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Approximation of Picture Fuzzy Bézier Curve Model

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Abstract This paper presents a new picture fuzzy set approach to modeling of Bézier curve approximation. To produce the picture fuzzy control point, the notion is used to describe the point relation of the picture fuzzy set (PFS). The new picture fuzzy control point is combined with the Bézier function to create a model of the picture fuzzy Bézier curve. Subsequently, the approximation curves comprising the positive, neutral, negative, and refusal membership curves are displayed. A numerical example has been used to approximate the picture fuzzy Bézier curve model for the illustration. Consequently, in the conclusion of this paper, an approach to obtain picture fuzzy Bézier curve is described.

Keywords Approximation; Bézier curve; picture fuzzy set; picture fuzzy Bézier curve; picture fuzzy control point.

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1 Introduction

An expansion of the fuzzy set (FS) from Zadeh [1] and intuitionistic fuzzy set (IFS), Li [2], Das and Coung [3-7] have presented picture fuzzy set. By providing the neutral and refusal membership degrees, it can effortlessly handle the ambiguity inherent in human mental processes. The two components of the IFS hesitation margin in PFS are the neutral membership degree and the refusal membership degree. The PFS is reduced to IFS when the neutral and refusal membership degrees are both zero. It is challenging to describe circumstances in real life where the human mind involves more than only the choices “yes,” “abstain,” “no,” and “refusal.”. When handling situations of this nature, PFS is the better option because it offers positive, neutral, negative, and refusal membership degrees. One instance of a situation where voters can express their thoughts is during a national general election, where they can “vote for,” “abstain,” “vote against,” or “refuse” to participate in the election [5-7]. Since FS and IFS oppose neutrality, situations like this one are possible in reality and fall outside of their purview. Let’s consider an example where an expert gets someone’s opinion about an object.

The individual may now state that there are four possible outcomes: 0.4 is the object may be good, 0.2 is the thing may not be good, 0.1 is the object may be both good and bad, and 0.3 is the object may not be known to the person. Neither the FSs nor the IFSs are in charge of this problem.

Inspired by the implementation of PFS in the process of making decisions, Garg [8] put forth several operators of aggregations inside the framework of PFS and demonstrated a method of constructing decisions utilizing the suggested operators of aggregations. Wang [9] presented a geometric aggregation operator based on picture fuzzy sets and used score and accuracy functions to compare two picture fuzzy numbers (PFNs). Guiwu [10] suggested the cross entropy of PFSs and employed PFS in a decision-making problem. PFS correlation coefficient was established by Singh [11] and used to solve the clustering analysis problem. The fact that the neutral membership degree is regarded in the same way as a positive or negative membership degree raises another significant difficulty. However, in real life, there are situations that are completely different, and the degrees of membership, non-membership, and neutrality all have varied functions and roles in the process of making decisions and ranking them. The main motivation behind the study of PFS is their ability to handle more nuanced and complex uncertainty in data, especially in cases where traditional FS and even IFS fall short. PFS allowing for the simultaneous representation of positive, neutral, negative and refusal membership. This additional dimension of refusal makes PFS particularly powerful in various applications where uncertainty is multifaceted and cannot be simply captured by binary or even ternary logic.

Nevertheless, there are also fuzzy issues in mathematical modeling, particularly when it comes to curve and surface concerns. To accurately represent an actual data point, curve and surface are required [12]. But due to ambiguity and uncertainty, the data generated point is hard to interpret. In many fields, there is a lot of discussion around the uncertainty issue. The primary challenge in creating a smooth curve and surface model in geometric modeling is uncertainty in data collection. Fuzzy set theory is applied to resolve this issue. The development of a fuzzy Bézier curve model with integrated uncertainty is covered by Talibe [13]. The integrated-uncertainty data in this model were determined by combining type-2 (T2FST) and type-1 (T1FST) fuzzy set theory. The term “integrated-uncertainty data” refers to the combination of complicated and uncertainty data that arises when a group of data sets that need to be modelled have intrinsic degrees of ambiguity. Thus, to obtain the crisp integrated-uncertainty fuzzy Bézier curve, three steps were required: fuzzification, type-reduction, and defuzzification. Tania [14] talks about building a membership function using a Bézier curve.

