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Triangular Z-Numbers Using Centroid Method: A Case Study of Flood Mitigation Measures

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Abstract Z-numbers offer a robust framework for modeling human reasoning under uncertainty by integrating both a constraint and a reliability component. Despite their theoretical strengths, the practical application in Fuzzy Multi-Criteria Decision Making (FMCDM) has been limited due to the complexity of Z-numbers arithmetic and the lack of an effective defuzzification method that can preserve information during conversion to crisp values. To address these challenges, this study proposes a novel FMCDM methodology that employs triangular Z-numbers, a simplified representation to improve computational efficiency. The centroid method is adopted for defuzzification, selected for its ability to preserve the information during defuzzification process. This proposed methodology enhances both the computational feasibility and practical usability of Z-numbers, allowing decision-makers to express their judgments and confidence levels without relying on complex mathematical formulations. The effectiveness of the proposed methodology is demonstrated through a case study on flood mitigation measures, showing its ability to produce reliable and consistent rankings of alternatives. Comparative analysis with existing methods further validates the robustness and applicability of the proposed methodology, making it a viable tool for complex decision-making processes.

Keywords Z-number; Triangular Z-numbers; Centroid method; Multi-criteria decision making; FMCDM method.

Mathematics Subject Classification 03E72, 28E10, 90C70.

1 Introduction

Z-numbers, initially proposed by Zadeh [1], extend classical fuzzy set theory to address imprecise, incomplete, and unreliable information by incorporating two ordered pairs of fuzzy numbers: a restriction component and a reliability component [1, 2]. This dual structure offers a more comprehensive approach to capturing the vagueness and uncertainty inherent in many real-world scenarios [3–5]. Notably, Z-numbers are capable of modeling both the uncertainty conveyed through natural language [6, 7] and the reliability that reflects the confidence in the accuracy of the information [8]. In this manner, Z-numbers enhance the representation of subjective judgment by modeling what is believed and the strength of that belief. For instance, in the context of supplier selection, an expert might express an opinion as, "Supplier A is superior to Supplier B, and I am very confident in this judgment." In this statement, the first part represents the assessment (restriction), while the second part expresses the confidence level (reliability), which collectively forms a Z-number.

Since their introduction, Z-numbers have experienced significant theoretical advancements, enhancing their functionality and broadening their range of applications. Researchers have demonstrated a keen interest in refining this theory further. Aliev *et al.* [9] made a noteworthy contribution to the arithmetic of discrete Z-numbers by introducing essential operations, including addition, subtraction, multiplication, division, and square root. Building on their previous work, Aliev *et al.* [10] expanded the arithmetic framework of Z-numbers by utilizing horizontal membership functions, which yielded more informative outcomes. Recently, Massanet *et al.* [11] introduced mixed-discrete Z-numbers to help alleviate the computational complexity associated with Z-number operations. Additionally, to better align the evaluation of Z-numbers with human intuition regarding confidence and precision, Mohamad *et al.* [12] have proposed ordered Z-numbers. This, ultimately, offers a structured total ordering framework that maintains both the restrictions and reliability components, enabling more intuitive ranking.

The notion of Z-numbers has evolved from discrete to continuous elements. For example, Aliev *et al.* [13] developed arithmetic operations for continuous Z-numbers, encompassing addition, subtraction, multiplication, division, maximum, minimum, and square root. Meanwhile, Qiu *et al.* [14] introduced the generalized difference operation for continuous Z-numbers and proposed a ranking function based on the generalized centroid approach. Abdullahi *et al.* [15] further defined both ordered discrete and continuous Z-numbers, establishing a relationship between these Z-numbers and arbitrarily ordered subsets of real numbers, which is crucial for the advancement of temporal Z-numbers.

In addition, recent advancements have highlighted the potential for integrating Z-numbers with the extension of fuzzy sets, resulting in more effective tools for managing imprecise or uncertain information. For instance, Sari and Kahraman [16] as well as Alam *et al.* [17] have successfully incorporated Z-numbers into Intuitionistic fuzzy sets, utilizing the membership and non-membership functions to capture uncertainty and reliability more effectively. Similarly, Yong *et al.* [18] and Du *et al.* [19] have introduced Z-numbers into Neutrosophic sets, enhancing the representation of truth, indeterminacy, and falsity membership functions. This was achieved by incorporating layers of restriction and reliability components into each element.

The concept of Z-numbers highlights the significance of representing both uncertainty and reliability in decision-making processes [20]. This enables a more transparent and intuitive representation of various types of uncertainty in input data, applicable to both quantitative and qualitative information, which is crucial for effective decision-making [21]. Several studies, including those by Aliev *et al.* [9], Tarmudi and Abdullah [22], and Poleshchuk [23], have employed the concept of Z-information to represent information using Z-numbers.

Despite their advantages, Z-numbers remain challenging to interpret and apply, mainly due to the probabilistic nature of the reliability component [7]. To address these issues and enhance practical usability, one common approach is to represent them in terms of fuzzy numbers [24]. Recently, Li *et al.* [25] proposed triangular Z-numbers, using an extension of the triangular distribution to reduce the calculation complexity of classical Z-numbers. Their findings, which include basic operations (i.e., addition, subtraction, multiplication, division, minimum and maximum, and negation), demonstrated that triangular Z-numbers offer higher precision and lower entropy than the classical Z-numbers.

