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Modelling and Pricing Temperature Using Ornstein-Uhlenbeck Process with Stochastic Speed of Mean Reversion

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Abstract In this paper, we focus on Ornstein-Uhlenbeck (OU) process with stochastic speed of mean reversion to model the temperature variations. The OU model may explains well the temperature's behaviour, but some empirical studies evidenced the problem of the constant mean reversion rate. Thus, this study suggests to represent the speed of mean reversion as an OU-type stochastic process driven by an independent pure-jump increasing Lévy process. The analytical solution of the process is defined under Skorohod integral and the change of the (log) OU process is defined under normal inverse Gaussian family of distribution. Next, we price the cumulative average temperature (CAT) futures analytically using the spot-forward relationship framework. In addition, we also provide numerical pricing of the CAT futures using Monte-Carlo simulation method. The empirical findings in this study are based on the temperature data collected in Subang, Malaysia.

Keywords Mean reversion; OU process; Levy process; futures price; normal inverse Gaussian; incubation period.

Mathematics Subject Classification 39A50, 60H35.

1 Introduction

Temperature is naturally a non-storable commodity. We cannot store and sell it later in the usual way. Thus, there is no trading for temperature in the spot but in the derivatives market. Temperature evolves dynamically in a seasonal pattern, but its change is subjected to random forces which cannot be easily predicted. In addition, temperature has its long term average level, or more intuitively it is a mean reverting process. Temperature may increases to a certain degree over certain period of time, but it will later reverts back to the average level. Similarly, it shall turns to its mean level after negatively deviates. The time it takes for temperature to arrive at its long term mean level is arbitrary. It does not necessarily moves in a periodic pattern and its dynamics is totally depending on how fast is the rate of reversal. This rate is sometimes called the speed of mean reversion which measures how fast for any mean reverting process to revert back to the average level.

To manage risks associated with temperature fluctuations, weather derivatives have been developed as financial instruments such as discussed by Alaton et al. [1]. Pricing these derivatives requires accurate modeling of temperature dynamics, capturing key statistical properties such as seasonality, mean reversion, and stochastic fluctuations. Various studies have explored different modeling approaches, with early research focusing on mean-reverting stochastic processes by Benth and Ŝaltytė Benth [9], while more recent advancements incorporate multifactor models [15] and continuous-time autoregressive moving average (CARMA) models (Darus and Taib [12]. Additionally, studies such as Benth and Ŝaltytė Benth [10] have explored fractional Brownian motion, while Groll and Meyer-Brandis [14] investigated Hawkes processes to better capture extreme events in temperature dynamics. A fundamental component of these models is mean reversion. This mean reversion plays a crucial role in describing how temperature fluctuates around a long-term average. The OU process is widely used to model this behavior, as it provides a mathematical framework for capturing mean-reverting dynamics.

Let us introduce B as a Brownian motion which has a Gaussian stationary distribution and $\sigma > 0$ is a constant parameter. In an OU process X, given in a form of stochastic differential equation

$$dX(t) = -\alpha X(t)dt + \sigma dB(t), \tag{1}$$

the speed of mean reversion of the process is normally represented by a parameter α. This process assumes that the speed of mean reversion is constant over time. However, this looks contradictory with the nature of temperature since its mean reversion rate is much depending on its current state or obviously not constant. This is supported by the findings in Zapranis and Alexandridis [17] where the mean reversion rate of daily average temperature of Paris changes considerably over time. A similar study in electricity spot market by Barlow et al. [2] found large uncertainty in the estimation of mean reversion rate. We conclude that the constant speed of mean reversion in many stochastic processes is not well-suited for capturing real-world dynamics. Since the mean reversion rate changes over time, we may explain its dynamics by: (1) time-dependent function or (2) stochastic process.

In this study, we enrich Equation (1) by changing the constancy assumption of α to the dynamical structure described by a stochastic process. However, the challenge is to determine appropriate model or process that best describes the mean reversion behaviour. We may simply refer to any stationary process for the stochastic speed of mean reversion due to the mean-reverting property of the rate. Following Benth and Khedher [6], we choose to describe α dynamics using the OU process driven by an independent pure-jump Lévy process or sometimes called subordinator. We suppose α follows an inverse Gaussian process so that α takes only positive value.

Our ultimate goal is the formulation of futures price using the spot-forward relationship framework. In particular, we price the temperature futures based on the Cumulative Average Temperature (CAT) index, that is the index for the weather derivatives traded at the Chicago Mercantile Exchange (CME). Such index is used as underlying in the summer season for cities in Canada, Europe and Asia (refer to Benth and Ŝaltytė Benth [9]). We have used an Esscher transform for the dynamics of the speed of mean reversion to ensure that the pricing process becomes martingale after discounting. Under such transformations, the constant called the market price of risk alters the dynamics of OU process and mean reversion rates.

The remaining of the paper is structured as follows. The temperature data is discussed in Section 2. We have analyzed historical temperatures of Subang, Malaysia which is not so reliable in the derivatives pricing purpose. However, we have at least show that our proposed model is applicable in pricing temperature derivatives. Our proposed model of OU-type with stochastic speed of mean reversion is introduced in Section 3. In Section 4, we present the formulation of temperatures futures. The Monte-Carlo simulation is discussed together with the effect of changing the parameter. Finally, some conclusions are offered in Section 5.

