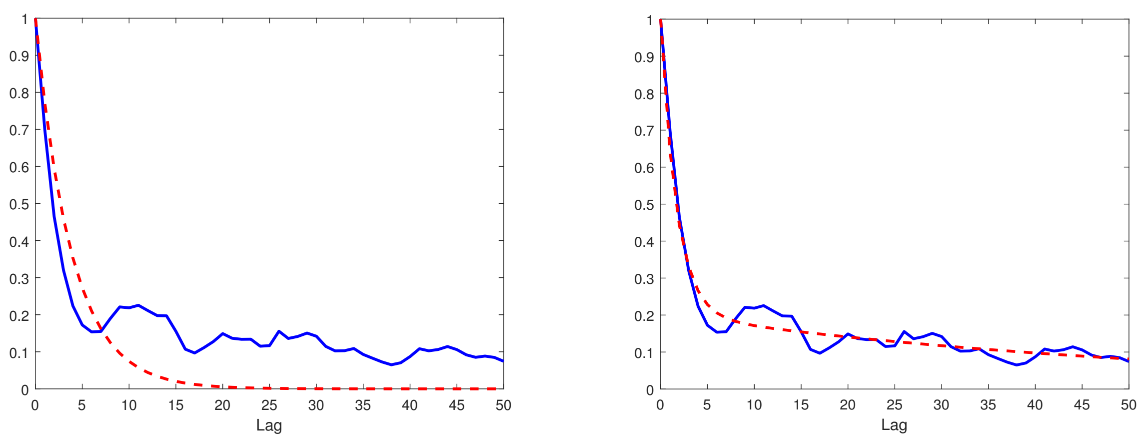

Figure 4: The autocorrelation function of the squared logreturns (complete line) of the Panamax spot freight rates, along with the fitted exponential function (dashed line) on the left, while the two exponential functions on the right.

Table 3: The estimated constant λ with the fitted ACF.

One exponential function	Two exponential functions			
$\hat{\lambda}$	$\hat{\omega}_1$	$\hat{\omega}_2$	$\hat{\lambda}_1$	$\hat{\lambda}_2$
0.2602	0.7967	0.2033	0.5850	0.0184

3 Forward Prices Formulas

A forward contract's price will provide insight into future spot price behaviour. It will also be evident for the underlying spot price and the stochastic volatility state. To this end, we introduce forward prices based on our proposed models, as denoted in the prior section.

Let us introduce the arbitrage-free price of a forward contract $F(t,T)$ at time t with delivery time T whereby $0 \leqslant t \leqslant T < \infty$. This contract settled against the spot price of freight at $t \geqslant 0$, as $S(t)$. Referring to Mohtar and Taib [15] and Taib [18], we shall consider the following propositions.

Proposition 1 *The forward contract price at time t with delivery time $0 \leqslant t \leqslant T$ for the geometric Brownian motion model is given by*

$$F(t,T) = S(t)\exp(\lambda(T-t)), \tag{11}$$

whereby $\lambda = \mu + \sigma\theta$ with μ is the constant of drift and σ is the constant of volatility. Here, θ is a real-valued function that relies on time and has often interpret as the market price of risk.

Proposition 2 *The forward contract price at time t with delivery time $0 \leqslant t \leqslant T$ for the NIG Lévy model is defined by*

$$F(t,T) = S(t)\exp(\Lambda(T-t)), \tag{12}$$

whereby $\Lambda = \phi_L(\theta_L + 1) - \phi_L(\theta_L)$ with ϕ_L is the logarithm of the moment generating function under Lévy process L (also known as the cumulant function). The constant θ_L has regarded as the market price of risk.

Proposition 3 *The forward contract price at time t with delivery time $0 \leqslant t \leqslant T$ for the BNS stochastic volatility model is derived by*

$$\begin{aligned} F(t,T) = S(t)\exp\left((\mu+\theta)(T-t) + \sum_{j=1}^{n}\Psi(T-t)Y_j(t)\right) \\ \times \exp\left(\sum_{j=1}^{n}\int_t^T \{\phi_U(\Psi(T-v)+\theta_V) - \phi_U(\theta_V)\}\,dv\right), \end{aligned} \tag{13}$$

whereby $\Psi(\varphi) = \frac{\omega_j}{\lambda_j}(\beta+0.5)(1-\exp(-\lambda_j\varphi))$, $Y_j(t)$ is the Ornstein-Uhlenbeck processes and the constant θ_V being the market price of volatility risk.