Nevertheless, a key challenge lies in finding a suitable defuzzification (or ranking) method that can convert Z-numbers into crisp values without incurring significant information loss [26–28]. Thus, developing an effective defuzzification method is critical for the successful application of Z-numbers in real-world decision-making. While most existing defuzzification methods, such as the maximum, weighted average, center of sums, mean of max membership, and center of largest area, rely on converting Z-numbers into regular fuzzy numbers, often leading to a loss of information, particularly from the reliability component [29].

In contrast, the Center of Gravity (COG), also known as the centroid method, is widely regarded as one of the most reliable and intuitive techniques for determining the center of mass. For example, Bakar and Gegov [30] employed a centroid point and spread-based multi-layer decision methodology, demonstrating more consistent ranking results than the existing methods. Similarly, Abdullahi *et al.* [31] also studied a centroid-based defuzzification technique for trapezoidal Z-numbers. However, their approach was limited to single-dimensional comparisons and did not consider the Multi-Criteria Decision Making (MCDM) context, which is crucial in real-world applications involving complex evaluation. Furthermore, Alam *et al.* [17] proposed an intuitive centroid method for defuzzifying Intuitionistic Fuzzy Z-Numbers. The study also indicated that this approach enhanced the accuracy and interpretability of the ranking of alternatives by effectively preserving both uncertainty and reliability information.

However, to date, the direct use of the centroid method for defuzzifying Z-numbers in the decision-making scenarios remains unexplored. In this paper, we address this gap by proposing a novel centroid-based defuzification method tailored for triangular Z-numbers in a multi-criteria decision-making context. Specifically, we adopt the centroid method introduced by Hadi-Vencheh and Mokhtarian [32] to transform the triangular Z-numbers for each criterion and alternative into crisp values. The structure of this paper is as follows: Section 1 introduces the topic, followed by the preliminaries in Section 2, Section 3 outlines the proposed methodology, Section 4 presents a numerical example, and Section 5 concludes the paper.

2 Preliminaries

This section presents key concepts related to fuzzy set theory, including the defuzzification of Z-numbers using the centroid method and the MCDM method.

2.1 Fuzzy Set Theory

A fuzzy set is the extension of the classical sets that allows partial membership degrees to be assigned to elements of a particular set [33].

Fuzzy Set [33]: A fuzzy set A defined on a universe of discourse X may be expressed as:

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

where the value $\mu_A(x)$ describes the degree of membership of $x \in X$ in A and $\mu_A(x) : X \to [0, 1]$.

A type of fuzzy set commonly used to quantify fuzzy values is known as fuzzy numbers. Generally, fuzzy numbers are defined as follows:

Fuzzy Numbers [34]: A fuzzy number $\widetilde{\mathrm{A}}$ with a membership function $\mu_{\widetilde{\mathrm{A}}} : R \to [0, 1]$ can be defined as follows:

$$\mu_{\widetilde{\mathrm{A}}}(x) = \begin{cases} \mu^L_{\widetilde{\mathrm{A}}}(x), & a_1 \le x \le a_2 \\ \omega, & a_2 \le x \le a_3 \\ \mu^R_{\widetilde{\mathrm{A}}}(x), & a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases},$$

where $\mu^L_{\widetilde{\mathrm{A}}}(x)$ is the left membership function: $[a_1, a_2] \to [0, \omega]$ and $\mu^R_{\widetilde{\mathrm{A}}}(x)$ is the right membership function: $[a_3, a_4] \to [0, \omega]$. Here, $\mu^L_{\widetilde{\mathrm{A}}}(x)$ and $\mu^R_{\widetilde{\mathrm{A}}}(x)$ are monotonic and continuous mappings from the real line, R, to the closed interval $[0, \omega]$. If $\omega = 1$, $\widetilde{\mathrm{A}}$ is deemed a normal fuzzy number, and if $\mu_{\widetilde{\mathrm{A}}}(x)$ is piecewise linear, then $\widetilde{\mathrm{A}}$is referred to as a Trapezoidal Fuzzy Number (TrFN) and can be denoted as follows:

$$\widetilde{A} = (a_1, a_2, a_3, a_4; \omega).$$

If $\omega = 1$, $\widetilde{A} = (a_1, a_2, a_3, a_4)$. Besides, if $a_2 = a_3$, $\widetilde{A}$ is referred to as a Triangular Fuzzy Number (TFN).

Triangular Fuzzy Numbers [35]: A TFN defined by a triplet (a_1, a_2, a_3), where the membership function can be expressed and illustrated as follows (see Figure 1):

$$\widetilde{\mu}_A(x) = \begin{cases} 0, & x \in (-\infty, a_1) \\ \dfrac{x - a_1}{a_2 - a_1}, & x \in (a_1, a_2) \\ \dfrac{a_3 - x}{a_3 - a_2}, & x \in (a_2, a_3) \\ 0, & x \in (a_3, +\infty) \end{cases}.$$

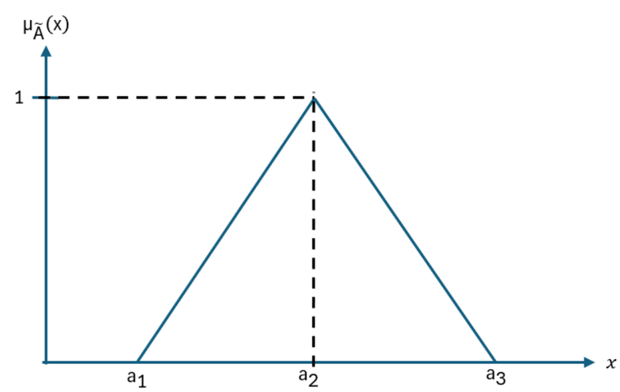

Figure 1: Triangular Fuzzy Number

Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be two TFNs. The basic arithmetic operations of TFNs, Kaufman and Gupta [36], are presented as:

- Addition: $\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,
- Subtraction: $\widetilde{A} \ominus \widetilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$,
- Multiplication: $\widetilde{A} \otimes \widetilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$,
- Division: $\dfrac{\widetilde{A}}{\widetilde{B}} = \left(\dfrac{a_1}{b_1}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_3}\right)$ such that, $b_1, b_2, b_3 \neq 0$.