2 The Temperature Data

We use historical daily average temperatures (DATs) data measured in degree of Celsius (°C). The data are collected from Subang station, Malaysia within the period of 1 January 1997 until 9 June 2016. A total number of 7100 data is recorded but that amount has been reduced to 7095 after the record on 29 February for every leap year have been removed. We do this to synchronize the number of days in each year, which equal 365 days. There are no missing data in the record. Figure 1 plot the time series of the daily average temperature data.

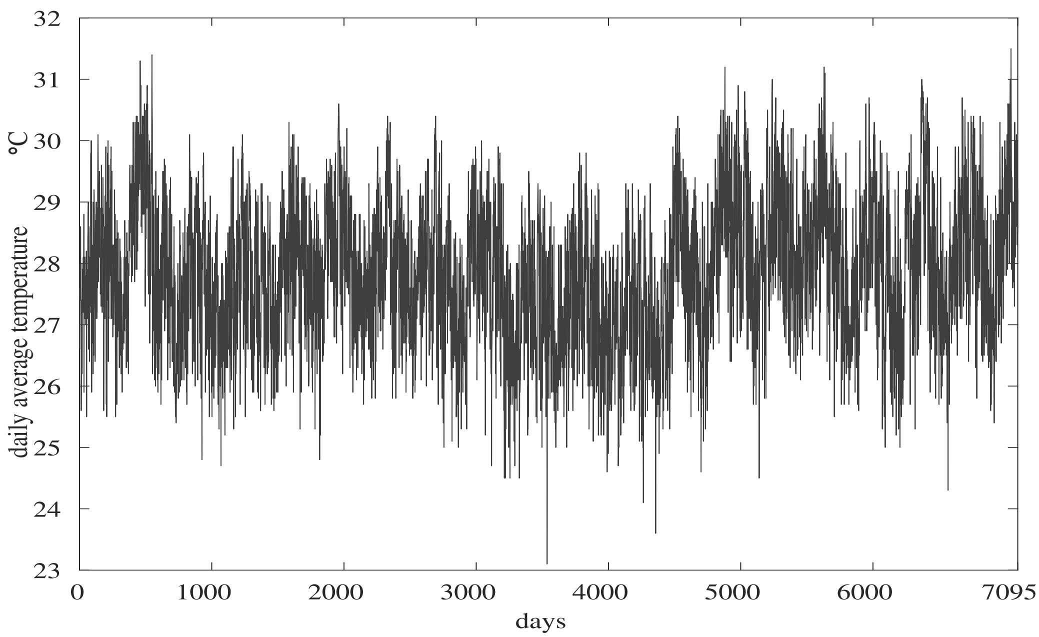


Figure 1: DATs in Subang for period starting 1 January 2011 to 9 June 2016

We see a clear seasonal pattern in the time series. Throughout the time series, we can see a seasonal pattern. Temperature moves in a regular cycle within small range of fluctuation. As reported in Table 1, the lowest and highest temperatures are 23.1 and 31.5 respectively. Estimated mean of temperature is 27.88 and volatility equals 1.15 which indicates a small variation of temperature.

The dynamics of temperature shall be explained using the OU process together with the presence of seasonality effect. Thus, we decompose the temperature $T(t)$ for time $t \geq 0$ to the

Table 1: Descriptive statistics of DATs

Min	Max	Q1	Med	Q3	Mean	Std.
23.10	31.50	27.10	27.90	28.70	27.88	1.15

deterministic seasonal function $\Lambda(t)$ and stochastic process $X(t)$ given as

$$T(t) = \Lambda(t) + X(t). \tag{2}$$

The seasonality pattern is explained by

$$\Lambda(t) = a_0 + a_1 t + a_2 \sin \frac{2\pi(t - a_3)}{365}, \tag{3}$$

which captures the linear trend and seasonal variation. We fitted temperatures data with seasonality function $\Lambda(t)$ and the parameters are estimated using nonlinear least squares method as given in Table 2. The average level of temperature is given by a_0, which equals 27.68. The

Table 2: Estimated parameters for seasonal mean function

a_0	a_1	a_2	a_3
27.6828	0.0001	0.6659	58.4473

slope of linear trend a_1 is close to zero, indicating a very small positive increment of temperature dynamics. We also observe small variation of temperature based on the amplitude parameter a_2, and is deviating approximately 0.7°C from the mean level a_0. While a_3 is a phase shift parameter that adjusts the seasonal cycle to align with the actual temperature patte rn, its value indicates a shift of approximately 58 days. The DATs together with fitted seasonal mean function are plotted in Figure 2. The figure of the last five years is presented to illustrate the fitted seasonal pattern with DATs more clearly.