The methods described in [15] were expanded upon by Anile et al. to address data modeling and data reduction challenges [16-18]. They start by converting a large amount of data into fuzzy integers that have appropriate membership functions. They developed fast techniques to compute spline alpha values and fuzzy B-splines to understand fuzzy data. The fuzzy B-splines methodology was developed by Anile and Spinella [18], who applied fuzzy arithmetic concepts to uncertain sparse data that resulted from mistakes in measurement, problems with data reduction, and modeling faults.

The concept of IFS especially in surface and curve for splines was presented by Zulkifly [19], who conducted research on the subject and based on the Bézier spline, where intuitionistic fuzzy control point (IFCP) is utilized to combine the curve and surface. In discussion of the intuitionistic fuzzy Bézier model, Wahab [20] created the intuitionistic fuzzy Bézier curve (IFBC) using an interpolation technique. By combining the defined Bernstein polynomial IFCP, they were able to visualize IFBC. Wahab [21] created an approximation-based intuitionistic fuzzy B-spline curve and cubic Bézier curve by interpolation method using IFCP. PFS theory is a mathematical representation that aims to provide ideas and methods for dealing with ambiguous problems, where curve modeling, on the other hand, is a technique for producing geometric mathematical representations. The primary goal of this research is to develop a geometric model that can handle uncertain data. This research suggests a new method for creating and exhibiting PFBC through approximation method by employing PFCP.

The arrangement of this paper according to the introduction and earlier research on the picture fuzzy set method including geometric modeling were covered in Section 1. Preliminary information, including definitions of image fuzzy sets, picture fuzzy numbers, and picture fuzzy relations, are provided in Section 2. PFPR and PFCPR are introduced in section 3. In Section 4, PFBC approximation using PFCP is introduced. In contrast, Section 5 presents a numerical illustration and PFBC visualization. Section 6 represents the research's conclusion.

2 Preliminaries

Some fundamental ideas about fuzzy sets, intuitionistic fuzzy sets, and picture fuzzy sets have been covered in this section.

Denition 1. [1] *Assume that is a nonempty set. A fuzzy set drawn from $X$is known as*

$$A = \{(x, \mu_A(x)) : x \in X\}$$

where, $\mu_A(x) : X \to [0,1]$ denotes the membership of the fuzzy set A. Fuzzy set is a collection of objects with graded membership.

Denition 2. [22-23] *An intuitionistic fuzzy set (IFS) on a universe is an object of the*

$$A^* = \{(x, \mu_{A^*}(x), v_{A^*}(x)) | x \in X\}$$

where $\mu_{A^*}(x) \in [0,1]$ is called the degree of membership of $X \in A^*$, $v_{A^*}(x) \in [0,1]$ is the degree of non-membership of Xin A^* while μ_{A^*} and v_{A^*} satisfy $0 \leq \mu_{A^*}(x) + v_{A^*}(x) \leq 1$ for all $x \in X$.

Hesitancy degree denoted as $\pi_{A^*}(x) = 1 - [\mu_{A^*}(x) + v_{A^*}(x)]$ is call the hesitancy degree of an element $x \in A^*$ or uncertainty associated with the membership or non-membership or both in A^*. The set of all intuitionistic fuzzy sets in X will be denoted by $IFS(X)$. The concept of picture fuzzy sets is a generalization of fuzzy sets and intuitionistic fuzzy sets.

Denition 3. [5-7] *A picture fuzzy set (PFS) set $\hat{A}$ on a universe of discourse X is of the form*

$$\hat{A} = \{x, \mu_{\hat{A}}(x), \eta_{\hat{A}}(x), v_{\hat{A}}(x) | x \in X\}$$

where, $\mu_{\hat{A}}(x) \in [0,1]$ is called the "degree of positive membership of x in $\hat{A}$", $\eta_{\hat{A}}(x) \in [0,1]$ is called the "degree of neutral membership of x in $\hat{A}$", $v_{\hat{A}}(x) \in [0,1]$ is called the "degree of negative membership of x in $\hat{A}$ ", while $\mu_{\hat{A}}(x)$,$\eta_{\hat{A}}(x)$ and $v_{\hat{A}}(x)$ must also satisfy the following condition:

$$0 \leq \mu_{\hat{A}}(x) + \eta_{\hat{A}}(x) + v_{\hat{A}}(x) \leq 1; \forall x \in X.$$

In above definition, for all $x \in X, \rho_{\hat{A}}(x) = 1 - [\mu_{\hat{A}}(x) + \eta_{\hat{A}}(x) + v_{\hat{A}}(x)]$ could be called the "degree of refusal membership of X in $\hat{A}$. When $\mu_{\hat{A}}(x) = 0$ for all $x \in X$, then the PFS reduces into IFS [22].