With this, we have provided the analytic equations of the forward price under the freight market employing three proposed spot models. In this paper, however, we put forward only the proposition of price modelling of freight forwards for each model. Readers may refer to Mohtar and Taib [15] or Taib [18] for detailed proofing of each.

Further, we shall briefly describe the shape of the *forward curve*[5]. The forward curve's form has generally set on by today's spot price, $S(t)$, together with the contribution of some constant, such as the risk premium, θ in the exponential, wherewith leads to a fixed shape. The risk premium has no impact other than scaling the original curve. Observe that in the GBM model, the forward curve will exponentially increase or decrease, known as *contango*[6] or *backwardation*[7], respectively, relying on the positivity or negativity of the λ value. Meanwhile, for the NIG Lévy spot model, integrands of cumulant function give another contribution to the forward price. We may directly calculate the integral employing the normal inverse Gaussian cumulant function as in equation (6).

Nevertheless, the forward price for BNS stochastic volatility spot model consists of the contribution of stochastic volatility, particularly as weighted by the parameter Ψ for the second expression of the equation (13). The last phrase of such an equation involves the integrands of the inverse Gaussian cumulant function, which is not stochastically varying. Notice that under the BNS stochastic volatility spot model, time-varying volatility $Y(t)$ is the most crucial part of the forward price. This $Y(t)$ that emerges in equation (13) also embraces the change of randomness in the price. Accordingly, one must first acquire the stochastic volatility present state to identify the forward shape.

4 Forward Curves

We analyse the performance of the models proposed in the previous section by comparing the actual forward prices from the *Baltic Exchange*[8] with the theoretical ones obtained from those models. Figure 5 presents the forward curves of the Baltic Forward Assessments (BFA) Panamax Time Charter at several given time points for different times of delivery. We can see that the market is in contango on the observed date, January 2, 2009, and being in backwardation on January 4, 2010. The forwards' prices on January 4, 2011, and January 3, 2012, stay relatively similar, indicating flat forward curves.

The samples of BFA Panamax Time Charter forward prices observed from January 2, 2009, to December 24, 2009, cover 2,500 data. For the sake of simplicity, we may refer forward prices as BFA Panamax. Table 4 presents the BFA Panamax data for January 2, 2009. Those values then being plotted in Figure 6, with the x-axis corresponding to the delivery time measured in days, while the BFA Panamax price on the y-axis measured in USD/day. We can spot that the forward prices become more expensive for the delivery period longer than the short one. This issue may be due to the uncertainty contributing to the price dynamics, making the price costly for the long-term contract.

Based on the prior section, the forward price formulas have been attained utilizing three

[5]The forward curve (also known as the future curve) reflects the relationship between the forward contract price and the time to maturity of the contract graphically.

[6]A market is said to be in contango if the forward (or future) contract price increases over time to the spot price as the delivery date approaches.

[7]In contrast with the contango, a market is to be backwardation if the forward (or future) contract price drops over time to the spot price when the delivery date approaches.

[8]See www.balticexchange.com for further information about the forward price data.

