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Adjacency Matrix and Energy of Graph of a Ring Associated to Kapal Layar Pattern in Tudung Saji

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Abstract Tudung saji is a traditional food cover which is commonly used in Malaysia. In previous research, some triaxial patterns of tudung saji have been shown to be isomorphic to some groups. One of the common tudung saji pattern is called Kapal Layar (Sailboat), and this pattern is isomorphic to cyclic group of order six. Then, a new graph called tudung saji graph is introduced depending on the sections of the triaxial template of the tudung saji pattern. This graph is constructed with the set of vertices consisting of the elements of the Kapal Layar pattern in which two vertices are connected by an edge if the corresponding strands for both vertices are equal. Next, from the structure of the tudung saji graph of Kapal Layar pattern and the isomorphism of the pattern to the ring of integers modulo six, the adjacency matrix of the graph is constructed. Finally, the energy of graphs of the ring associated to Kapal Layar pattern is computed.

Keywords tudung saji; graph theory; ring; adjacency matrix; graph energy.

Mathematics Subject Classification 65F15, 05C90, 17A01.

1 Introduction

Tudung saji is a conical-shaped food cover which is woven in three directions. Previously, a two-dimensional triaxial template has been introduced by mimicking the three directional weaving to present the patterns of tudung saji [1]. Some of these patterns are called Kapal Layar (Sailboat), Bunga Tanjung (Cape Flower), Jari Ketam (Crabs Claw) and Pati Sekawan (Flock of Pigeons), as shown in Figure 1, Figure 2, Figure 3 and Figure 4.

Figure 1: Kapal Layar

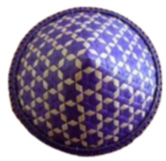

Figure 2: Bunga Tanjung

Figure 3: Jari Ketam

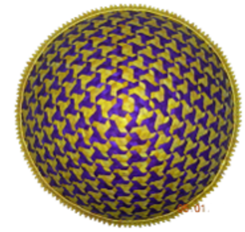

Figure 4: Pati Sekawan

This triaxial template consists of three sections, namely A, B and C to represent the insertion technique from three different directions. Figure 5 shows the formation of Kapal Layar pattern on the triaxial template.

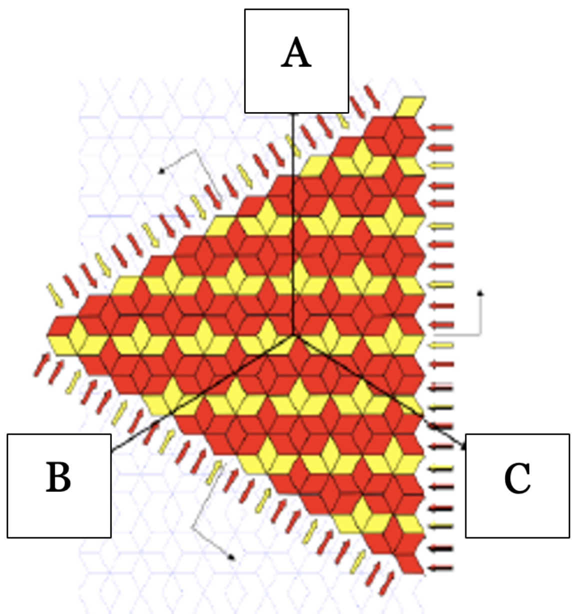

Figure 5: Formation of Kapal Layar pattern on the triaxial template

Based on Figure 5, the insertion of three-strand block with two colors, which are red, red and yellow, starts intermittently from the arrow at section A in an anticlockwise direction and ends right before the second arrow. For section B, it started with red followed by yellow and red, and ends right before the third arrow. Lastly in section C, another insertion starts with red followed by red and yellow which ends before the first arrow. Eventually, this insertion is labelled as 0 for red, red and yellow, and labelled as 1 for yellow, red and yellow (i.e. 0=RRY, 1=YRY). Therefore, this Kapal Layar pattern is formed and introduced as (010) element. For other three-strand block patterns, another labelling system is introduced. For example, 2 represent the strand insertion of yellow, red and red (i.e. 2=YRR). Thus, another element such as (012) can be produced for another pattern of three-strand block with two colours.