Denition 4. [6] *For a fixed $x \in \hat{A}$, $(\mu_{\hat{A}}(x), \eta_{\hat{A}}(x), v_{\hat{A}}(x), \rho_{\hat{A}}(x))$ is call picture fuzzy number (PFN) where*

$$\mu_{\hat{A}}(x) \in [0,1], \eta_{\hat{A}}(x) \in [0,1], v_{\hat{A}}(x), \rho_{\hat{A}}(x) \in [0,1]$$

and $\mu_{\hat{A}}(x) + \eta_{\hat{A}}(x) + v_{\hat{A}}(x) + \rho_{\hat{A}}(x) = 1$. Simply, PFN is represented as

$$(\mu_{\hat{A}}(x), \eta_{\hat{A}}(x), v_{\hat{A}}(x)).$$

Denition 5. [6] *Assume that X, Y and Z be typical non-empty sets. A relation of picture fuzzy (PFR), $\hat{R}$ is a subset of picture fuzzy, $X \times Y$ provided by*

$$\hat{R} = \{((x,y), \mu_{\hat{R}}(x,y), \eta_{\hat{R}}(x,y), v_{\hat{R}}(x,y)) : x \in X, y \in Y\},$$

in which $\mu_{\hat{R}} : X \times Y \to [0,1]$, $\eta_{\hat{R}} : X \times Y \to [0,1]$ and $v_{\hat{R}} : X \times Y \to [0,1]$ satisfied the condition $0 \leq \mu_{\hat{R}}(x,y) + \eta_{\hat{R}}(x,y) + v_{\hat{R}}(x,y) \leq 1$ for every $(x,y) \in (X,Y)$.

The set of all pictures' fuzzy relation in $X \times Y$ will be denoted by $PFR(X \times Y)$.

3 The Relation of Picture Fuzzy Point and Picture Fuzzy Control Point

One of the key concepts in fuzzy systems theory and fuzzy set theory is fuzzy relations. In fuzzy control theory, one method that is frequently used in approximation theory and inference is the Zadeh's composition rule. Researchers found that intuitionistic fuzzy relations produced a lot of outcomes. By definition, [24] defined several new intuitionistic preference relations, such as the consistent and incomplete intuitionistic preference relations and their properties were examined. As a result, Coung [5-7] presents some initial PFR results. Based on the idea of PFS, the point relation of picture fuzzy set (PFPR) is described in Definition 6.

Denition 6. *Assume that P and Q be a collection of points non-empty set and $P, Q, I \subseteq \mathbb{R}^3$ then PFPR is described as*

$$\begin{aligned}\hat{S} = \{\langle (p_i,q_j), \mu_{\hat{S}}(p_i,q_j), \eta_{\hat{S}}(p_i,q_j), v_{\hat{S}}(p_i,q_j), \rho_{\hat{S}}(p_i,q_j)\rangle \mid \\ (p_i,q_j), \mu_{\hat{S}}(p_i,q_j), \eta_{\hat{S}}(p_i,q_j), v_{\hat{S}}(p_i,q_j), \rho_{\hat{S}}(p_i,q_j) \in I\}\end{aligned}$$

where, (p_i,q_j) are point in the ordered pair and $(p_i,q_j) \in P \times Q$. $\mu_{\hat{S}}(p_i,q_j)$, $\eta_{\hat{S}}(p_i,q_j)$, $v_{\hat{S}}(p_i,q_j)$ and $\rho_{\hat{S}}(p_i,q_j)$ are the classifications of positive, neutral, negative and refusal membership of the corresponding ordered pair of points in $[0,1] \in I$. The degree of refusal is indicated by

$$\rho_{\hat{S}}(x) = 1 - [\mu_{\hat{S}}(p_i,q_j) + \eta_{\hat{S}}(p_i,q_j) + v_{\hat{S}}(p_i,q_j)]$$

and the condition of $0 \leq \mu_{\hat{S}}(p_i,q_j) + \eta_{\hat{S}}(p_i,q_j) + v_{\hat{S}}(p_i,q_j) \leq 1$ is followed.