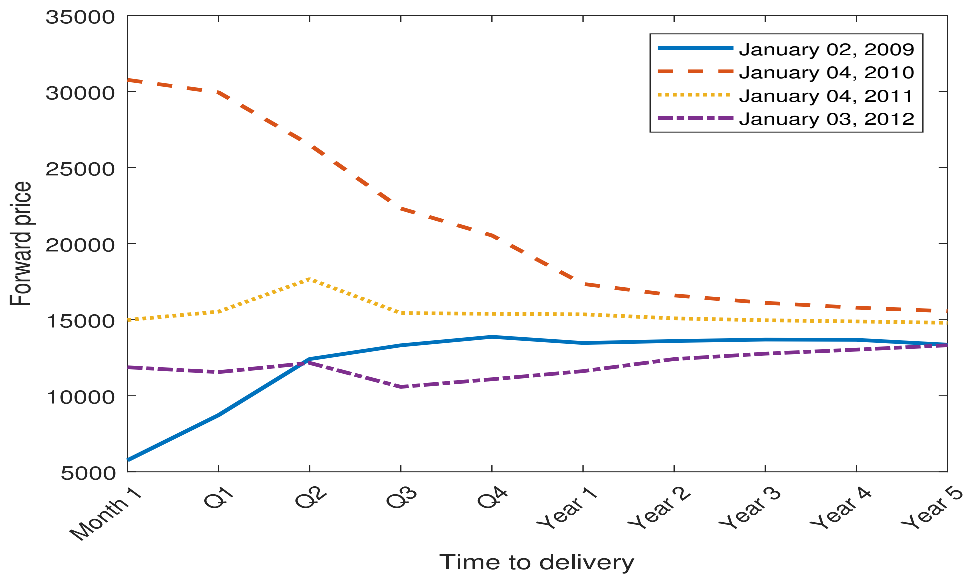

Figure 5: The slope of the forward curves for Panamax vessels with different delivery times.

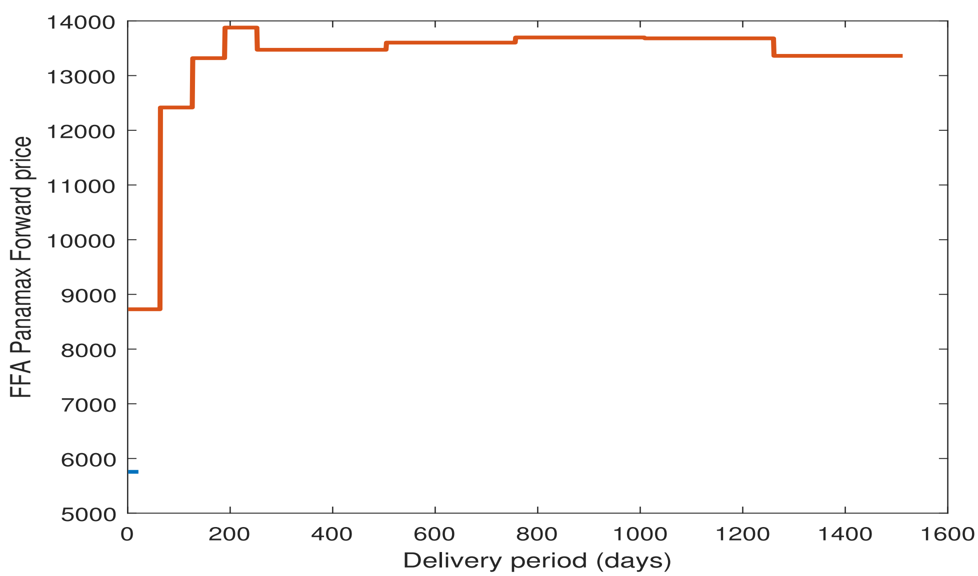

Figure 6: The BFA Panamax actual forward curve was observed on January 2, 2009.

Table 4: The BFA Panamax on January 2, 2009.

No. of contract	BFA Panamax	
	FFA maturity	FFA rate (USD/day)
1	January (Month 1)	5,756.00
2	Quarter 1 (Q1)	8,728.00
3	Quarter 2 (Q2)	12,417.00
4	Quarter 3 (Q3)	13,319.00
5	Quarter 4 (Q4)	13,878.00
6	Year 1	13,472.00
7	Year 2	13,603.00
8	Year 3	13,697.00
9	Year 4	13,681.00
10	Year 5	13,361.00

continuous-time stochastic models. Generally, the forward price at time t with delivery time T can be formulated as

$$F(t,T) = g(t, S(t), p, \theta), \tag{14}$$

where $S(t)$ is the spot price and θ is the market price of risk (or market price of volatility risk). The spot model parameters, such as the drift and the volatility, are grouped in p, including four parameters for NIG distribution and others, which have been empirically measured in Section 2. Still, θ is the unknown parameter.

