MATEMATIKA, MJIM, Volume 40, Number 2, 49–60


# Determining Fire Insurance Premium in Indonesia based on Severity and Frequency Claim Distributions

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Article history
Received: 6 April 2023
Received in revised form: 30 May 2024
Accepted: 30 June 2024
Published on line: 1 August 2024

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**Abstract** This research utilizes a fitted distribution for calculating fire insurance premiums in Indonesia. This research aims to study, analyze, and estimate the premium of some coverage on fire insurance data in Indonesia. Here, we study data on fire insurance in Indonesia from the period of 2006 to 2016. Property that can be insured based on the type of building used is called occupation. This is regulated in the 2017 *Otoritas Jasa Keuangan* (OJK) Circular Letter. Furthermore, one type of occupation is selected namely occupation number 2937 and as a result, we analyze 530 claims in total with three types of coverage; (i) building, (ii) stock and content, and (iii) building, stock, and content including severity of the claim. For the number of claims, we assume the data following the Poisson distribution. In contrast, for the severity of claim, two of the three candidate distributions are heavy-tailed distributions i.e. Weibull, Lognormal, and the latter is light-tailed distribution Gumbel. At first, it is assumed that the severity is Weibull distribution, considering that fire is a rare occurrence and can be considered as a Non-Homogeneous Poisson process. To obtain the best-fit distribution from these distributions, the traditional method Kolmogorov-Smirnov is applied for each distribution. We also estimate each parameter for a best-fit distribution. Those parameters are used to estimate the premium for each coverage above. According to the probability density function's graph, the most fitted distribution for (i) and (ii) is Gumbel while the most fitted distribution for (iii) is Lognormal. The Indonesia's fire insurance data fits the heavy-tail distributions.

**Keywords** Claims; distribution fitting; fire insurance; Kolmogorov-Smirnov; premium.

**Mathematics Subject Classification** 62P05, 91B02

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## 1 Introduction

Building and residential fires are very common in Indonesia, especially in Java and Bali Islands during the dry season. Threats arise due to human carelessness in constructing buildings which do not follow the applicable building safety standards. There are some areas in Indonesia which need more attention to this threat. These include cities like Special Capital Region (*Daerah Khusus Ibukota* (DKI)) Jakarta, Bogor, Tangerang, Bekasi, Depok, and the surrounding areas that are very densely populated. It is also necessary to pay attention to big cities with very high population numbers, such as Surabaya, Bandung, and other cities with an equivalent density level in addition to densely populated industrial areas that use fuel and hazardous materials [1].

Fires in the building are generally caused by electrical problems due to aging infrastructure, and the use of substandard electrical equipment. Dense urban environments common in developing countries contribute to the rapid spread of fires and difficulty in accessing the burning buildings. However, Indonesia also faces a unique challenge of informal settlements being constructed with flammable materials like wood and corrugated metal, making fires spread faster. Many buildings, especially in slum areas, are lack of proper fire safety features such as fire escapes and fire-resistant materials. On the other hand, Indonesia might see a higher proportion of industrial fires due to the existence of some sectors such as palm oil production.

Fires cause enormous losses including damage to buildings, disruption of production processes, social losses, environmental damage, material losses, and even death. Based on the data compiled by the National Disaster Management Agency (BNPB), there were 355 fire incidents with victims reaching 1,388 people and losses of up to 6 billion rupiahs [2] in Indonesia from 2016 to 2021. The awareness of fire risk shall then be increased. In 2021, the government allocated 417 trillion Rupiah for infrastructure development, which has been increased by around 48% from 281.1 trillion Rupiahs allocated in 2020. This infrastructure development targeted the construction and reconstruction of roads, new airports, irrigation networks, bridges, and railroads [3]. As a result, there will be an increase in the number of buildings which is in line with the likelihood of a fire risk. Thus, buildings will be constructed intensively; consequently, the risk of fire will increase. Hence, there will be more opportunities for insurance companies, especially fire insurance companies, to offer their services in this field. Insurance companies can provide property fire risk coverage in line with the Indonesian government's infrastructure development effort.