In this paper, the fuzzy weighted average of fuzzy numbers will be used to aggregate the rating criteria and their corresponding importance weights.

Fuzzy weighted Average of Fuzzy Numbers [37]: Let there exist $\omega > 0$ and an index $i \in N_m$ such that $W_i(\omega) = 1$. A fuzzy weighted average of fuzzy numbers U_1, U_2, ..., U_m with fuzzy weights W_1, W_2, ..., W_m can be defined as:

$$U^W(u) = \max\left\{ \min\left\{ \begin{array}{l} U_1(u_1), U_2(u_2), ..., U_m(u_m), \\ W_1(\omega_1), W_2(\omega_2), ..., W_m(\omega_m) \end{array} \right\} \right\}.$$

Such that, $u = \dfrac{\omega_1 u_1 + \omega_2 u_2 + \cdots + \omega_m u_m}{\omega_1 + \omega_2 + \cdots + \omega_m}$ and $\sum_{i=1}^{m} \omega_i > 0$, where U^W represents the fuzzification of u.

As one of the fuzzy set extensions introduced by Zadeh, Z-numbers enhance existing fuzzy set extensions by combining both uncertainty and reliability elements, which were previously considered separately in earlier research on fuzzy sets.

Z-numbers [1]: A Z-number is an ordered pair of fuzzy numbers denoted as $Z = (\widetilde{A}, \widetilde{B})$ such that, the first component $\widetilde{A}$ is a restriction on the values, is a real-valued uncertain variable X , and the second component $\widetilde{B}$ is a measure of reliability for the first component.

Discrete Z-numbers [9]: A discrete Z-number is an ordered pair of $Z = (\widetilde{A}, \widetilde{B})$, where $\widetilde{A}$ is a discrete fuzzy number representing a fuzzy restriction on values that a random variable X may take, i.e., X is $\widetilde{A}$ and $\widetilde{B}$ is a discrete fuzzy number with a membership function $\mu_{\widetilde{\mathrm{B}}} : \{b_1, \ldots, b_n\} \to [0, 1]$, $\{b_1, \ldots, b_n\} \subset [0, 1]$, representing a fuzzy restriction on the probability measure of $\widetilde{A}$, $P(\widetilde{A})$ is $\widetilde{B}$.

Triangular Z-numbers [38]: Let a Z-number denoted by $Z = (\widetilde{A}, \widetilde{B})$, where $\widetilde{A} = \{a_1, a_2, a_3\}$ and $\widetilde{B} = \{b_1, b_2, b_3\}$, be two TFNs. The membership functions can be expressed and illustrated as (see Figure 2):

$$\mu_{\widetilde{\mathrm{A}}}(x) = \begin{cases} 0 & , \ x \in (-\infty, a_1) \\ \dfrac{x - a_1}{a_2 - a_1} & , \ x \in [a_1, a_2] \\ \dfrac{a_3 - x}{a_3 - a_2} & , \ x \in [a_2, a_3] \\ 0 & , \ x \in (a_3, +\infty) \end{cases} , \quad \mu_{\widetilde{\mathrm{B}}}(x) = \begin{cases} 0 & , \ x \in (-\infty, b_1) \\ \dfrac{x - b_1}{b_2 - b_1} & , \ x \in [b_1, b_2] \\ \dfrac{b_3 - x}{b_3 - b_2} & , \ x \in [b_2, b_3] \\ 0 & , \ x \in (b_3, +\infty) \end{cases} .$$

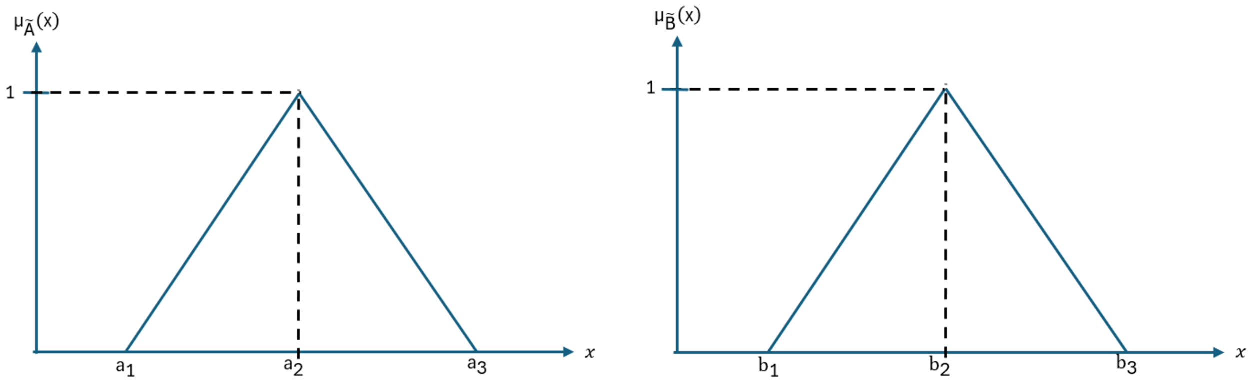

Figure 2: Triangular Z-numbers

Previous researchers have converted Z-numbers into classic fuzzy numbers to reduce the complexity of Z-number operations. To apply Z-numbers to the decision-making process, Kang *et al.* [24] proposed fuzzy expectation as a method to convert Z-numbers into regular fuzzy numbers.