5 The Market Price of Risks

Herewith, we investigate the values of $\hat{\theta}$, namely the estimated market price of risks (representing the market price of risk and market price of volatility risk) for the theoretical price to match the observed one. Referring to Benth and Šaltylė Benth [24] and Benth [25], this $\hat{\theta}$ can be found by adjusting theoretical prices to the observed forward curve, which also minimizes the gap between the two. We may now estimate θ as given by

$$\hat{\theta} := \min_{\theta} \sum_{i=1}^{N_t} \| \, F(t, T_{t,i}) - \hat{F}(t, T_{t,i}) \, \| \ , \tag{15}$$

whereby minimize against measurable functions θ being uniformly bounded by a constant for each delivery time i until N_t, the maximum delivery period at any particular day t. Here, $\hat{F}(t, T_{t,i})$ are the observed prices, also known as the actual forward prices quoted in the market. The gap (or distance) is calculated by $\| \cdot \|$, usually the Euclidean distance or some weighted version. This established technique has been demonstrated by Härdle and López Cabrera [26], in which the market price of risk for temperature forwards is studied.

The implied market price of risks for each day t is computed using *non-linear least squares regression*[9]. We started the calculation of $\hat{\theta}$ on January 2, 2009, for time charter contracts with

[9]The calculation was done using the `nlinfit`-command in Matlab.

delivery periods of one month, quarter-1, quarter-2, quarter-3, quarter-4, year-1, year-2, year-3, year-4 and year-5. Then we moved to January 3, 2009, and the same procedure has done. We do this for each business day until December 24, 2009, giving us a total of $t = 250$ days. The input data to value the market price of risks $\hat{\theta}$, such as the estimated parameters under the proposed spot models, are summarized in Table 5. Observe that herein the '$*$' refers to the estimated parameters for the logreturns, while '$\star$' is the corresponding value in annual terms.

Table 5: Summary of the parameters for the spot models.

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\omega}_1$	$\hat{\omega}_2$	$\hat{\lambda}_1$	$\hat{\lambda}_2$
GBM*	–	–	–	0.0001 (0.0243⋆)	0.0245 (0.3891⋆)	–	–	–	–	–
NIG Lévy*	30.7049	−0.5472	0.0184	0.0004	–	–	–	–	–	–
BNS*	30.7049	−0.5472	0.0184	0.0004	–	0.2602	0.7967	0.2033	0.5850	0.0184

Note: *indicates the estimated parameters for the logreturns; ⋆indicates the corresponding value in annual terms.

Let us compare the empirical $\hat{\theta}$ values obtained from GBM with the non-stationary Lévy-based dynamics and BNS stochastic volatility spot models. The estimated $\hat{\theta}$ and their corresponding density are plotted in Figure 7.

Referring to Benth *et al.* [10], one of the commodities markets' oddities is that the market price of risk may be either negative or positive, subject to the horizon of time considered. For instance, a study by Schwartz [5] reports that the one-factor model calibration towards futures prices of copper and oil experienced negativity for market prices of risk in both samples. Lucia and Schwartz [6] also find a negative market price of risk associated with the non-stationary term in their two-factor models when analysing data from the Nord Pool market. Then Cartea and Figueroa [7] also estimated a negative market price of risk under England and Wales wholesale electricity prices modelling. Further, Cartea and Williams [9] discovered that there is positivity on the market price of risk for long-term contracts under modelling gas prices and forward contracts. In contrast, for short-term contracts, the market price of risk changes signs across time despite being positive on average.

Besides, in their study, Bakshi and Kapadia [16] and Doran and Ronn [17] have also attempted to demonstrate a negative market price of volatility risk. In our case study, notice that the value of the estimated market price of risks, namely $\hat{\theta}$, decreases across time, as in Figure 7. However, the $\hat{\theta}$ value under NIG Lévy spot model is positive for the short term and turns negative in the long term. Meanwhile, for both GBM and BNS spot models, the empirical $\hat{\theta}$ has produced the negativity values for the entire duration.