Fire insurance is part of general insurance. It covers loss or damage from property and/or interests that are insured directly because of fire, lightning, explosion, falling aircraft, and smoke derived from the insured property/interests of Indonesian Fire Insurance Standard Policy (*Polis Standar Asuransi Kebakaran Indonesia* (PSAKI)). In this paper, the insurance company is called the insurer and the policyholder is the insured. For property that can be insured, it is grouped by types of building use called occupancy [4]. Losses due to fires, as stated before, are often quite large and this can be anticipated by insuring it on a fire insurance policy.

Research on fire insurance has been done a lot. Ho [5] studied the application of data mining in constructing risk factors to determine claims on this insurance at small and medium levels. It generates claim odds and correlations of each variable that will be used to determine claims. Fire insurance claims need to be studied more to understand the properties of fire insurance data. The severity of the claim is high; meanwhile, the claim accrues unpredictably. Exploration of original data will give acute insight into calculating the fire insurance premium, especially in Indonesia. Many phenomena in the fire insurance industry need to be studied based on the varied natural characteristics in Indonesia. So it is necessary to review reinsurance data on fire insurance which covers cases throughout Indonesia. Estimating future claim events and severity of claims accurately will be significant in the fire insurance industry in Indonesia.

The data on fire insurance claims are very likely to have outliers, so it is clear that the distribution considered suitable for the data is the distribution with heavy-tail or light-tail [6–8]. Previous studies generally considered distributions with heavy-tails. For example, Nadarajah and Bakar [9] modeled Danish fire insurance data through a composite lognormal distribution model. They write down that the results obtained give a better fit for the data. Meanwhile, Asgharzadeh et al. [10] apply a non-composite model to Danish fire insurance data using the Generalized inverse Lindley distribution, which is uncommon. Then Brazauskas and Kleefeld [11] modeled the Norwegian fire insurance severity claim data parametrically using five distributions, namely Lognormal-Pareto (two versions), Weibull-Pareto (two versions), and Folded-t. In the case of certain data, the value claim difference could be very large. Therefore, as the first novelty, this paper considers Gumbel distribution as a representative of the light-tail distribution. Heavy-tail distributions are also considered for comparison, namely Lognormal and Weibull.

Furthermore, the determination of the premium of fire insurance in Indonesia still needs to be studied and applied to the original data. This paper will discuss this as a second novelty considering that there is no research or literature that calculates how to determine fire premiums in Indonesia. Hereafter, we present our research method, data, and data preparation procedures. Then, we would explain the descriptive statistics and identify the heavy-tailed distribution of fire insurance data in the result and discussion section.

## 2 Method

The data used in this study are provided by a reinsurance company in Indonesia. The data consist of fire insurance claims for occupation 2937 that is classified into property of private and public warehouse. The variable used is the severity of claim, coverage, and the time of claim accrued. These data are: (i) confidential because there are some customer data that should not be known by parties other than those authorized, and (ii) a collection of transactions from several insurance companies from all over Indonesia. Therefore, the size and attributes of these data are quite large (more than 10 million records). It is not an easy and simple procedure to organize such big data that are ready to analyze statistically. One reason is each insurance company has its own terms and format for recording. So, it takes considerable effort to adapt them into a common format that is appropriate for all, with minimal missing values. Another consequence is that data writing becomes inconsistent. Hence, preliminary data cleansing was executed as follows:

1. Understanding and studying data. A serial of discussions was done with the authorized team to ensure the data validity on:

    a. uniformity of date format

    b. uniformity of floating value format (representing the value of money)

    c. uniformity of term

   d. validity of dates (chronological events)
   e. validity of nominal value
   f. validity of enumeration or text value
   g. validity of aggregate column
   h. meaning of each column data

2. Recording each item that breaks the above criteria or data that are ambiguous.

3. Confirming every finding in 2) and recording any additional information concerning the semantics of the data.

4. Fixing the "error" data.

5. Iterating steps 1), 2), 3) and 4) until the data are considered well established by the authorized team.

6. Saving the final data in a common and interoperable format.

These full data consist of the insured's information in fire insurance for private and public warehouses in 2006-2016 given several claim criteria. Furthermore, one type of occupation is selected namely occupation number 2937 and as a result, we analyze 530 claims in total with three types of coverage; (i) building, (ii) stock and content, and (iii) building, stock, and content including total claims. The method applied to obtain severity estimation was through Kolmogorov Smirnov test. The Kolmogorov Smirnov test is a test to compare two empirical distributions based on the difference between two distribution functions [12].