Splines are specified by a set of control points in geometric modeling. These points change the spline's shape, and the curve smoothly interpolates through or near these points. The notion of fuzzy control point from Wahab's earlier research [20][21] is used in this part to develop the picture fuzzy control point relation (PFCPR).

Denition 7. *Let $\hat{S}$be a PFPR then PFCPR is viewed as a group of points $n+1$ and it is used to characterize the curve and is represented by coordinates and location as*

$$\begin{aligned}
\hat{C}_i^{\mu} &= \left\{ \hat{C}_0^{\mu}, \hat{C}_1^{\mu}, \hat{C}_2^{\mu}, .., \hat{C}_n^{\mu} \right\} \\
\hat{C}_i^{\eta} &= \left\{ \hat{C}_0^{\eta}, \hat{C}_1^{\eta}, \hat{C}_2^{\eta}, .., \hat{C}_n^{\eta} \right\} \\
\hat{C}_i^{v} &= \left\{ \hat{C}_0^{v}, \hat{C}_1^{v}, \hat{C}_2^{v}, .., \hat{C}_n^{v} \right\} \\
\hat{C}_i^{\rho} &= \left\{ \hat{C}_0^{\rho}, \hat{C}_1^{\rho}, \hat{C}_2^{\rho}, .., \hat{C}_n^{\rho} \right\}
\end{aligned}$$

where, $\hat{C}_i^{\mu}$, $\hat{C}_i^{\eta}$, $\hat{C}_i^{v}$ and $\hat{C}_i^{\rho}$ are picture fuzzy control point for negative, neutral, positive and refusal membership and i is one less than n.

4 Model of the Picture Fuzzy Bézier Curve (PFBC)

In geometric modeling, a parametric curve is a Bézier curve whose control polygon determines its polynomial function of scalar t. The number of points needed to specify the degree of the polynomial. As seen in the following definition, PFBC is produced by combining PFCP with the basis function or Bernstein polynomial.

Denition 8. *Let $\hat{C}_i = \left\{ \hat{C}_i^{n} \right\}$ for $i = 0, 1, 2, .., n$ represent the PFCP as well PFBC be represented by $\hat{B}_i(t)$with a vector of position across the curve as a function of the scalar t. The combination of PFBC with $\hat{C}$is*

$$\hat{B}_i(t) = \sum_{i=0}^{n} \hat{C}_i \beta_i^n(t), \; 0 \leq t \leq 1 \tag{1}$$

where $\beta_i^n(t) = \begin{pmatrix} n \\ i \end{pmatrix} t^i (1-t)^{n-i}$ is the blending function with $\begin{pmatrix} n \\ i \end{pmatrix} = \dfrac{n!}{i!\,(n-i)!}$ is the binomial coefficient. The geometric coefficient, $\hat{C}$ that defines the control polygon for the PFBC degree n is the i^{th} PFCP. The notation for PFBC with the n^{th} degree is

$$\hat{B}(t) = \hat{C}_0\beta_0^n + \hat{C}_1\beta_1^n + \hat{C}_2\beta_2^n + .. + \hat{C}_n\beta_i^n$$

The PFBC in (1) is represented as follows and consists of positive membership, neutral membership, negative membership, and refusal curve:

$$\hat{B}^{\mu}(t) = \sum_{i=0}^{n} \hat{C}_i \beta_i^n(t)$$

$$\hat{B}^{\eta}(t) = \sum_{i=0}^{n} \hat{C}_i \beta_i^n(t)$$

$$\hat{B}^{v}(t) = \sum_{i=0}^{n} \hat{C}_i \beta_i^n(t)$$

$$\hat{B}^{\rho}(t) = \sum_{i=0}^{n} \hat{C}_i \beta_i^n(t)$$

A polygon known as the PFCP for PFBC is created by joining the PFCP in the proper sequence. The number of PFCP is greater than the degree of the polynomial defining the PFBC segment by one. PFBC typically takes the form of PFCP. The initial and final points of PFCP align with the initial and final PFCP.

5 Numerical Example

Consider PFBC with four PFCP and degree three ($n = 3$) as in Table 1 to demonstrate the picture fuzzy Bézier curve (PFBC) approximation. In this instance, PFBC is visualized using simply the x value.

Table 1: Visualization of the picture fuzzy control point relationship and its corresponding degree.