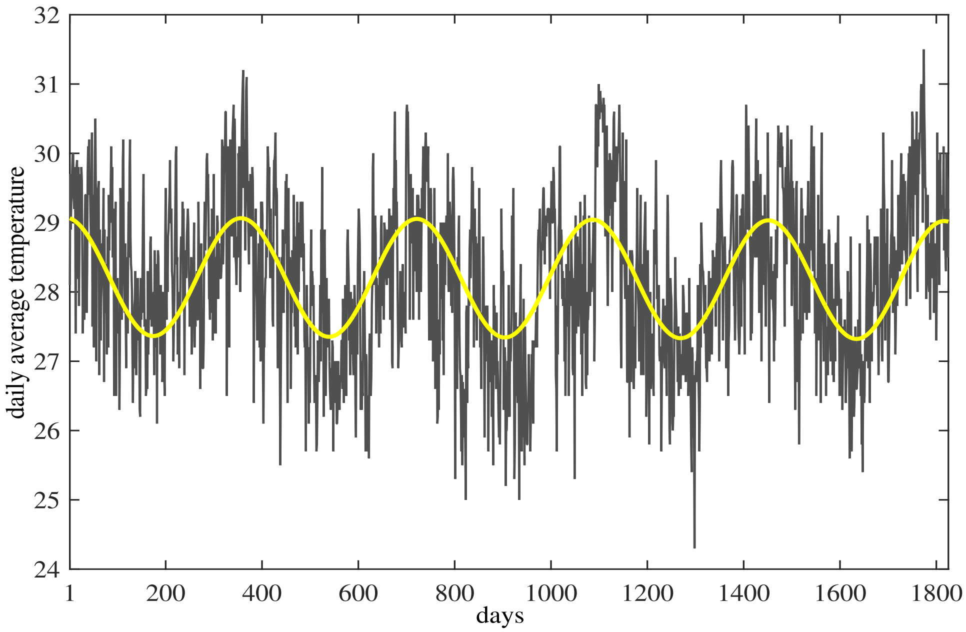


Figure 2: DATs in Subang with fitted seasonal function

The yellow curve represents the fitted seasonal mean function, which models the periodic seasonal pattern, while the gray curve shows the actual recorded DATs in Subang, Malaysia. The close alignment between the two curves indicates that the seasonal mean function effectively captures the underlying seasonal trends in the temperature data.

3 Specification of the Pricing Model

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space equipped with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$. For one dimensional Lévy process $L(t)$, we define the OU process $\{X(t)\}_{0\leq t\leq T}$ as a solution of

$$dX(t) = -\alpha(t)X(t)dt + \sigma dL(t), \tag{4}$$

where σ is a constant volatility and $\alpha(t)$ is a stochastic process describing the mean reversion level. The speed of mean reversion measures how fast the process $X(t)$ reverts back to the long-term mean level. A high value of $\alpha(t)$ indicates a fast reverting and on the contrary, a low $\alpha(t)$ value meaning slow reverting level. Thus, we restrict the $\alpha(t)$ to have only positive value, contrasted to the suggestion by Benth and Khedher [6] that the mean reversion may take negative values as well. By allowing the mean reversion rates to behave stochastically, the specification of such stochastic process becomes more complicated. One may simply choose any stationary process for its dynamics. In this paper, we define $\alpha(t)$ to be the OU process given by

$$d\alpha(t) = -\beta\alpha(t)dt + \eta\, dZ(t), \tag{5}$$

where $Z(t)$ is an independent pure-jump increasing Lévy process or sometimes called a subordinator. We have in this process a constant parameter $\beta > 0$ which represent the mean reverting rate of the mean reversion process.

Specification of Equation (4) and (5) implies that the filtration $\mathcal{F}_{t\geq 0}$ are generated by the Lévy processes L and Z. Based on the Remark 3.13 in Benth and Khedher [6], the solution of Equation (4) under Malliavin derivative in the direction of B is given by

$$X(s) = X(t)\exp\left(-\int_t^s \alpha(u)du\right) + \int_t^s \sigma \exp\left(-\int_v^s \alpha(u)du\right)\delta B(v),$$

where $\int_t^s Y\delta B(v)$ is defined under Skorohod integral. Furthermore, we have

$$\int_t^s \alpha(u)du = \beta^{-1}\left(1 - e^{-\beta(s-t)}\right) + \int_t^s \beta^{-1}\left(1 - e^{-\beta(s-u)}\right)dZ(u),$$

by assuming that α is integrable on $[t,s]$ for $s < \infty$ and

$$\mathbb{E}\left[\exp\left(2\int_t^s \alpha(u)du\right)\right] < \infty.$$

The choice of $Z(t)$ being subordinator is such that the process has only positive increments and no drift, and hence $\alpha(t)$ is defined as positive stochastic process for which the speed of mean reversion makes sense. Following Taib [16], we let $\alpha(t)$ follow the inverse Gaussian law, therefore the increments of Equation (4) would approximately be NIG$(\tilde{\alpha},\tilde{\beta},\tilde{\delta},\tilde{\mu})$ distributed. Briefly, the NIG is a class of generalized hyperbolic distributions with density functions given by