There are several principles in calculating the risk premium [13]. To obtain a risk premium, security loading is needed where each insurance company's security loading can be different from one another. Reinsurance company's data consist of data from several insurance companies, so that the authors prefer estimating the pure premium to risk premium. Thus, the pure premium can be estimated with the formula [14]:

$$E[S_N] = E[N]E[X] \tag{1}$$

where $N$ denotes the discrete random variable of claim frequency/number of claims and $X$ denotes the continuous random variable of severity of claim. Thus, we are estimating the premium of the fire insurance by using severity estimation claim and frequency estimation claim. Here, we consider only three distributions as mentioned before:

a. Weibull Distribution $f(x) = \dfrac{\beta}{\alpha}\left(\dfrac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}$

b. Lognormal Distribution $f(x) = \dfrac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\dfrac{(\ln x-\mu\ )^2}{2\sigma^2}\right)$

c. Gumbel Distribution $f(x) = \dfrac{1}{\beta}e^{-(z+e^{-z})}, z = \dfrac{x-\mu}{\beta}$

with parameter notation $\beta$ for scale, $\mu$ for location, $\sigma$ for scale parameter of the included Normal distribution, and $\alpha$ as the shape parameter. Because the fire incident is a non-homogeneous Poisson process, and the condition of Indonesia is a large archipelagic country, then following generality, we hypothesize that the total claims will have a Weibull distribution. These two alternatives are provided for consideration: (i) the Lognormal distribution as a representation of the well-known heavy-tailed distributions in almost all fields of science (not only insurance), and (ii) the Gumbel distribution as a representation of the commonly used light-tailed distribution in the actuarial field. The two alternative distributions are simpler distributions than the Weibull distribution because there is no multiplication with the random variable $X$. The parameter estimation uses the Kolmogorov-Smirnov test.

The Kolmogorov-Smirnov test is defined by (i) hypothesis null, that each of the data follows a specified distribution (Gumbel, Lognormal, or Weibull) and (ii) hypothesis alternative, that the data do not follow the specified distribution. The hypothesis regarding the distributional form is rejected if the test statistic of Kolmogorov-Smirnov is greater than the critical value at the significance level $\alpha = 5\%$.

## 3 Result and Discussion

The initial hypothesis that the distribution is not normally distributed can be easily concluded based on the number of outliers shown in Figure 1. To provide more details on these data, Table 1 presents descriptive statistics for each type of coverage. In addition to the number of observations, indicators for the first four moments (mean, standard deviation, skewness, excess kurtosis), and minimum and maximum, we also present the 99% quantile and the mean loss, if the loss is above 99%. The 99% quantile is the value at risk (at 99% confidence level) and the mean loss exceeding value is the 99% quantile of tail value at risk.

The descriptive statistics show that the mean value of each coverage severity is greater than the median, indicating that the severity data are not symmetrical and gathered at a low value. The data is skewed to the right as shown by the skewness values for building, stock and content, building, stock, and content coverage 16.72, 5.05, 9.88, respectively. This skewness value indicates that the data are not normally distributed. Furthermore, the kurtosis value is higher than 3, indicating that the fittest distribution may have a heavier tail than the Normal distribution.

Based on the statistical descriptions below, it can be concluded that a special case occurs in the data. There are outliers whose values are very far apart (see Table 2). Therefore, the hypothesis regarding the distribution of claims will be Weibull distributed shift to the light-tailed distribution. The explanation for this large range of claim values is that the data are obtained from many insurance companies in Indonesia which offer the amount of claim values ranging from low to high. The case data used in this paper are different from the common insurance data which generally have a heavy-tailed distribution as in [15].