Fuzzy Expectation of Z-Numbers [24]: Let a Z-number, $Z = (\widetilde{A}, \widetilde{B})$ such that $\widetilde{A} = \left\{\left\langle x, \mu_{\widetilde{A}}(x)\right\rangle | x \in [0,1]\right\}$ and $\widetilde{B} = \left\{\left\langle x, \mu_{\widetilde{B}}(x)\right\rangle | x \in [0,1]\right\}$ where $\mu_{\widetilde{A}}(x)$ is a trapezoidal membership function and $\mu_{\widetilde{B}}(x)$ is a triangular membership function. The formula to convert the reliability component, $\widetilde{B}$, into crisp numbers, to calculate the weighted Z-numbers $\left(E_{\widetilde{A}^{\alpha}}(x)\right)$, and to transform the irregular fuzzy number to a regular fuzzy number, $\widetilde{Z}'$, can be described as follows:

$$\alpha = \frac{\int x \mu_{\widetilde{B}}(x) dx}{\int \mu_{\widetilde{B}}(x) dx},$$

$$\widetilde{Z}' = \left\{\left\langle x, \mu_{\widetilde{Z}^{v}}(x)\right\rangle | \mu_{\widetilde{Z}^{\alpha}}(x) = \mu_{\widetilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), x \in [0,1]\right\}.$$

2.2 Defuzzification Method: Centroid Method

The defuzzification method is a technique utilized to convert fuzzy numbers into precise crisp values, thereby facilitating a more accurate decision-making process. This study employs the COG, also referred to as the centroid method, due to its theoretical robustness and widespread application in fuzzy decision-making approaches. Notably, the centroid method for fuzzy numbers was first introduced by Yager in 1980 as a method for ranking fuzzy numbers. It entails determining the balance point of the area under the curve of the membership function of a fuzzy number.

Centroid Concept of Fuzzy Numbers [39]: Let x be the horizontal coordinate as the ranking index, the centroid for a fuzzy number, $\widetilde{A}$ is expressed as:

$$x_{\widetilde{A}} = \frac{\int_0^1 \omega(x) f_{\widetilde{A}}(x) dx}{\int_0^1 f_{\widetilde{A}}(x) dx},$$

where $\omega(x)$ denotes the weight function that reflects the significance of the value x, and $f_{\widetilde{A}}(x)$ denotes the membership function of the fuzzy numbers.

Centroid Method for Trapezoidal Fuzzy Numbers [40]: Let $\widetilde{A} = (a_1, a_2, a_3, a_4; \omega)$ be the TrFNs. The centroid method of the TrFNs, $\widetilde{A}$ is defined as:

$$x_0\left(\widetilde{A}\right) = 1/3\left(a_1 + a_2 + a_3 + a_4 - \frac{a_4a_3 - a_2a_1}{(a_4 + a_3) - (a_2 + a_1)}\right),$$

$$y_0\left(\widetilde{A}\right) = \omega/3\left(1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_2 + a_1)}\right).$$

When $a_2 = a_3$, the centroid method of the TFNs can be calculated as:

$$x_0\left(\widetilde{A}\right) = 1/3\left(a_1 + a_2 + a_4\right),$$

$$y_0\left(\widetilde{A}\right) = 1/3\omega.$$

The centroid method (also known as the centroid formula for Z-numbers) has been introduced by Abdullahi *et al.* [31].

Centroid Formula for Z-numbers [31]: The centroid formulae of Z-numbers are converted into regular fuzzy numbers, $\widetilde{Z}'$ corresponding to a value x on the horizontal axis and a value y on the vertical axis. The centroid-point, $C_{\widetilde{Z}'} = \left(x\left(\widetilde{Z}'\right), y\left(\widetilde{Z}'\right)\right)$ of a regular fuzzy number $\widetilde{Z}' = (a_1, a_2, a_3, a_4)$ can be calculated as follows:
Trapezoidal fuzzy numbers:

$$x\left(\widetilde{Z}'\right) = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{a_3a_4 - a_1a_2}{(a_3 + a_44) - (a_1 + a_2)}\right] \text{ and}$$

$$y\left(\widetilde{Z}'\right) = \frac{1}{3}\left[1 + \frac{a_3 - a_2}{(a_3 + a_4) - (a_1 + a_2)}\right].$$

Triangular fuzzy numbers: When $a_2 = a_3$, the TrFNs will become TFNs, and the centroid point, $C_{\widetilde{Z}'}$ can be calculated as follows:

$$x\left(\widetilde{Z}'\right) = 1/3\left(a_1 + a_2 + a_4\right) \text{ and } y\left(\widetilde{Z}'\right) = 1/3.$$

2.3 Fuzzy Multi-Criteria Decision Making (MCDM) Method

The general procedure of the MCDM method consists of three simple steps: identifying and selecting the criteria, determining the weights of alternatives, and ranking the alternatives using a suitable MCDM method [41]. Since the decision-making process requires decision-makers to address the uncertainty aspects, fuzzy set theory is integrated into the MCDM method. In this section, the step-by-step Fuzzy MCDM (FMCDM) method introduced by Hadi-Vencheh and Mokhtarian [32] is presented.