$$f(x;\tilde{\alpha},\tilde{\beta},\tilde{\delta},\tilde{\mu}) = k\exp(\tilde{\beta}(x-\tilde{\mu}))\frac{K_1(\tilde{\alpha}\sqrt{\tilde{\delta}^2+(x-\tilde{\mu})^2})}{\sqrt{\tilde{\delta}^2+(x-\tilde{\mu})^2}},$$

where $k = \tilde{\delta}\tilde{\alpha}\exp(\tilde{\delta}\sqrt{\tilde{\alpha}^2-\tilde{\beta}^2})/\pi$ and $K_1(x)$ is the modified Bessel function of the third kind with index 1. Parameters $\tilde{\alpha}$ and $\tilde{\beta}$ measure the tail heaviness and skewness respectively. The $\tilde{\delta}$ is a scale parameter and $\tilde{\mu}$ is a location parameter of the distribution. Reader is encouraged to read Barndorff-Nielsen [3] or Benth et al. [9] for more detail on the NIG distribution.

The stochastic process can be classified as NIG Lévy process if $L(1)$ is distributed according to the NIG distribution. The Lévy measure of $L(t)$ is given by (see Barndorff-Nielsen and Shephard [4])

$$l(dz) = \frac{\tilde{\alpha}\tilde{\delta}}{\pi|z|}e^{\tilde{\beta}z}K_1(\tilde{\alpha}|z|)dz, \tag{6}$$

and the cumulant function is

$$\phi(\lambda) = i\lambda\tilde{\mu} + \tilde{\delta}(\sqrt{\tilde{\alpha}^2-\tilde{\beta}^2} - \sqrt{\tilde{\alpha}^2-(\tilde{\beta}+i\lambda)^2}). \tag{7}$$

For the purpose of derivatives pricing, we have to translate the real world measure $\mathbb{P}$ into the risk neutral measure $\mathbb{Q}$. Since the models are driven by class of Lévy processes (which in particular Brownian motion for the OU process Equation 4), we rely on the Girsanov and Esscher transformations. To specify the risk neutral probability measure $\mathbb{Q}$ for stochastic speed of mean reversion, we require an exponential integrability condition on the Lévy as follows. The following Lemma take from Benth and Šaltytė-Benth [8], ensure the finite moment condition exists.

Condition 1 (Benth and Šaltytė-Benth [8])
There exist a constant $k > 0$ such that the Lévy measure satisfies the integrability condition

$$\int_1^\infty e^{kz}\ell(dz) < \infty.$$

Lemma 1 *If $g : [0,t] \mapsto \mathbb{R}$ is bounded and measurable function and Condition 1 holds for $k := \sup_{s\in[0,t]} |g(s)|$, then*

$$\mathbb{E}\left[\exp\left(\int_0^t g(u)dL(u)\right)\right] = \exp\left(\int_0^t \phi(g(u))du\right),$$

given that $\phi(u) = \psi(-i\lambda)$.

Proof See Benth and Šaltytė-Benth [8].

We introduce $\pi_Z(t)$ to be the density process of a measure $\mathbb{Q} \sim P$ defined under Radon-Nikodym derivatives as

$$\frac{d\mathbb{Q}}{dP} = \pi_Z(t), \tag{8}$$

where

$$\pi_Z(t) = \exp\left(\theta Z(t) - \phi_Z(\theta)\, t\right), \tag{9}$$

for a constant market price of risk, θ.

4 The Price of Temperature Derivatives

We consider the Cumulative Average Temperature (CAT) index in pricing the temperature futures. The CAT index is a measurement of cumulative amount of temperatures for a certain period $[\tau_1, \tau_2]$. It can be simply defined as

$$\text{CAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T(t)dt. \tag{10}$$

Temperature futures can be entered at any time $t \leq \tau_1$ with delivery or settlement at time τ_2. We pay nothing to enter the contract which implies that

$$e^{-r(\tau_2 - t)}\mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2} T(s)ds - F(t, \tau_1, \tau_2) \mid \mathcal{F}_t\right] = 0, \tag{11}$$

from the arbitrage pricing theory (see Duffie [13]). Hence, the price of CAT futures for any $0 \leq t \leq \tau_1 \leq \tau_2 \leq \infty$ is given as

$$F(t, \tau_1, \tau_2) = \mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2} T(s)ds \mid \mathcal{F}_t\right], \tag{12}$$

where r is a constant risk-free rate of the return and $\mathbb{E}_Q[\cdot]$ is the expectation operator with respect to the pricing measure Q. Note that in this derivation, we set the driving factor L be the Brownian motion B.

Proposition 1 *The price of a CAT futures at time t for contract with delivery at time $0 \leq t \leq \tau_1$ is given as*

$$\begin{aligned} F(t,\tau_1,\tau_2) = &\int_{\tau_1}^{\tau_2} \Lambda(s)ds \\ &- \int_{\tau_1}^{\tau_2} X(t) \exp\left(-\kappa(s-t) + \int_t^s \{\phi_{Z_2}(\theta_{Z_2} + \kappa(s-u)) - \phi_{Z_2}(\theta_{Z_2})\}du\right) ds. \end{aligned}$$

where $\kappa(x) = \beta^{-1}\left(1 - e^{-\beta x}\right)$.