As mentioned earlier, the Kapal Layar pattern is formed based on a three-strand block with two colors. Since the Kapal Layar pattern can be formed from six different types of aforementioned insertion strand, the elements of Kapal Layar (KL) pattern can be presented as $KL = \{(010), (011), (001), (100), (101), (110)\}$. A few years back, Kapal Layar pattern has been shown to be isomorphic to cyclic group of order six [2]. Since the Kapal Layar pattern is considered to have six-fold rotational symmetry, it can also be presented as $KL = \{R_0, R_{60}, R_{120}, R_{180}, R_{240}, R_{300}\}$, where $(010) \cong R_0, (011) \cong R_{60}, (001) \cong R_{120}, (100) \cong R_{180}, (101) \cong R_{240}, (110) \cong R_{300}$.

Next, the Cayley Table for Kapal Layar pattern is constructed as follows.

Table 1: Cayley Table for Kapal Layar pattern.

$\circ$	R_0	R_{60}	R_{120}	R_{180}	R_{240}	R_{300}
R_0	R_0	R_{60}	R_{120}	R_{180}	R_{240}	R_{300}
R_{60}	R_{60}	R_{120}	R_{180}	R_{240}	R_{300}	R_0
R_{120}	R_{120}	R_{180}	R_{240}	R_{300}	R_0	R_{60}
R_{180}	R_{180}	R_{240}	R_{300}	R_0	R_{60}	R_{120}
R_{240}	R_{240}	R_{300}	R_0	R_{60}	R_{120}	R_{180}
R_{300}	R_{300}	R_0	R_{60}	R_{120}	R_{180}	R_{240}

Extending the result in [3], in this paper, the Kapal Layar pattern $KL = \{(010), (011), (001), (100), (101), (110)\}$ is shown to be a ring associated to the ring of integers modulo six, Z_6, due to the isomorphism of the cyclic group of order six with Z_6.

In this paper, a new graph called tudung saji graph is introduced to present the connection of the tudung saji and graph theory. This graph is formed whereby the set of vertices consist of the elements of the tudung saji pattern in which two vertices are connected by an edge if the corresponding strands for both vertices are equal. Then, based on the new definition, the tudung saji graph is constructed for the ring associated to the Kapal Layar pattern by taking the elements of $KL = \{(010), (011), (001), (100), (101), (110)\}$ as the vertices of the graph and the vertices are connected based on the adjacency property defined.

Previously, various research has been done to associate rings with algebraic graph such as [4] where the authors explore how zero-divisor graphs can be employed to investigate non-commutative rings. The paper also offers various instances of how the graph structure may be utilized to analyze algebraic properties of the ring. Other than that, a broad perspective on how graph theory can be used to analyze algebra is given, which encompasses the study of zero-divisor graphs, as well as the use of spectral graph theory for the exploration of the algebraic properties of rings [5]. In addition, the study of a matrix with entries from a ring, where the entries represent the weights of the edges in the graph called adjacency matrix has also been explored [6].

In addition to forming the tudung saji graph for the ring associated to the Kapal Layar pattern, the energy of this graph is also determined in this paper. Firstly, the definitions of adjacency matrix [7] and energy of a graph [8] are given as follows:

Definition 1 ***Adjacency Matrix*** *Let* Γ *be a graph of order* n *and size* m*, where* $V(\Gamma) = \{v_1, v_2, ..., v_n\}$ *and* $E(\Gamma) = \{e_1, e_2, ..., e_m\}$*. The adjacency matrix of* Γ *is the* $n \times n$ *matrix* $A = [a_{ij}]$*, where* $a_{ij} = 1$ *if* $v_i v_j \in E(\Gamma)$ *and* $a_{ij} = 0$*, otherwise.*

Definition 2 ***Energy of Graph*** *For any graph,* Γ*, the energy of the graph is defined as*

$$\varepsilon(\Gamma) = \sum_{i=1}^{n} |\lambda_i|$$

where $\lambda_1, \lambda_2, ..., \lambda_n$ *are the eigenvalues of the adjacency matrix of* Γ*.*

In the following section, some literature reviews on the tudung saji, triaxial template, algebraic graph theory and energy of the graph associated to ring are presented.