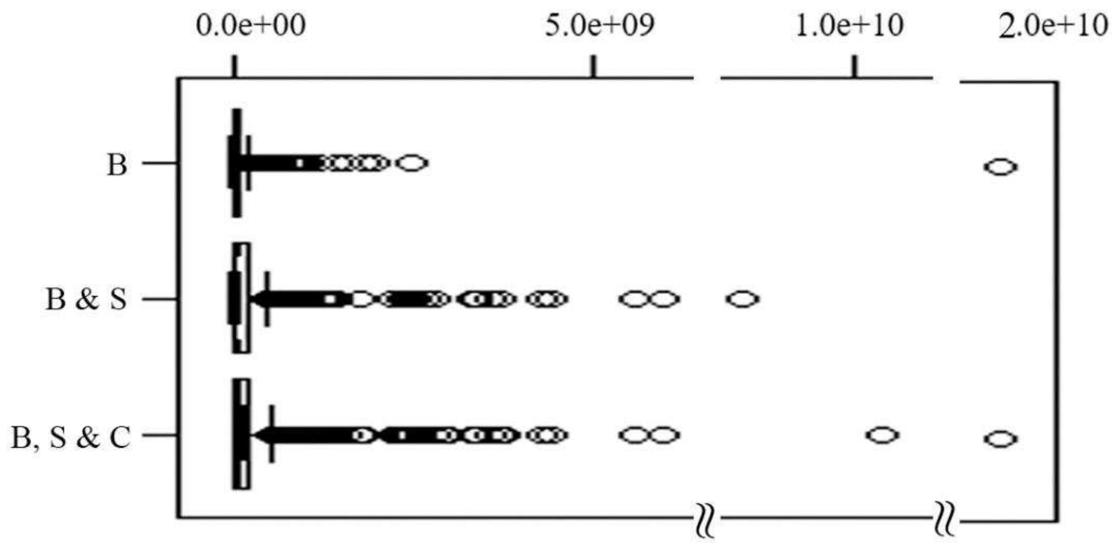



Figure 1: Boxplot for severity in fire insurance of private and public warehouses in 2006-2016. The number of outliers shows that the distribution is not Normal distribution.

Table 1: Summary statistics of severity for each type of coverage

| **Statistics** | **Building** | **Stock and content** | **Building, stock, and content** |
|---|---|---|---|
| Count | 344 | 371 | 515 |
| Range | 18,454,003,424.00 | 8,427,470,196.00 | 18,450,709,916.00 |
| Mean | 166,221,106.60 | 326,501,835.80 | 346,237,362.70 |
| Variance ($\times 10^{17}$) | 10.5 | 7.55 | 13.2 |
| Standard Deviation | 1,025,286,468.00 | 869,167,479.00 | 1,149,924,912.00 |
| CV | 6.16 | 2.66 | 3.32 |
| Standard Error | 55,279,741.60 | 45,124,927.11 | 50,671,745.87 |
| Skewness | 16.72 | 5.05 | 9.88 |
| Kurtosis | 297.45 | 31.98 | 132.88 |

Table 2: Percentile of severity for each type of coverage

| **Percentile** | **Building** | **Stock and content** | **Building, stock, and content** |
|---|---|---|---|
| Minimum | 226,015.8 | 8,404.8 | 3,519,524.2 |
| $5^{th}$ Percentile | 1,534,217.7 | 2,640,979.0 | 5,775,352.0 |
| $15^{th}$ Percentile | 5,025,366.8 | 6,891,326.0 | 9,831,548.0 |
| Q1 | 8,667,320.0 | 10,479,169.0 | 16,010,870.0 |
| Median | 24,810,387.0 | 43,540,928.0 | 50,295,256.0 |
| Q3 | 81,068,624.0 | 185,081,987.0 | 222,981,690.0 |
| $85^{th}$ Percentile | 187,332,074.7 | 44,5938,376.0 | 430,147,586.0 |
| $95^{th}$ Percentile | 443,572,105.9 | 1,605,274,315.0 | 1,479,078,266.0 |
| Maximum | 18,454,229,440.0 | 8,427,478,600.0 | 18,454,229,440.0 |
| Interquartile (IQ) | 72,401,304.0 | 174,602,818.0 | 206,970,820.0 |
| No. of Weak Outliers (1.5*IQ) | 51 | 56 | 139 |
| No. of Moderate Outliers (3*IQ) | 34 | 40 | 109 |
| No. of Strong Outliers (4.5*IQ) | 24 | 28 | 89 |
| No. of Very Strong Outliers (6*IQ) | 16 | 25 | 67 |