Subsequently, Table 6 reports the results respectively, namely the actual forward prices of the BFA Panamax for each time to maturity of the contracts and the theoretical forward prices of each model obtained employing the $\hat{\theta}$ in the equations (11), (12) and (13). From the results, it can be observed herein that all the forward prices between GBM and NIG Lévy spot models under theoretical forward price show a low-lying deviation compared to BNS stochastic volatility spot model. A similar issue is also discovered in Figure 8, that the theoretical forward curve of the GBM and exponential NIG Lévy spot models show no change rate between those two forward curves. This issue occurs because those curves for GBM and exponential NIG Lévy spot models are fixed, unlike the BNS spot model, which contains stochastic volatility factors.

Additionally, all of the theoretical forward curves in Figure 8 also decrease with time due to the decreasing of the empirical $\hat{\theta}$ values as inscribed above (refer to Figure 7). It can be

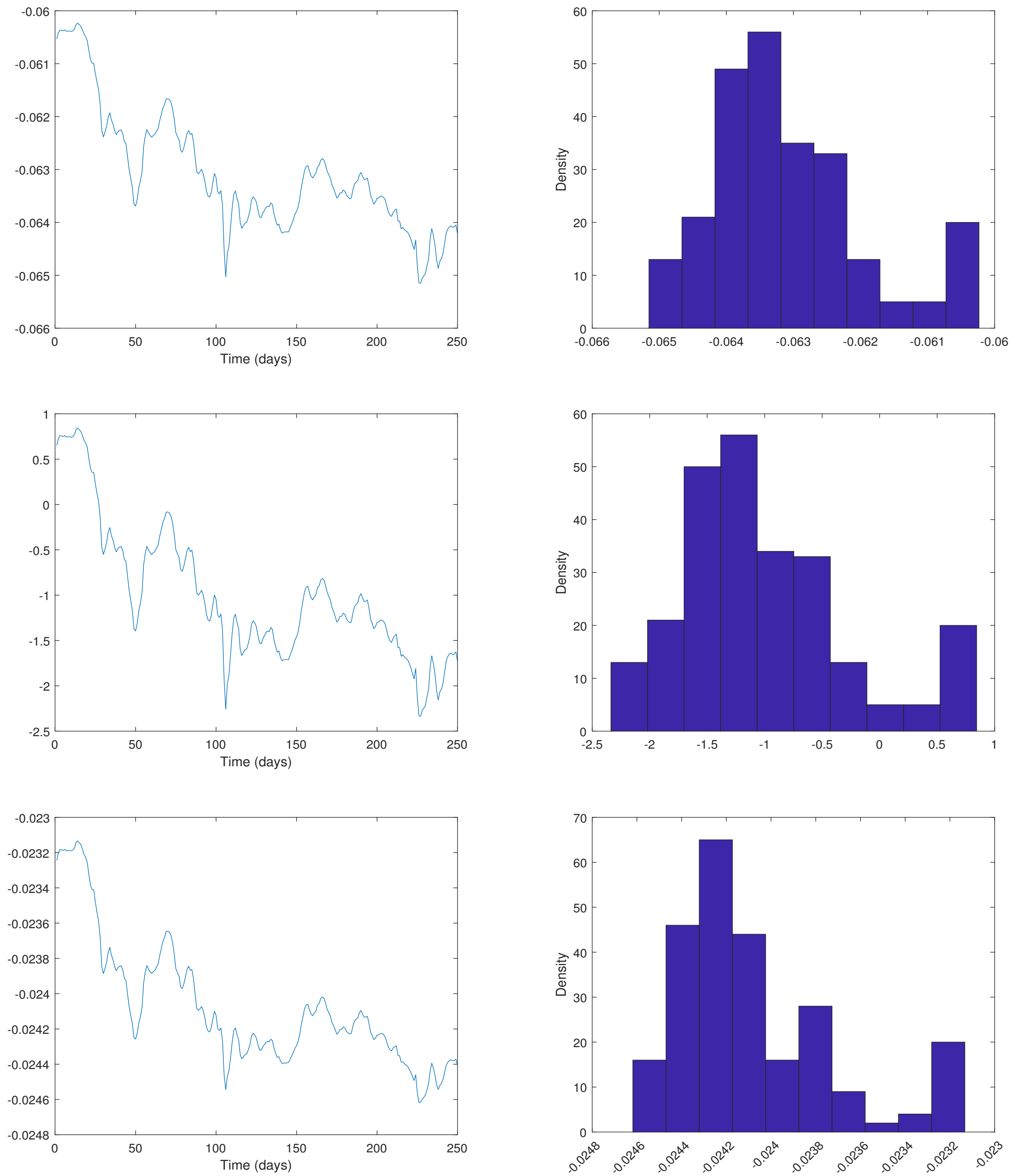

Figure 7: The estimated values of $\hat{\theta}$ on the left while their corresponding density on the right: GBM (top), NIG Lévy (middle) and BNS (bottom) models.