2 Literature Review

Ethnomathematics is an interdisciplinary study that connects arts and mathematics. The elements of algebra and geometry in ethnomathematics can be related to the weaving of tudung saji. Tudung saji is a Malay version of food cover which is widely used in Nusantara. Weaving by hand is a method of forming pliable tracks by interlacing them rectangularly. The technique used by the weaver of tudung saji is called triaxial weave, the weave technique from three directions.

The investigation of isomorphism between selected elements in ethnomathematics and certain groups in group theory is essential as it preserve the structure between both. Previously, [3] have shown the isomorphism for one of the tudung saji pattern namely Kapal Layar with cyclic group of order six, C_6. The isomorphisms between Kapal Layar pattern has been established with the group due to their symmetrical operations which are considered as the elements that could be mapped with the elements of groups. The triaxial templates of tudung saji pattern also hold other symmetrical operations like rotations, reflection, and inversion [2, 3].

In addition, a graph is often used to represent the structure of a ring [9]. Meanwhile, a ring is a mathematical structure that consists of a set of elements together with two operations: addition and multiplication. Therefore, the use of graph theory in the study of rings provides a powerful tool for understanding the algebraic structure of rings. In addition, the isomorphism of the pattern to the ring of integers is a bijective function that preserves the operations, relations, and structures of the pattern and the ring of integers. A study on isomorphism of a graph to the ring of integers shows that the structure of the graph is like that of the integers. Therefore, by using the structure of the graph and the relation of isomorphism to the ring of integers we find the sum of the absolute values of the eigenvalues of its adjacency matrix, which is helpful to compute the energy of graph. This provides a measure of how well-connected the graph is [10].

The study of the energy of a graph was first introduced by a very well-known mathematician [11] who was inspired through Huckel Molecular Orbital (HMO) theory which was proposed around the 1930s. Generally, it is defined as the summation of the absolute values of its eigenvalues of the adjacency matrix of the graph. Many researchers are inspired to do the research on the energy of graph because of its chemical implications and its other characteristics. Research on energy of graph has expanded on laplacian energy [12], resolvent energy [13], seidel energy and laplacian energy [14, 15], as well as its connection with matrices [16]. Other recent studies on the energy of graphs can also be found in [17–20].

In this paper, the results are focused on the formation of the tudung saji graph of Kapal Layar pattern which is also a ring, the construction of their adjacency matrix, as well as the energy of the graph.

3 Results and Discussion

In this section, the results and findings of the research are discussed. We start with the formation of The Tudung Saji Graph.

3.1 The Tudung Saji Graph

Recall from preliminary result [3] that Kapal Layar pattern $KL = \{(010), (011), (001), (100), (101), (110)\}$ is isomorphic to the cyclic group of order six, C_6. Based on [3], it was also proved that $C_6 \cong Z_6$, the ring of integers modulo n. Since the elements of Z_6 can be mapped to elements of Kapal Layar pattern, this proved that Kapal Layar pattern is also a ring.

Firstly, we discuss the results on the tudung saji graph based on Figure 5. A Tudung Saji Graph associated to a three-strand tudung saji pattern is defined as follows.

Definition 3 *Suppose Δ is a three-strand tudung saji pattern. A tudung saji graph associated to Δ, denoted by Γ_Δ, is a graph with vertices containing the elements $(u_1u_2u_3)$ of Δ where $u_i = 0, 1, 2$ for i=1,2,3 with the corresponding strands of the triaxial template of three sections. Two vertices $u = (u_1u_2u_3)$ and $v = (v_1v_2v_3)$ are adjacent if $u_i = v_i$.*

Based on Definition 3, the tudung saji graph of Kapal Layar pattern is constructed.

The result on the graphical representation of Kapal Layar pattern is presented in Proposition 1.

Proposition 1 *The tudung saji graph of Kapal Layar pattern is a union of two complete graphs of three vertices, $\Gamma_\Delta = K_3 \cup K_3$.*

Proof Let the six elements of Kapal Layar pattern be $KL = \{(010), (011), (001), (100), (101), (110)\}$. Based on Definition 3, two vertices $u = (u_1u_2u_3)$ and $v = (v_1v_2v_3)$ are adjacent if $u_i = v_i$ for $i = 1, 2, 3$. For Kapal Layar pattern, we only have $u_i = 0, 1$ for labelling system of 0=RRY and 1=RYR. First, we let $u_i = 0$, which represent the first labelling system with 0. For the first section (i.e $u_1 = 0$), these three elements {(010),(011),(001)} are found to be adjacent to each other. Next, we let $u_i = 1$ which represent the first labelling system with 1. For the first section (i.e. $u_1 = 1$), these three elements {(100),(101),(110)} are found to be adjacent to each other. This adjacency formed the union of complete graph with three vertices, $K_3 \cup K_3$. Next, similar graph is formed for the second section (u_2) and the third section (u_3). The proof then follows.