Consider an FMCDM problem that has m alternatives, $P_1, \ldots, P_m$ $(i = 1, \ldots, m)$ and n benefit criteria, $Q_1, \ldots, Q_n$ $(j = 1, \ldots, n)$. Each alternative is evaluated with respect to the n criteria. Suppose $\widetilde{R} = (\widetilde{r}_{ij})_{m\times n}$ be a fuzzy decision matrix and $\widetilde{S} = (\widetilde{s}_1, \ldots, \widetilde{s}_m)$ be fuzzy weights. Consequently, the fuzzy weighted average for each alternative, $\widetilde{\theta}_i$, can be computed as follows:

$$\widetilde{\theta}_i = \frac{\widetilde{s}_1\widetilde{r}_{i1} + \widetilde{s}_2\widetilde{r}_{i2} + \ldots + \widetilde{s}_m\widetilde{r}_{im}}{\widetilde{s}_1 + \widetilde{s}_2 + \ldots + \widetilde{s}_m}.$$

The author proposed a more straightforward equation to calculate the centroid, $\widetilde{\theta}_i$, as follows:

Suppose $\widetilde{x}_{ij} = (\alpha_{ij}, \beta_{ij})$ and $\widetilde{w}_j = (\gamma_j, \delta_j)$ be the centroid of fuzzy numbers $(\widetilde{x}_{ij})$ and the centroid of fuzzy weights $(\widetilde{w}_j)$, respectively. Correspondingly, the centroid of the fuzzy weighted average of each alternative, $\widetilde{\theta}_i$, can be calculated as follows:

$$\widetilde{\theta}_i = (\eta_i, \lambda_i) = \frac{(\mu, v)}{\left(\sum_{j=1}^{n} \gamma_j\right)^2 + \left(\sum_{j=1}^{n} \delta_j\right)^2},$$

where,

$$\mu = \sum_{j=1}^{n} [\gamma_j\alpha_{ij} - \delta_j\beta_{ij}] \sum_{j=1}^{n} \gamma_j + \sum_{j=1}^{n} [\delta_j\alpha_{ij} + \beta_{ij}\gamma_j] \sum_{j=1}^{n} \delta_j \text{ and}$$

$$v = \sum_{j=1}^{n} [\gamma_j\alpha_{ij} + \gamma_j\beta_{ij}] \sum_{j=1}^{n} \gamma_j - \sum_{j=1}^{n} [\gamma_j\alpha_{ij} - \delta_j\beta_{ij}] \sum_{j=1}^{n} \delta_j.$$

The score of the i^{th} alternative, P_i , can be calculated as follows:

$$P_i = \sqrt{\eta_i^2 + \lambda_i^2}.$$

The alternative with the highest score of P_i is selected as the best alternative.

3 Proposed MCDM Method based on Triangular Z-Numbers

In this section, we propose an MCDM method based on triangular Z-numbers. Let d_k represent the decision-makers $d_1, \ldots, d_q$ who have to evaluate m alternatives, $A_1, \ldots, A_m$ based on n decision criteria $C_1, \ldots, C_n$. The general procedure of the proposed MCDM based on triangular Z-numbers is illustrated in the following Figure 3.

Figure 3 presents the step-by-step procedure of MCDM Method based on Triangular Z-Numbers. The steps consist of six main steps: Step 1- Construct, Step 2 - Convert, Step 3 – Aggregate, Step 4 and 5 – defuzzify, and Step 6 - Rank. The details of each step are described as follows:

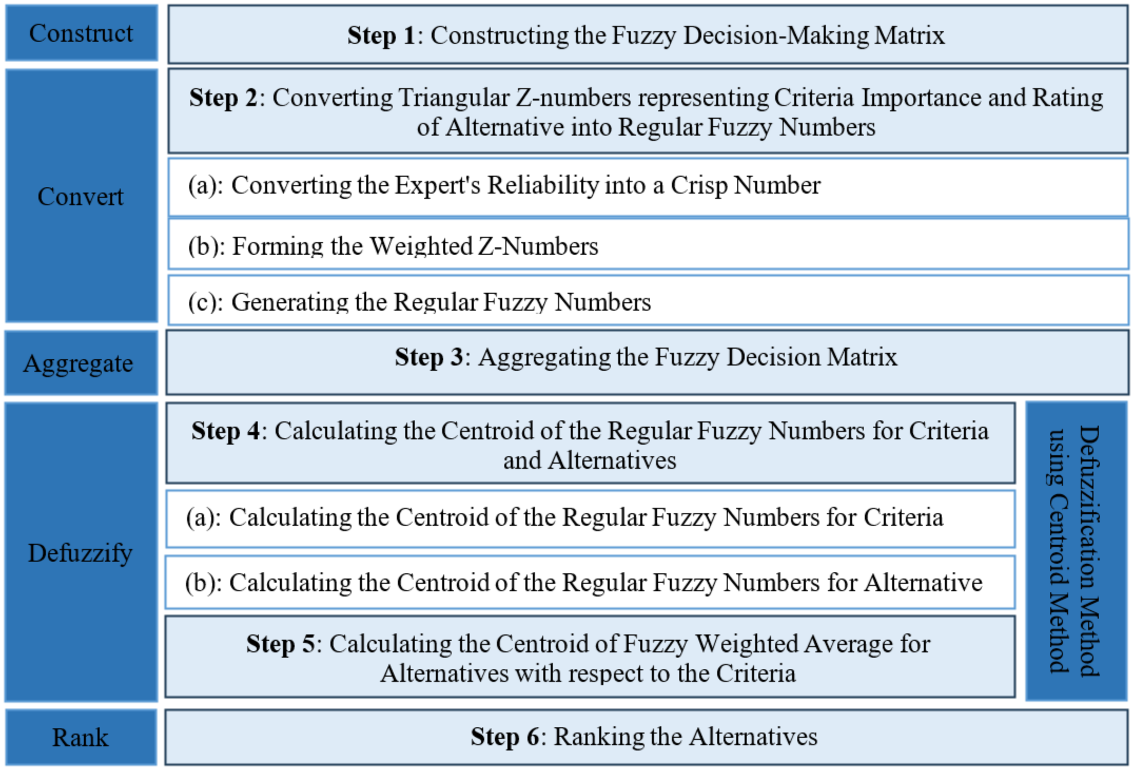

Figure 3: The procedure of MCDM Method based on Triangular Z-Numbers

Step 1: Constructing the Fuzzy Decision-Making Matrix

The evaluation scale for each criterion and alternative using triangular Z-numbers will be established based on the linguistic variables and the corresponding TFNs, as exemplified in Table 1 and Table 2. Table 1 exemplifies the linguistic variable and the corresponding TFNs for restriction. Meanwhile, Table 2 outlines the linguistic variable and the corresponding TFNs for the expert's confidence level (reliability). Table 1 and Table 2 were adopted from Tarmudi and Abdullah [22].

Table 1: Linguistic Variable for Restriction

Linguistic Variable	**Triangular Fuzzy Numbers,** $\widetilde{A}_k = \left(a_1^{d_k}, a_2^{d_k}, a_3^{d_k}\right)$
Very High (VH)	$(0.9, 1.0, 1.0)$
High (H)	$(0.7, 0.8, 0.9)$
Medium High (MH)	$(0.5, 0.7, 0.9)$
Medium (M)	$(0.3, 0.5, 0.7)$
Medium Low (ML)	$(0.1, 0.3, 0.5)$
Low (L)	$(0.0, 0.1, 0.3)$
Very Low (VL)	$(0.0, 0.0, 0.1)$

Table 2: Linguistic Variable for Expert's Confidence Levels (Reliability)

Linguistic Variable	Triangular Fuzzy Numbers, $\widetilde{B}_k = \left(b_1^{d_k}, b_2^{d_k}, b_3^{d_k}\right)$
Very Sure (VS)	$(0.9, 1.0, 1.0)$
Sure (S)	$(0.5, 0.7, 0.9)$
Neutral (N)	$(0.3, 0.5, 0.7)$
Not Sure (NS)	$(0.1, 0.3, 0.5)$
Not Very Sure (NVS)	$(0.0, 0.0, 0.1)$

Here, decision-makers will access the relative importance weights of each criterion using the linguistic variables to form the fuzzy decision matrix for criteria in the form of triangular Z-numbers, $Z = \left(\widetilde{A}^{d_k}, \widetilde{B}^{d_k}\right)$. Accordingly, $\widetilde{A}^{d_k}$ represents the restriction such that $\widetilde{A}^{d_k} = \left(a_1^{d_k}, a_2^{d_k}, a_3^{d_k}\right)$ and $\widetilde{B}^{d_k}$ represents the expert's confidence level such that $\widetilde{B}_{dk} = \left(b_1^{d_k}, b_2^{d_k}, b_3^{d_k}\right)$. The decision-makers rate each alternative with respect to the criteria to form the fuzzy decision-making matrix for the alternative. Let the linguistic variable given by the decision maker $d_k(k = 1, \ldots, q)$ for criteria is represented as $C_j^{d_k} = \left(\widetilde{A}_j^{d_k}, \widetilde{B}_j^{d_k}\right) = \left(a_1^{d_k}, a_2^{d_k}, a_3^{d_k}, b_1^{d_k}, b_2^{d_k}, b_3^{d_k}\right)$, $(j = 1, \ldots, n)$ and the linguistic variable for each alternative given by the decision maker (d_k) with respect to the criteria is represented as $A_{ij}^{d_k} = \left(\widetilde{A}_{ij}^{d_k}, \widetilde{B}_{ij}^{d_k}\right) = \left(a_1^{d_k}, a_2^{d_k}, a_3^{d_k}, b_1^{d_k}, b_2^{d_k}, b_3^{d_k}\right)$, $(j = 1, \ldots, n; i = 1, \ldots, m)$.

As an illustration, suppose a decision-maker rates Alternative 1 with respect to Criteria 1 as "*I am sure the performance of Alternative 1 towards Criteria 1 is very high,*" the Z-number representation is "*(Very High (VH), Sure (S)).*"

Step 2: Converting Triangular Z-numbers representing Criteria Importance and Rating of Alternative into Regular Fuzzy Numbers

In this step, the crisp number $\left(\alpha_{\widetilde{\beta}_k}\right)$ and regular fuzzy number $\left(\widetilde{Z}_r^{d_k\prime}\right)$ can be calculated using the definition in Section 2.2.