Proof From Equation (12) and Fubini's theorem, we have

$$\mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2} T(s)ds \mid \mathcal{F}_t\right] = \int_{\tau_1}^{\tau_2} \Lambda(s)ds + \int_{\tau_1}^{\tau_2} \mathbb{E}_Q\left[X(s) \mid \mathcal{F}_t\right] ds. \tag{13}$$

Referring to Remark 3.13 in Benth and Khedher [6], the second term in Equation (13) can be written as

$$\begin{aligned} \mathbb{E}_Q\left[X(s) \mid \mathcal{F}_t\right] = &X_t \exp\left(-\beta^{-1}\left(1 - e^{-\beta(s-t)}\right)\right) \\ &\times \mathbb{E}_Q\left[-\exp\left(\int_t^s \beta^{-1}\left(1 - e^{-\beta(s-u)}\right) dZ(u)\right) \mid \mathcal{F}_t\right]. \end{aligned}$$

Appealing Bayes' formula to the conditional expectation, we have

$$\begin{aligned} &\mathbb{E}_Q\left[-\exp\left(\int_t^s \beta^{-1}\left(1 - e^{-\beta(s-u)}\right) dZ(u)\right) \mid \mathcal{F}_t\right] \\ &= \mathbb{E}\left[-\exp\left(\int_t^s \beta^{-1}\left(1 - e^{-\beta(s-u)}\right) dZ(u)\right) \frac{\pi_Z(s)}{\pi_Z(t)} \mid \mathcal{F}_t\right] \\ &= -\exp\left(-\int_t^s \phi_Z(\theta_Z)du\right) \mathbb{E}\left[\exp\left(\int_t^s \left\{\theta_Z + \beta^{-1}\left(1 - e^{-\beta(s-u)}\right)\right\} dZ(u)\right)\right] \\ &= -\exp\left(-\int_t^s \phi_Z(\theta_Z)du\right)\left[\exp\left(\int_t^s \phi - Z\left(\theta_Z + \beta^{-1}\left(1 - e^{-\beta(s-u)}\right)\right) du\right)\right] \\ &= -\exp\left(\int_t^s \left\{\phi_Z\left(\theta_Z + \beta^{-1}\left(1 - e^{-\beta(s-u)}\right)\right) - \phi_Z(\theta_Z)\right\} du\right). \end{aligned}$$

Hence, the proposition follows.

4.1 Pricing of Temperature Futures

Temperature can be classified as non-storable commodity which does not require us to have an equivalent martingale measure. It can just be an equivalent measure which implies that there is more than one choice for $\mathbb{Q}$. In the sequel, we omit the $\mathbb{Q}$ in the associated expectation $\mathbb{E}_{\mathbb{Q}}$.

Now, we want to solve Equation (12) using Monte Carlo simulation method. Initially, we discretize Equation (4) together with Equation (5) using Euler scheme

$$\begin{aligned} X(t+1) &= e^{-\alpha(t)}X(t) + e^{-\alpha(t)}\sigma(t)\Delta B(t), \\ \alpha(t+1) &= e^{-\beta}\alpha(t) + e^{-\beta}\Delta Z(t). \end{aligned} \tag{14}$$

We model the deseasonalized temperature $X(t) = T(t) - \Lambda(t)$ by an autoregressive process, AR(1) given by

$$x(t) = \hat{\beta}x(t-1) + \epsilon(t),$$

where $X(t) \approx x(t)$, $\hat{\beta}$ is a constant and ϵ is $i.i.d$ residuals. By fitting $X(t)$ with AR(1), we obtain the value of $\hat{\beta}$ equal 0.6029. This $\hat{\beta}$ value indicates that the speed of mean reversion of the deseasonalized temperature is rather fast. This is considered fast because it determines how quickly the temperature deviations return to their long-term average. A higher value of $\hat{\beta}$ leads to a shorter half-life, which means that the process corrects itself more quickly after a disturbance. Referring to Clewlow and Strickland [11], we calculate the half life of temperature with mean reversion $\hat{\beta}$ for OU process driven by Brownian motion using

$$\tau_\alpha = \frac{\ln(2)}{\alpha}.$$

By converting the discrete speed of mean reversion $\hat{\beta}$ into continuous mean reversion rate, we find $\alpha = -\ln(\hat{\beta}) = 0.51$. Therefore, the estimated half life of temperature is equal 1.37 which can be translated as on average it takes 1 day and 9 hours for the deseasonalized temperature to revert half way back to its long term mean level.

In Figure 3, we show the histogram of residuals from the regression.