Next, the tudung saji graph of three different cases, the first, second and third sections (refer to section A, B, and C in Figure 5) are presented in Figure 6, Figure 7 and Figure 8, respectively.

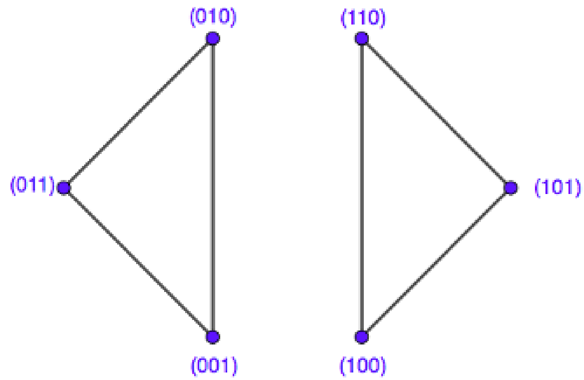

Figure 6: The tudung saji graph formed based on adjacency on the first section

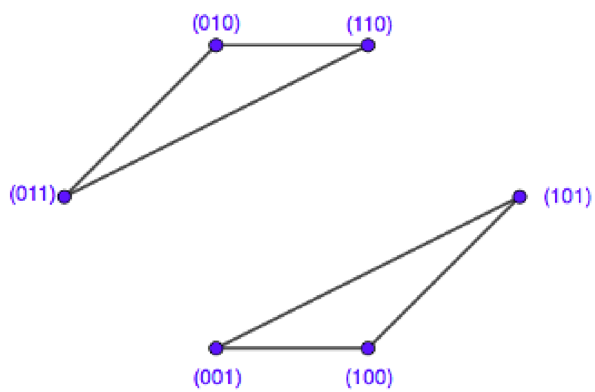

Figure 7: The tudung saji graph formed based on adjacency on the second section

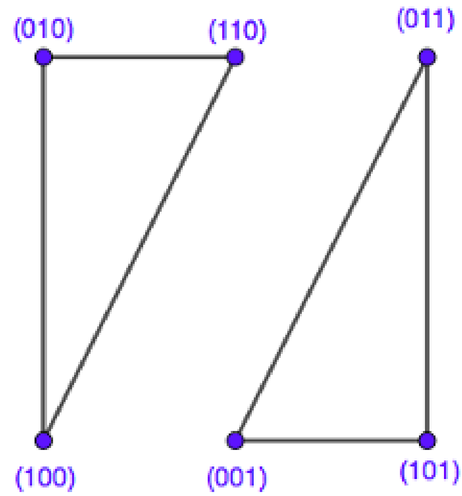

Figure 8: The tudung saji graph formed based on adjacency on the third section

Next, the adjacency matrix of the tudung saji graph of Kapal Layar pattern is discussed.

Based on Definition 3, the adjacency matrix of the tudung saji graph of Kapal Layar pattern is determined, presented in Proposition 2.

Proposition 2 *The adjacency matrix of the tudung saji graph of Kapal Layar pattern is given as follows*

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

where $A_{11} \cong (010)$, A_{12}, $A_{21} \cong (011)$, A_{13}, $A_{31} \cong (001)$, A_{14}, $A_{41} \cong (100)$, A_{15}, $A_{51} \cong (101)$, *and* A_{16}, $A_{61} \cong (110)$.

Proof Based on the adjacency of the graph given in Figure 4, 5, and 6, each element of Kapal Layar pattern is connected only between two edges, u_i and v_j, for a chosen $j = 1, 2, 3$. Therefore, three graphs in Figure 4, 5, and 6 holds similar adjacency matrix. Thus, each arrow and column of the matrix holds at most two entries of value 1.

Next, the energy of tudung saji graph is determined, given in Theorem 1.