Step 2 (a): Converting the Expert's Reliability into a Crisp Number

First, the expert's reliability will be converted into a crisp number (weight of reliability, $\alpha_{\widetilde{\beta}_k}$) using the equation as follows:

$$\alpha_{\widetilde{\beta}_k} = \frac{\int x\mu_{\widetilde{B}_k}(x)dx}{\int \mu_{\widetilde{B}_k}(x)dx}, \tag{1}$$

such that, $\mu_{\widetilde{B}_k}$, $k = 1, \ldots, h$ is a triangular membership function.

Step 2 (b): Forming the Weighted Z-Numbers

The weight of reliability is added to the restriction element $(\widetilde{A}_k)$ to form the weighted Z-numbers $(\widetilde{Z}_r^{d_k})$, as presented in the equation as follows:

$$\widetilde{Z}_r^{d_k} = \left(a_1^{d_k}, a_2^{d_k}, a_3^{d_k}; \alpha_{\widetilde{\beta}_s}\right). \tag{2}$$

Step 2 (c): Generating the Regular Fuzzy Numbers
Third, the regular fuzzy numbers ($\widetilde{Z}_r^{d_k\prime}$) can be formed using the equation as follows:

$$\widetilde{Z}_r^{d_k\prime} = \left\{ \left\langle x, \mu_{\widetilde{Z}_r^{\prime\alpha}}(x) \right\rangle | \mu_{\widetilde{Z}_r^{\prime\alpha}}(x) = \mu_{\widetilde{A}_r} \left(\frac{x}{\sqrt{\alpha_{\widetilde{B}_k}}} \right), x \in [0,1] \right\}. \tag{3}$$

The regular fuzzy numbers for each criterion ($\widetilde{Z}_{\widetilde{C}_j}^{d_k\prime}$) can be presented as:

$$\widetilde{Z}_{\widetilde{C}_j}^{d_k\prime} = \left(\widetilde{a}_1^{d_k}, \widetilde{a}_2^{d_k}, \widetilde{a}_3^{d_k} \right) = \left(\sqrt{\alpha_{\widetilde{B}_s}} \left(a_1^{d_k} \right), \sqrt{\alpha_{\widetilde{B}_s}} \left(a_2^{d_k} \right), \sqrt{\alpha_{\widetilde{B}_s}} \left(a_3^{d_k} \right) \right). \tag{4}$$

The regular fuzzy numbers for alternatives ($\widetilde{Z}_{\widetilde{A}_{ij}}^{d_k\prime}$) can be presented as:

$$\widetilde{Z}_{\widetilde{A}_{ij}}^{d_k\prime} = \left(\widetilde{a}_1^{d_k}, \widetilde{a}_2^{d_k}, \widetilde{a}_3^{d_k} \right) = \left(\sqrt{\alpha_{\widetilde{B}_s}} \left(a_1^{d_k} \right), \sqrt{\alpha_{\widetilde{B}_s}} \left(a_2^{d_k} \right), \sqrt{\alpha_{\widetilde{B}_s}} \left(a_3^{d_k} \right) \right). \tag{5}$$

Step 3: Aggregating the Fuzzy Decision Matrix
The fuzzy decision matrix formed in the previous step will be aggregated using the fuzzy weighted average of fuzzy numbers. The aggregated fuzzy decision matrix for the criteria provided by the decision makers can be denoted as:

$$\begin{aligned} \widetilde{Z}_{\widetilde{C}_j}^{\prime} &= \left(\widetilde{Z}_{\widetilde{C}_j}^{d_1\prime} + \widetilde{Z}_{\widetilde{C}_j}^{d_2\prime} + \ldots + \widetilde{Z}_{\widetilde{C}_j}^{d_q\prime} \right) /q, \\ &= \left(\left(\widetilde{a}_{11}^{d_1}, \widetilde{a}_{12}^{d_1}, \widetilde{a}_{13}^{d_1} \right) + \left(\widetilde{a}_{11}^{d_2}, \widetilde{a}_{12}^{d_2}, \widetilde{a}_{13}^{d_2} \right) + \ldots + \left(\widetilde{a}_{n1}^{d_q}, \widetilde{a}_{n2}^{d_q}, \widetilde{a}_{n3}^{d_q} \right) \right) /q, \\ &= \left(\widetilde{a}_{n1}, \widetilde{a}_{n2}, \widetilde{a}_{n3} \right), \end{aligned} \tag{6}$$

where $\widetilde{Z}_{\widetilde{C}_j}^{d_q\prime}$ represents the opinion given on the criteria by the decision maker, such that $\widetilde{Z}_{\widetilde{C}_j}^{d_q\prime} = \left(\widetilde{a}_{n1}^{d_k}, \widetilde{a}_{n2}^{d_k}, \widetilde{a}_{n3}^{d_k} \right)$.

The aggregated fuzzy decision matrix for the alternative given by the decision-makers can be denoted as:

$$\begin{aligned} \left[\widetilde{Z}_{\widetilde{A}_{ij}}^{\prime} \right] &= \left(\widetilde{A}_{11}^{d_1} + \widetilde{A}_{11}^{d_2} + \ldots + \widetilde{A}_{nm}^{d_q} \right) /q, \\ &= \left(\left(\widetilde{a}_{111}^{d_1}, \widetilde{a}_{112}^{d_1}, \widetilde{a}_{113}^{d_1} \right) + \left(\widetilde{a}_{111}^{d_2}, \widetilde{a}_{112}^{d_2}, \widetilde{a}_{113}^{d_2} \right) + \ldots + \left(\widetilde{a}_{nm1}^{d_q}, \widetilde{a}_{nm2}^{d_q}, \widetilde{a}_{nm3}^{d_q} \right) \right) /q, \\ &= \left(\widetilde{a}_{nm1}, \widetilde{a}_{nm2}, \widetilde{a}_{nm3} \right), \end{aligned} \tag{7}$$

where $\widetilde{A}_{ij}^{d_q}$ represents the opinion on the alternative given by the decision maker, such that $\widetilde{Z}_{\widetilde{C}_j}^{d_q\prime} = \left(\widetilde{a}_{nm1}^{d_k}, \widetilde{a}_{nm2}^{d_k}, \widetilde{a}_{nm3}^{d_k} \right)$.