Table 6: The forward prices (theoretical and actual) for the BFA Panamax data were observed on January 2, 2009.

Forward maturity	Actual forward price	Theoretical forward price driven by empirical $\hat{\theta}$		
		GBM model	NIG Lévy model	BNS model
January (Month 1)	5,756.00	28,047.16	28,047.17	23,249.84
Quarter 1 (Q1)	8,728.00	27,293.04	27,293.06	19,636.33
Quarter 2 (Q2)	12,417.00	26,199.70	26,199.74	17,782.59
Quarter 3 (Q3)	13,319.00	25,150.16	25,150.21	17,292.07
Quarter 4 (Q4)	13,878.00	24,142.66	24,142.73	17,176.35
Year 1	13,472.00	20,500.42	20,500.55	16,757.76
Year 2	13,603.00	17,407.67	17,407.83	16,414.82
Year 3	13,697.00	14,781.49	14,781.67	16,071.14
Year 4	13,681.00	12,551.51	12,551.70	15,728.97
Year 5	13,361.00	9,050.06	9,050.25	14,935.31

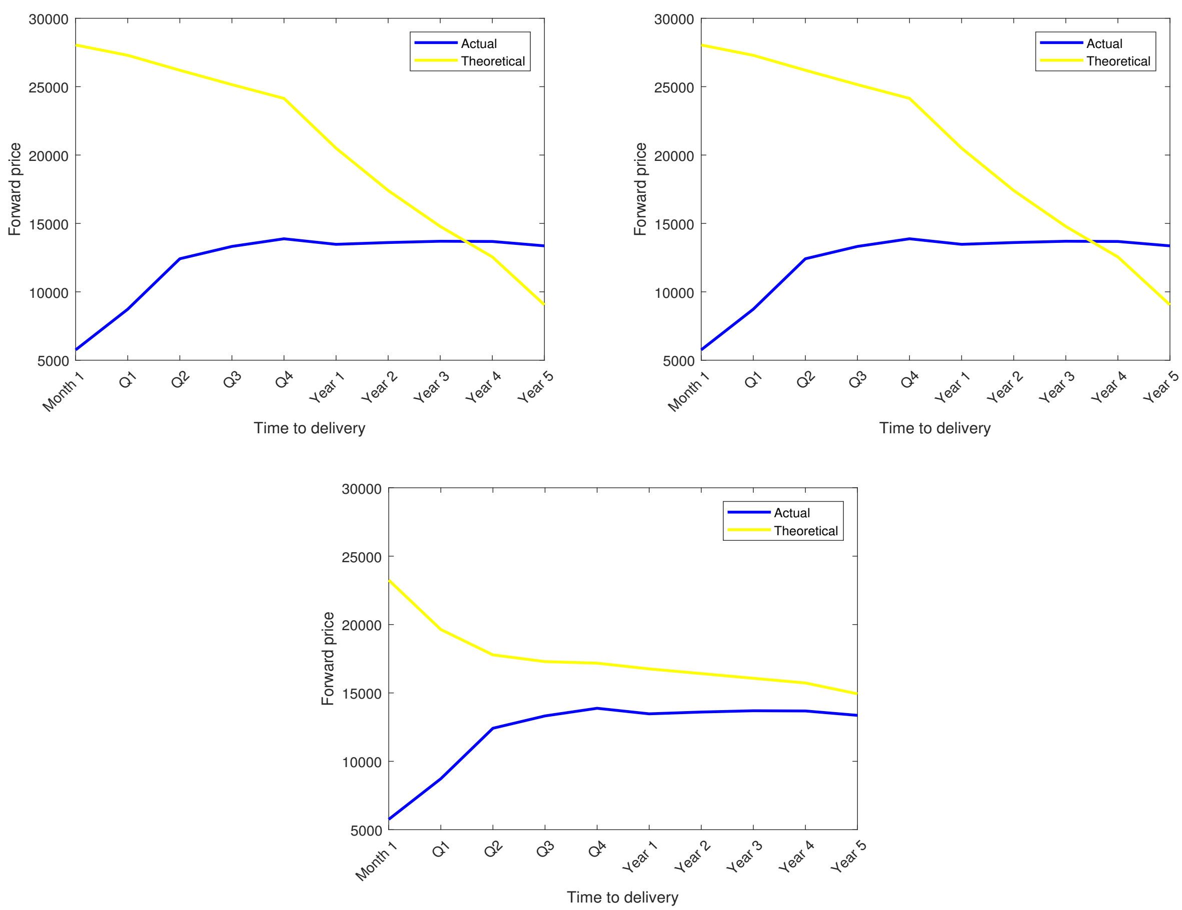

Figure 8: Forward curves for the Panamax prices that adopt our empirical $\hat{\theta}$ values - observed on January 2, 2009: GBM (top left), NIG Lévy (top right) and BNS (bottom) models.

observed that before the delivery period year-3, for the GBM and exponential NIG Lévy spot models, the theoretical forward curves gradually decline over time to the actual forward curves, such that they hit the intersection point as shown respectively in between the delivery period year-3 and year-4. This case means, driven by adopting our empirical $\hat{\theta}$ values observed on January 2, 2009, we managed to produce the minimization of the difference price gap between the theoretical and actual forward prices. Unfortunately, these difference price gaps later ascend after those intersection points under GBM and exponential NIG Lévy spot models since both curves of forward deviate from each other for those two models.

On the contrary, the plotted graph for the theoretical forward curve in the BNS stochastic volatility spot model varies drastically compared to others, as illustrated in Figure 8. It swiftly declines over time to the actual forward curve as early as before the delivery period Q2. Then, it gradually becomes horizontal afterwards and eventually gets closer to the actual forward curve. The depreciation of the difference price gap between the theoretical and actual forward prices under the BNS stochastic volatility spot model also occurs significantly at the beginning of the delivery period and later slowly shrinks over this horizontal period consistently. This condition mirrors that the BNS stochastic volatility spot model is fast and stable practically throughout the delivery period for governing the theoretical forward price becomes closer to the actual forward price since time is a valuable quantity.