### 3.1 Claim Severity Estimation

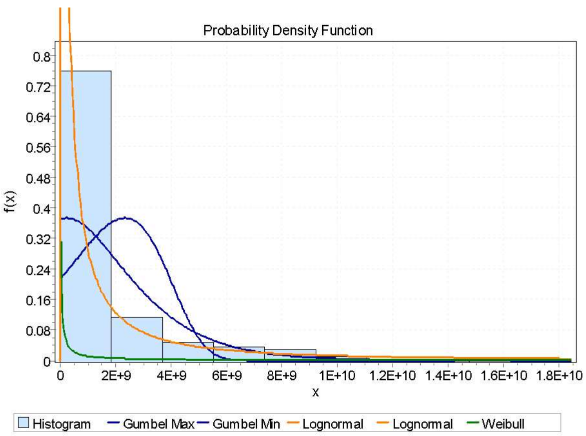
The amount of claim accrued is called claim severity which is calculated based on the insurance policy that the insured chooses when buying the fire insurance policy by considering the coverage. The distribution fitting is executed using the Kolmogorov-Smirnov test. The parameter values for each type of coverage are presented in Table 3. In this paper, up to 1% significance level is considered.

Gumbel-Max distribution is the best fit for building coverage, and also for stock and content coverage. However, building, stock, and content coverage fits Lognormal distribution. This also means that light-tail distribution is suitable for building coverage, and stock and content coverage. However, heavy-tail distribution is more suitable for building, stock, and content coverage. The mean of Gumbel-Max$(\beta, \mu)$ distribution, for $(\beta, \mu)$ are (1,817,587,512.62; 244,425,129.37) and (1,915,565,338.85; 478,105,956.18) consecutively are 1,293,565,113.98368 and 1,583,800,276.90663. Meanwhile, the mean of the Lognormal (2.276; 19.15) distribution is equal to 2,764,395,991.12. More details are shown in Table 3 and upheld in Figure 2.

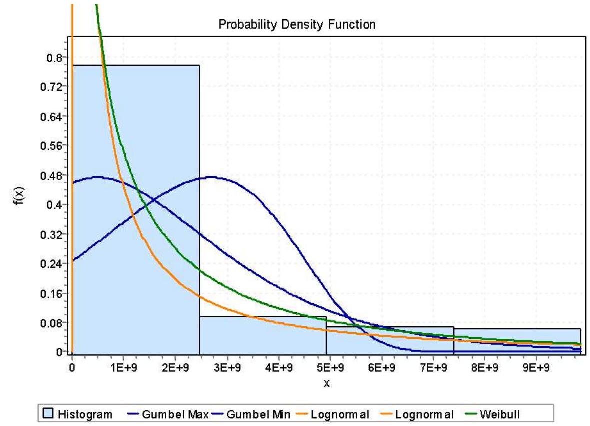
Table 3: The distribution fitting for severity of each coverage

| Type of coverage | Distribution | Kolmogorov Smirnov | | Parameters |
|---|---|---|---|---|
| | | Statistic | Rank | |
| Building | **Gumbel-Max*** | 0.31856 | **1** | $\boldsymbol{\beta = 1,817,587,512.62}$ $\boldsymbol{\mu = 244,425,129.37}$ |
| | Gumbel-Min | 0.33735 | 2 | $\beta$=1,817,587,512.62 $\mu = 2,342,705,098.60$ |
| | Lognormal** | 0.35133 | 4 | $\sigma$=2.3494 $\mu = 19.85$ |
| | Lognormal | 0.35133 | 5 | $\sigma$=2.3494 $\mu = 19.85$ |
| | Weibull*** | 0.35094 | 3 | $\alpha$=0.0172 $\beta$=824.69 |
| | Weibull | 0.35875 | 6 | $\alpha$=0.5445 $\beta$=1,258,761,416.43 |
| Stock and content | **Gumbel-Max** | **0.27707** | **1** | $\boldsymbol{\beta = 1,915,565,338.85}$ $\boldsymbol{\mu = 478,105,956.18}$ |
| | Gumbel-Min | 0.28727 | 2 | $\beta = 1,915,565,338.85$ $\mu = 2,689,494,597.64$ |
| | Lognormal | 0.32659 | 4 | $\sigma$=2.1681 $\mu = 20.25$ |
| | Lognormal | 0.32659 | 5 | $\sigma$=2.1681 $\mu = 20.25$ |
| | Weibull | 0.35695 | 6 | $\alpha$=0.01424 $\beta$=23338.21 |
| | Weibull | 0.32083 | 3 | $\alpha$=0.62236 $\beta$=1,673,519,777.22 |
| Building, stock, and content | Gumbel Max | 0.30747 | 4 | $\beta = 1,777,357,741.82$ $\mu = 293,258,542.08$ |
| | Gumbel Min | 0.33450 | 5 | $\beta = 1,777,357,741.82$ $\mu = 2,345,096,003.51$ |
| | Lognormal | 0.09836 | 2 | $\sigma$=2.276 $\mu = 19.15$ |
| | **Lognormal** | **0.09836** | **1** | $\boldsymbol{\sigma}$=2.276 $\boldsymbol{\mu = 19.15}$ |
| | Weibull | 0.30633 | 3 | $\alpha$=0.13735 $\beta$=3,115,082,156.06 |
| | Weibull | 0.97170 | 6 | $\alpha$=0.22892 $\beta$=2.0202E-5 |