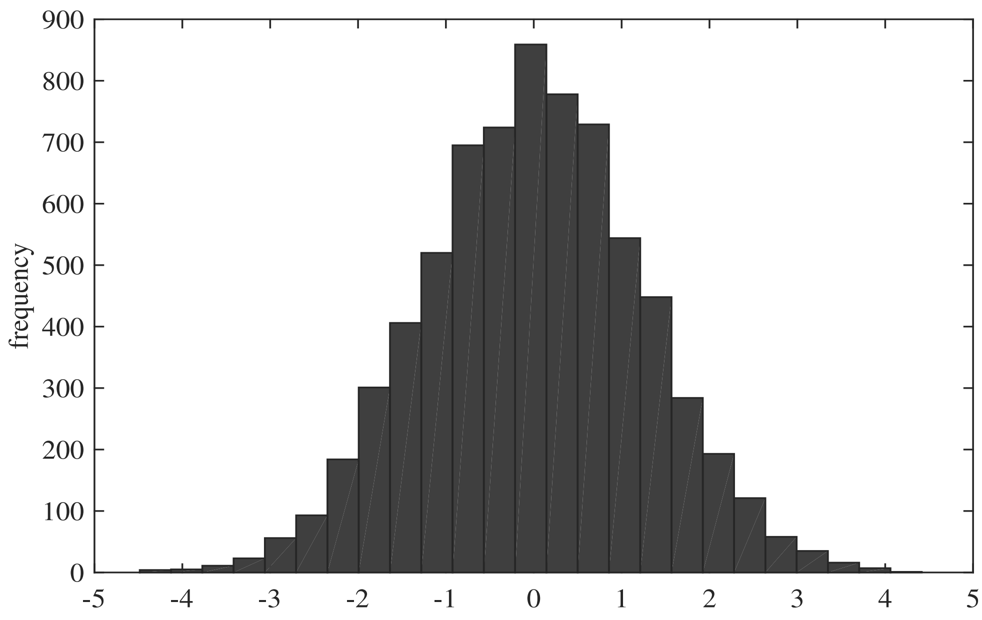


Figure 3: Histogram of residuals

We let $\alpha(t)$ be IG(δ, γ) distributed. By letting $\alpha(t) \sim$ IG(δ, γ) and assuming $\epsilon \sim N(0, 1)$, then $\Delta X(t)$ becomes approximately NIG$(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu})$ distributed. We take the residuals after removing the AR(1) effect to be fitted with NIG distribution. The estimates of NIG are given in Table 3.

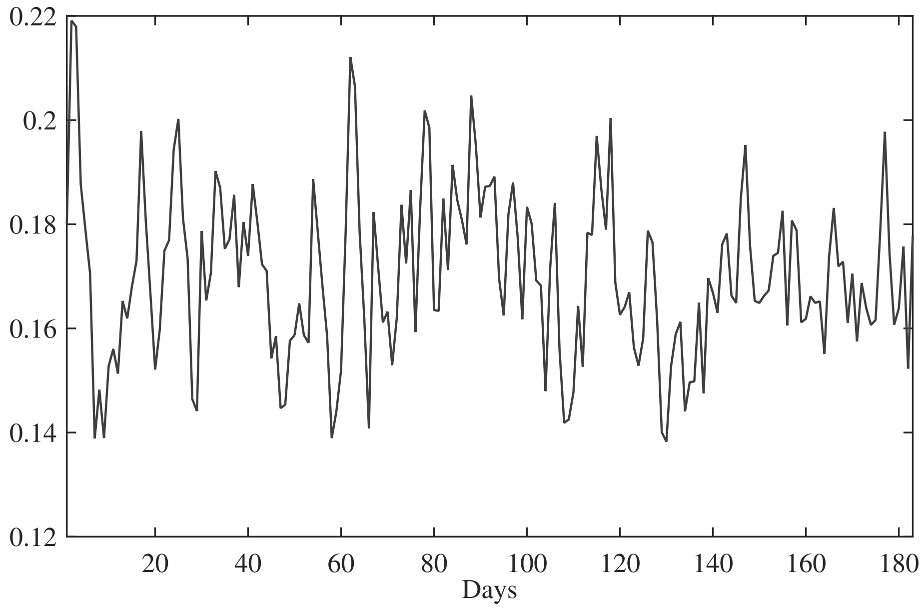
Table 3: Estimated parameters for NIG-fitted distribution

$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\mu}$	$\tilde{\delta}$
2.3763	0.6000	4.6326	-0.2736