Theorem 1 *The energy graph of the Tudung Saji Graph of Kapal Layar pattern is equal to eight,* $\varepsilon(\Gamma_\Delta) = 8$.

Proof Let the characteristic polynomial be $f(\lambda) = \det(\lambda I - A)$.

From Proposition 2,

$$\lambda I - A = \begin{bmatrix} \lambda & -1 & -1 & 0 & 0 & 0 \\ -1 & \lambda & -1 & 0 & 0 & 0 \\ -1 & -1 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & -1 & -1 \\ 0 & 0 & 0 & -1 & \lambda & -1 \\ 0 & 0 & 0 & -1 & -1 & \lambda \end{bmatrix}$$

Thus,

$$\begin{aligned} f(\lambda) &= \lambda^6 - 6\lambda^4 - 4\lambda^3 + 9\lambda^2 + 12\lambda + 4 \\ &= (\lambda + 1)\left(\lambda^5 - \lambda^4 - 5\lambda^3 + \lambda^2 + 8\lambda + 4\right) \\ &= (\lambda + 1)(\lambda + 1)\left(\lambda^4 - 2\lambda^3 - 3\lambda^2 + 4\lambda + 4\right) \\ &= (\lambda + 1)(\lambda + 1)(\lambda + 1)(\lambda - 2)(\lambda + 1)(\lambda - 2) \end{aligned}$$

By letting $f(\lambda) = 0$, the eigenvalues of the adjacency matrix A are $\lambda_1 = -1$ with multiplicity four and $\lambda_2 = 2$ with multiplicity two.

Hence, based on Definition 2, the energy of graph of Kapal Layar pattern, $\varepsilon(\Gamma_\Delta) = \sum_{i=1}^{n} |\lambda_i|$ where $\varepsilon(\Gamma_\Delta) = |-1| + |-1| + |-1| + |-1| + |2| + |2| = 8$.

Next, the use of Maple software in the construction of the characteristic polynomial and the eigenvalues are discussed in the following subsection.

3.2 The Maple Coding

In this paper, the Maple software is used to assist the computation of the characteristic polynomial and the eigenvalues of the adjacency matrix.

The Maple coding used is given as follows:

First, the adjacency matrix of the tudung saji graph for the ring associated to the Kapal Layar pattern is provided as the input;

with(LinearAlgebra)

A:=Matrix(6,6, [[0,1,1,0,0,0], [1,0,1,0,0,0], [1,1,0,0,0,0], [0,0,0,0,1,1], [0,0,0,1,0,1], [0,0,0,1,1,0]])

$$A := \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Then, using the 'Linear Algebra' package, the characteristic polynomial and the eigenvalues of the graph are obtained.

LinearAlgebra[CharacteristicPolynomial](A, lambda)

$$\lambda I = \begin{bmatrix} \lambda & 1 & 1 & 0 & 0 & 0 \\ 1 & \lambda & 1 & 0 & 0 & 0 \\ 1 & 1 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 1 \\ 0 & 0 & 0 & 1 & \lambda & 1 \\ 0 & 0 & 0 & 1 & 1 & \lambda \end{bmatrix}$$

$$\lambda I{-}\mathrm{A} = \begin{bmatrix} \lambda & -1 & -1 & 0 & 0 & 0 \\ -1 & \lambda & -1 & 0 & 0 & 0 \\ -1 & -1 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & -1 & -1 \\ 0 & 0 & 0 & -1 & \lambda & -1 \\ 0 & 0 & 0 & -1 & -1 & \lambda \end{bmatrix}$$

Eigenvalues $\lambda I - A$

$$\begin{bmatrix} \lambda I - 2 \\ \lambda I - 2 \\ \lambda I + 1 \\ \lambda I + 1 \\ \lambda I + 1 \\ \lambda I + 1 \end{bmatrix}.$$

4 Conclusion

In this research, a new graph called tudung saji graph has been introduced. From the construction of graph, it was found that the tudung saji graph of Kapal Layar pattern is a union of two complete graphs of three vertices, i.e. $\Gamma_\Delta = K_3 \cup K_3$. for any chosen values of j, where $j = 1, 2, 3$. Furthermore, based on the adjacency matrix and the characteristic polynomial of Kapal Layar pattern which is also a ring, it was found that the energy of tudung saji graph is equal to eight, $\varepsilon(\Gamma_\Delta) = 8$.

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