* Light-tailed distribution; ** Heavy-tailed distribution; *** Heavy-tailed distribution. Parameter notations: $\beta$ = scale; $\mu$ = location; $\sigma$ = scale parameter of the included Normal distribution; $\alpha$=shape.




(a)

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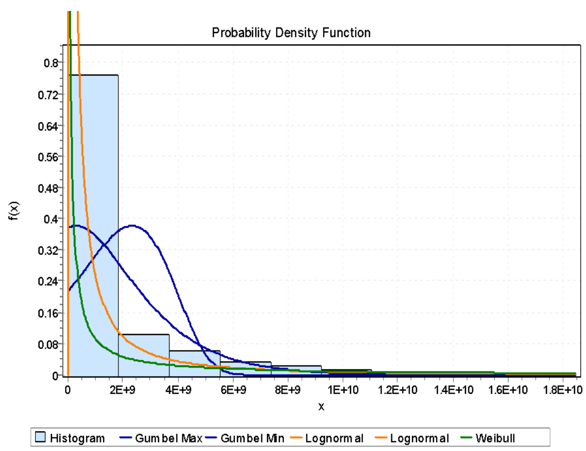


(c)

Figure 2: The distribution fitting of severity and its parameter estimation (a) building coverage (b) stock and content coverage, (c) building, stock, and content coverage

## 3.2 The Claim Frequency Estimation

The claim frequency refers to the number of claims in the block of insurance policies [16]. The frequency of claims is approximated by a Poisson distribution [17]. The Poisson rate is estimated by using the approximate inter-arrival times which have the exponential distribution for each claim data in days. For the Poisson rate, $t = 365$ is used since the fire insurance policy is valid for one year (365 days). The data used are those with claim amounts of greater than zero. The parameter estimations of multiple claims data are given in Table 4. Poisson rate is used because it makes more sense than the exponential rate [18]. It can be seen that the Poisson rate increases with increasing coverage.

Table 4: The Poisson approximation by using exponential distribution of each claim inter-arrival times data (in days) with $t = 365$ days

| Coverage | Exponential rate $\mu$ | Poisson rate $\lambda$ | Poisson rate $\lambda t$ |
|---|---|---|---|
| Building (b) | 10.4797 | 0.0954 | 35 |
| Stock and content (sb) | 9.8814 | 0.1012 | 37 |
| Building, stock, and content (sbc) | 7.1748 | 0.1394 | 51 |

### 3.3 The Premium Estimation

The premium is estimated by using Equation (1). The mean of claim amounts is calculated using the parameters estimator in Table 3 while the claim frequency is based on Table 4. Then the pure premium for each coverage is obtained as follows:

$$E[S_b] \cong 1,293,565,113.98368 \times 35 = 58,210,430,129.27;$$
$$E[S_{sc}] \cong 1,583,800,276.90663 \times 37 = 58,600,610,245.55;$$
$$E\left[S_{bsc}\right] \cong 2,764,395,991.12 \times 51 = 140,984,195,547.12.$$

This pure premium calculation does not take into account the interest rates. The premium offers from building coverage and stock and content coverage will attract consumers because the difference between pure premium amounts is not much.