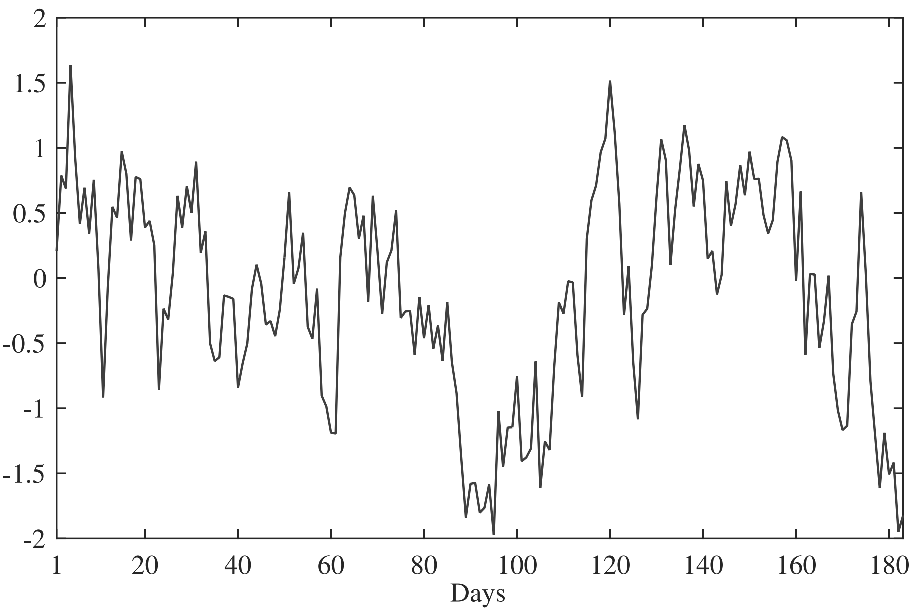
To obtain the price of temperatures futures, one should have an expectation of the cumulative average temperatures for a certain coverage period, i.e. $[\tau_1, \tau_2]$. Such expectation can be numerically solved by having paths of simulated temperatures, where certain number of realizations of temperatures can be obtained. This is precisely done by simulating $X(t)$, and by adding $\Lambda(t)$, we finally have the simulated temperatures at certain time t. However, simulation of $X(t)$ requires step-by-step procedures where one need to run simulations of mean reversion rate $\alpha(t)$, and later those simulated values will be used in simulating $X(t)$. In all simulations, we set the number of simulations, $n = 1000$, meaning that the procedures are repeated for 1000 times.

We use second equation in (14) to simulate the mean reversion rates. Since this process is not observable and the parameters of the model cannot be directly obtained, we do the simulation of speed of mean reversion $\alpha(t)$ using parameters reported in Table 3. The simulation provides us with the values of $\alpha(t)$ that omit the positive values of the rates. The simulation start by generating n random numbers from inverse Gaussian distribution, IG(δ, γ). By setting $\alpha(0) = 0.51$ (estimated α from fitting AR(1)) and parameter $\hat{\beta}$ equals 0.6029, we do the simulations for $t = 1, ..., 183$ days (this is approximately covers 6 months period). Next, we generate n random numbers from the standard normal distribution, $N(0, 1)$, and with the simulated $\alpha(t)$, we simulate $X(t)$ starting from $X(0) = 0$ with $\sigma = 0.5551$ (value obtained from fitting the residuals with normal distribution).

In Figure 4, we present the simulated paths of $\alpha(t)$ and $X(t)$. It is obvious that the simulated data of $\alpha(t)$ show positive values and $X(t)$ may takes negative values as well due to the negative random numbers generated from standard normal distribution. There exists time when $\alpha(t)$ value is small which reflects the slow speed of mean reversion towards its long term mean, and in the contrary, a high $\alpha(t)$ value represents a fast mean reversion rate.