PFCP, $\hat{C}_i$	$\mu_{\hat{C}}(x)$	$\eta_{\hat{C}}(x)$	$v_{\hat{C}}(x)$	$\rho_{\hat{C}}(x)$
2	(0.1)	(0.2)	(0.4)	(0.3)
7	(0.4)	(0.5)	(0.1)	(0.0)
-7	(0.1)	(0.2)	(0.4)	(0.3)
-5	(0.3)	(0.4)	(0.2)	(0.1)

The collection of data displayed in Table 1 represents each PFCPR's degree of positive membership ($\mu_{\hat{C}}(x)$), neutral membership ($\eta_{\hat{C}}(x)$), negative membership ($v_{\hat{C}}(x)$) and refusal membership ($\rho_{\hat{C}}(x)$). All values of PFCPR follow the conditions of a picture fuzzy number, as shown in Definition 4. By blending the PFCPR using the Bernstein blending function, the appropriate PFBC's approximation curve from Table 1 is shown individually in Figure 1 with their corresponding control points and picture control polygon.

Its picture fuzzy control polygon determines PFBC. Positive, neutral, negative, and refusal membership are the four curves that make up PFBCs, but the curve of Bézier just applies to the crisp of the curve where the intuitionistic fuzzy control polygon's first and end points match the first and end points of the PFBC. The fuzzy control polygon in Figures 1 and Figure 2 are represented by dashed lines, and the PFBC typically takes the shape of the picture fuzzy control polygon.

6 Discussion and Conclusion

Fuzzification in modeling is commonly used in computer graphics and industrial design for creating smooth curve. In industrial and real-life applications, picture fuzzy Bézier curve modeling approach can be valuable for tasks such as product design, manufacturing, quality control, supply chain management, databases, remote sensing, data mining, real-time tracking, the stock