## 4 Conclusions

This research utilizes a fitted distribution for calculating fire insurance premiums in Indonesia. This research studies the original data to estimate the premium by using frequency estimation claim and severity estimating claim of some coverage on fire insurance data in Indonesia. The frequency claim and the severity claim are estimated with statistical methods to find the best-fit distribution. Exploration of original data will give accurate insight into calculating the fire insurance premium. Accurately estimating future claim events and severity of claims will be significant in the fire insurance industry. Previous research does not provide the information of original data from Indonesia.

At first it is assumed that the observation data of the total claim random variable has a Weibull distribution. Based on the result of distribution fitting using Kolmogorov-Smirnov, the estimation of severity claim for private and public warehouses is extremely skewed to the right. Thus, it follows the Gumbel-Max and Lognormal distribution. The pure premium of maximum to minimum values of several coverages is also obtained.

## Acknowledgments

The authors would like to thank FMIPA ITB who has funded the work through the scheme of P2MI 2021.

## References

[1] Sufianto H and Green A R 2012 Urban Fire Situation in Indonesia Fire Technology 48 367-87

[2] National Disaster Management Agency (BNPB), “Fire inchident” 2021. https://dibi.bnpb.go.id/

[3] Wisudanto M S, Ramadhan B M, Widiastuti T, Andi I, and Al Mustofa M U 2019 Diversification of Insurance Companies’ Products as Supporting Infrastructure Development Institutions in Indonesia CSIS Journal of Infrastructure Development 2 161-7

[4] OJK 2017 Surat edaran Otoritas Jasa Keuangan Nomor 6/ SEOJK.05/2017

[5] Ho C L 2014 Data mining applied to construct risk factors for building claim on fire insurance, The International Journal of Engineering and Science 3(11) 45-61

[6] Reynkens T, Verbelen R, Beirlant J, and Antonio K 2017 Modelling censored losses using splicing: A global fit strategy with mixed Erlang and extreme value distributions Insurance: Mathematics and Economics 77 65-77

[7] Delong L, Lindholm M, and Wuthrich M V 2021 Gamma Mixture Density Networks and their application to modelling insurance claim amounts Insurance: Mathematics and Economics In Press.

[8] Boucher J P, Denuit M, and Guillén M 2008 Models of insurance claim counts with time dependence based on generalization of Poisson and negative binomial distributions Variance 2(1) 135-162

[9] Nadarajah S and Bakar S A A 2014 New composite models for the Danish fire insurance data Scandinavian Actuarial Journal 180-7

[10] Asgharzadeh A, Nadarajah S, and Sharafi F 2017 Generalized inverse Lindley distribution with application to Danish fire insurance data Communications in Statistics - Theory and Methods 46(10) 5001-21

[11] Brazauskas V and Kleefeld A 2016 Modeling severity and measuring tail risk of norwegian fire claims North American Actuarial Journal 20 1-16

[12] Tse K and Yiu 2009 Nonlife Actuarial Models Theory, Methods and Evaluation New York: University of Cambridge

[13] Gray R J and Pitts S M 2012 Risk Modelling in General Insurance Cambridge: Cambridge University Press

[14] Klugman S A, Panjer H H and Willmot GE 2012 Loss Models: From Data to Decisions New York: John Wiley & Sons

[15] Soohan A, Joseph H T, K, and Vaidyanathan R 2012 A new class of models for heavy tailed distributions in finance and insurance risk Insurance: Mathematics and Economics 51 43-52

[16] Y.-K. Tse, *Nonlife Actuarial Models Theory, Methods and Evaluation*. New Work: Cambridge University Press, 2009.

[17] Beirlant J, Maribe G, and Verster A 2018 Penalized bias reduction in extreme value estimation for censored Pareto-type data, and long-tailed insurance applications Insurance: Mathematics and Economics 78 114-22

[18] S. B. Vardeman, R. E. Walpole, R. H. Myers, I. Miller, and J. E. Freund, *Probability and Statistics for Engineers and Scientists.*, vol. 81, no. 393. Pearson Education International, 1986. doi: 10.2307/2288012.