(a) $\alpha(t)$

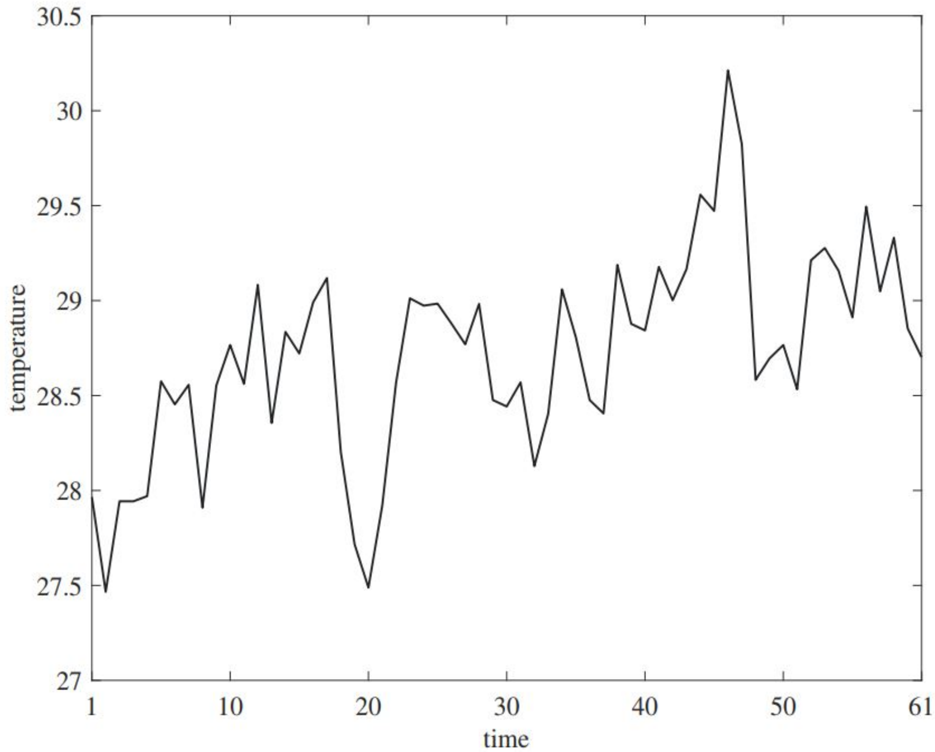


(b) $X(t)$

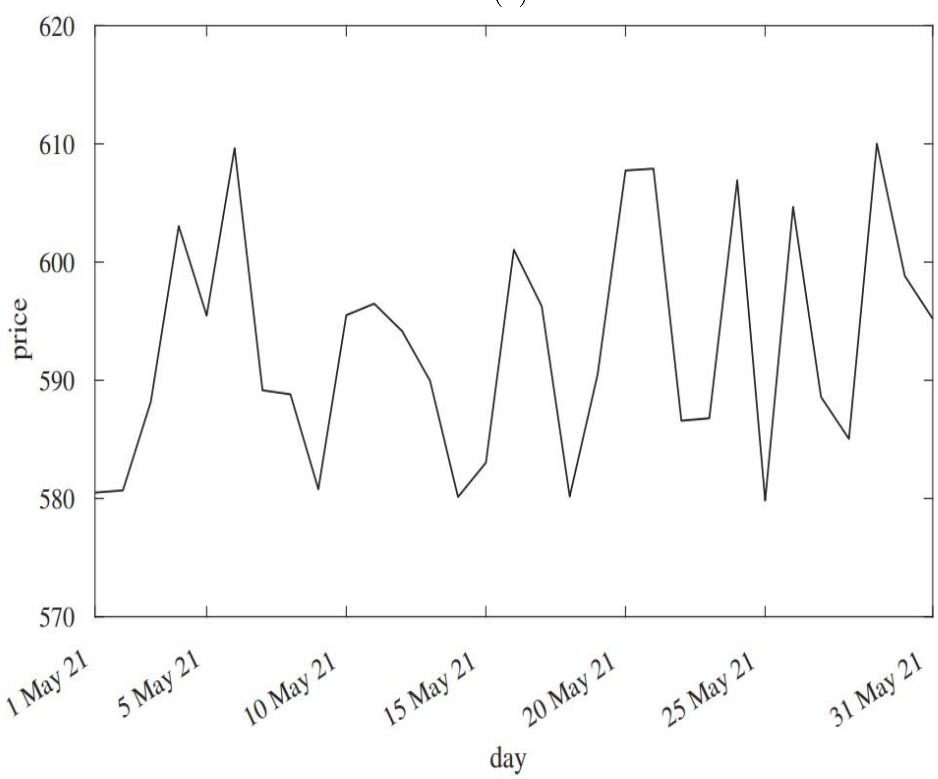
Figure 4: Time series of simulated (a) mean reversion rates, $\alpha(t)$ and (b) $X(t)$.

Looking at the trends and patterns observed in the simulation results, the mean reversion rate, $\alpha(t)$, fluctuates over time but always remains positive. This is to ensure that temperature deviations consistently revert to the mean. A higher value of $\alpha(t)$ indicates a faster return to equilibrium, while a lower value suggests a slower adjustment.

The simulated temperatures for May 2016 to June 2016 are plotted in Figure 5.



(a) DATs



(b) Price of temperature futures

Figure 5: Time series of simulated (a) DATs and (b) price of temperature futures.

To price the CAT futures, we discretize 12 and the settlement can be simply defined as

$$F(t, \tau_1, \tau_2) = \frac{k}{\tau_2 - \tau_1} \times \mathrm{CAT}(\tau_1, \tau_2), \tag{15}$$

where k is the money factor. For this pricing purpose, we set $k = 20$ (just for illustration). We take May 1, 2016 as the date of engaging in the contract. The contract period is specified to be June 2016. Based on our simulated temperatures, we sum the temperatures value from 1 June 2016 until 30 June 2016 which reflects the contract period for one month. We have obtained the price of temperature futures on 1 May 2016 for that particular period which equals 580.50. This is the price that should be paid if one want to buy the contract on 1 May 2016 for coverage in June 2016. We also plotted the price for 2 May 2016 until 31 May 2016 in Figure 5.

5 Conclusion

In this paper, we have introduced the OU process with the presence of stochastic speed of mean reversion driven by the Lévy process. We have specified the driving process for the mean reversion to be independent pure jump increasing Lévy process or subordinators to ensure its positivity. Since OU process is given in terms of stochastic differential equation, the challenge is to find its analytical solution which is possible by appealing to Malliavin calculus and solving the differential equation under Skorohod integral.

We have analyzed the time series of Subang's temperature data. It is hard to calibrate the parameters of the OU process that defines the speed of mean reversion since the mean reversion levels are not naturally observed. To ease the computation, we assume that speed of mean reversion follows the NIG distribution. This assumption may not be accurate in defining the mean reversion level empirically, but it helps us with numerical simulation of temperatures.

We have used our model to numerically price the temperature futures. The purpose of the pricing is just to show the applicability of the model even though there is no temperature derivatives traded for the selected data. We have first simulated the mean reversion rates and the OU process thereafter. Finally, we obtained the simulated temperatures and later find the price of temperature futures. However, there are still open questions left for future study. One may looks further on the stationary of the OU process for the case when volatility and speed of mean reversion are considered as stochastic processes simultaneously. The analytical solution of the stochastic differential equation for such process may also be investigated.

Acknowledgments

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