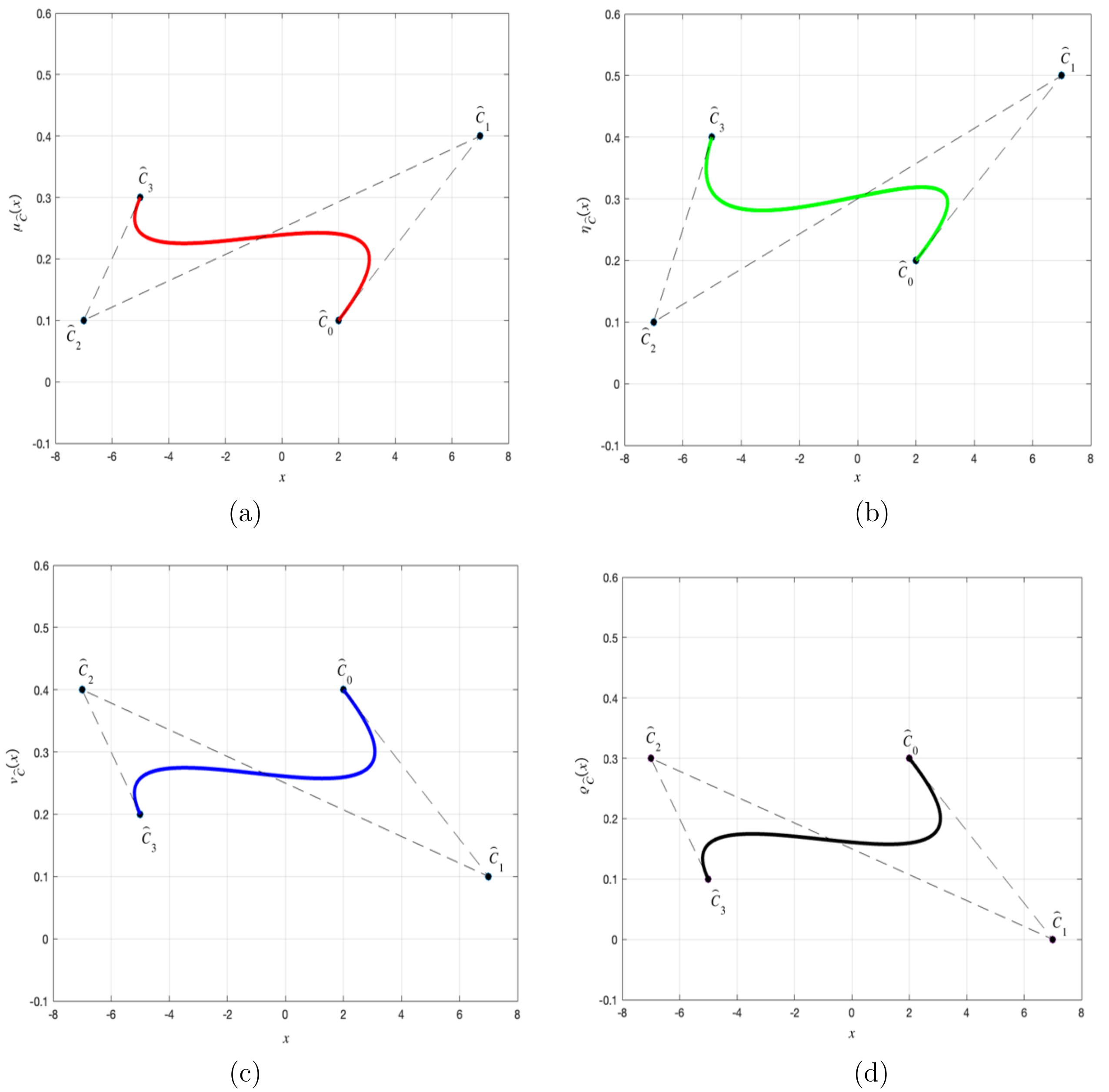

Figure 1: Being involved Bézier curve along with the corresponding control polygon and control points; (a) positive membership, (b) neutral membership, (c) positive membership and (d) refusal membership.

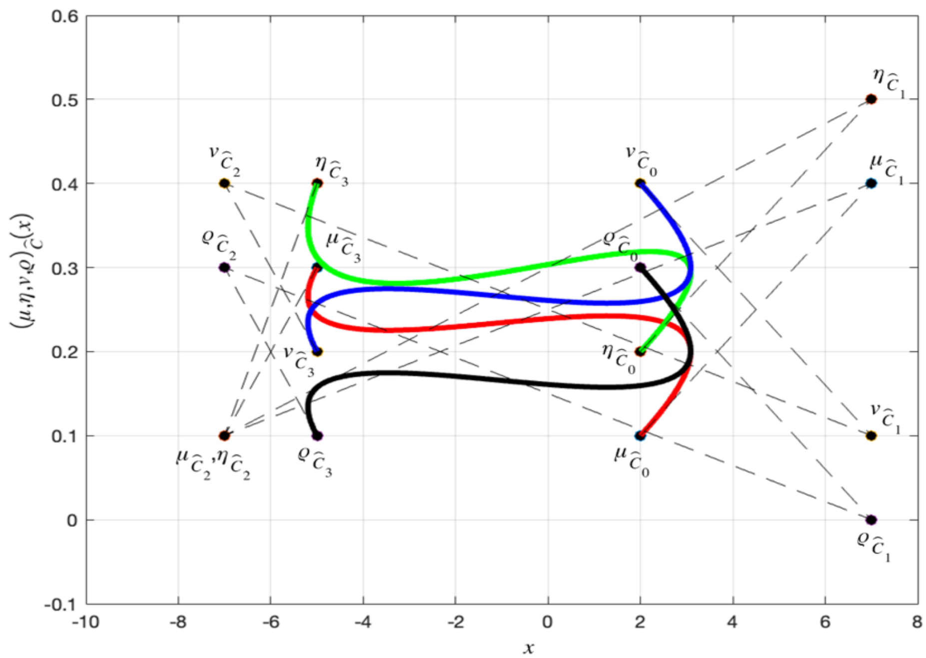

Figure 2: Picture fuzzy Bézier curve and the corresponding picture fuzzy control polygon with picture fuzzy control points.

market, managerial decision-making, economics, routing, wireless sensor networks and financial analysis. Some researcher [25-26] has shown PFS in real-life application. Overall, this research provides a powerful framework for representing and managing uncertainty in various industrial and real-life applications, enabling more informed decision making and enhancing system performance.

By introducing PFCP, this study has introduced an approximation of the PFBC model. Because the approximate PFBC model has a positive membership function, neutral membership function, negative membership function, and refusal function, it is the best method for modeling data with picture features. These functions will be used to process and analyze all the data provided in relation to a smooth Bézier curve. While combined with Bézier visualization, fuzzy picture data can provide comprehensive analysis and description of the types of problems under study, along with their explanation.

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