6 Conclusion

We establish the investigation of three distinct stochastic spot models developed for the evolution of freight rates in the dry bulk market. It covers GBM, NIG Lévy, and BNS stochastic volatility spot models, all of which shall be utilized in the forward pricing since the models are analytically tractable. Driven by the theory of arbitrage-free pricing, we acquire evident forward prices for all models in the study. We also describe the shape of the forward curves based on those formulas of the forward pricing. As a matter of fact, such curves are invariable for the GBM and exponential NIG Lévy spot models, whereas the curve for the BNS stochastic volatility spot model fluctuated.

Furthermore, we also analyse the forward curve performance of the non-stationary one-factor models. By doing that in such a way, we offer a method for estimating the market price of risk and the market price of volatility risk by adjusting theoretical prices to today's observed forward curve, which minimizes the distance between both of them. Our results are in line with most of the commodities literature (to exemplify, see Schwartz [5], Cartea and Figueroa [7], Kolos and Ronn [2]; Bakshi and Kapadia [16], Doran and Ronn [17]). We discover that the time-dependent parameters under the shipping freight market are negative either for the market price of risk or the market price of volatility risk. The impact goes that we manage to minimize the difference of price gap between the theoretical and actual forward prices.

The results of this study may help calculate the risk premium on an extension of future research. It is said that the market price of risks is closely related to the risk premium. In normal backwardation, a negative market price of risk is equivalent to a negative risk premium in the above context, as in Benth *et al.* [20]. We believe that such a procedure is not impossible to put into effect soon since the calculation of risk premium has been pointed out by Härdle and López Cabrera [26] for the weather market.

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