Step 4: Calculating the Centroid of the Regular Fuzzy Numbers for Criteria and Alternatives
The centroid of the regular fuzzy numbers for each criterion ($C_{\widetilde{Z}_{\widetilde{C}_j}^{\prime}}$) and alternative ($C_{\widetilde{Z}_{\widetilde{A}_j}^{\prime}}$) can be computed as follows:

$$\widetilde{Z}_r^{\prime} = \left(x \left(\widetilde{Z}_r^{\prime} \right), y \left(\widetilde{Z}_r^{\prime} \right) \right), \tag{8}$$

where $x\left(\widetilde{Z}'_r\right) = \dfrac{\widetilde{a}_1 + \widetilde{a}_2 + \widetilde{a}_3}{3}$ and $y\left(\widetilde{Z}'_r\right) = \dfrac{1}{3}$.

Step 4 (a): Calculating the Centroid of the Regular Fuzzy Numbers for Criteria

The centroid of the regular fuzzy numbers for the importance of criteria ($C_{\widetilde{Z}'_{\widetilde{C}_j}}$) can be presented as:

$$C_{\widetilde{Z}'_{\widetilde{C}_j}} = (\gamma_j, \delta_j). \tag{9}$$

Step 4 (b): Calculating the Centroid of the Regular Fuzzy Numbers for Alternative

The centroid of the regular fuzzy numbers for the performance of alternatives $\left(C_{\widetilde{Z}'_{\widetilde{A}_{ij}}}\right)$ can be presented as:

$$C_{\widetilde{Z}'_{\widetilde{A}_{ij}}} = (\alpha_{ij}, \beta_{ij}). \tag{10}$$

Step 5: Calculating the Centroid of Fuzzy Weighted Average for the Alternatives with respect to the Criteria

In this step, the centroid of the fuzzy weighted average, $\widetilde{\theta}_i = (\mu_i, \nu_i)$, for each alternative can be calculated as follows:

$$\mu_i = \frac{\sum_{i,j=1}^{m,n}\left[\gamma_j\alpha_{ij} - \delta_j\beta_{ij}\right]\sum_{j=1}^{n}\gamma_j + \sum_{i,j=1}^{m,n}\left[\delta_j\alpha_{ij} + \beta_{ij}\gamma_j\right]\sum_{j=1}^{n}\delta_j}{\left(\sum_{j=1}^{n}\gamma_j\right)^2 + \left(\sum_{j=1}^{n}\delta_j\right)^2}, \tag{11}$$

$$\nu_i = \frac{\sum_{i,j=1}^{m,n}\left[\delta_j\alpha_{ij} + \gamma_j\beta_{ij}\right]\sum_{j=1}^{n}\gamma_j - \sum_{i,j=1}^{m,n}\left[\gamma_j\alpha_{ij} - \delta_j\beta_{ij}\right]\sum_{j=1}^{n}\delta_j}{\left(\sum_{j=1}^{n}\gamma_j\right)^2 + \left(\sum_{j=1}^{n}\delta_j\right)^2}. \tag{12}$$

Step 6: Ranking the Alternatives

The score of the i^{th} alternative (S_{A_i}) can be computed as:

$$S_{A_i} = \sqrt{\mu_i^2 + \nu_i^2}. \tag{13}$$

Rank the alternatives based on the score of each alternative (i.e., the higher score indicates a better alternative). The alternative with the highest rank represents the best option to be selected.

4 Numerical Example: Flood Mitigation Measures

Consider the case study of prioritizing flood mitigation measures by Abd Rahman [42]. In this case study, the government aims to implement the most important flood mitigation measures. A committee of two decision-makers, d_k ($k = 1, \ldots, q$), is tasked with evaluating the flood mitigation measures (alternatives) based on four criteria. Three alternatives, A_i ($i = 1, \ldots, m$), and four criteria, C_j ($j = 1, \ldots, n$). The alternatives of flood mitigation measures are Elevation (A_1), Wet Flood Proofing (A_2), and Barriers (A_3). The criteria of flood mitigation measures are Technical (C_1), Economic (C_2), Social (C_3), and Environmental (C_4).

Step 1: Constructing the Fuzzy Decision-Making Matrix

Using the linguistic variables in Table 1 and Table 2, the decision-makers' evaluations of criteria and alternatives are exemplified in Table 3 and Table 4, respectively. Table 3 presents the triangular Z-numbers for the importance of the criteria by decision makers. Table 4 presents the triangular Z-numbers for decision makers' ratings on each alternative with respect to the criteria.

Table 3: Triangular Z-numbers for Criteria

Criteria (C_j)	**Decision Maker (d_k)**	**Triangular Z-number $\left(\widetilde{C}_j^{d_k}\right)$**
C_1	d_1	$\widetilde{C}_1^{d_1} = (\mathrm{VH}, \mathrm{S}) = (0.90, 1.00, 1.00;\ 0.50, 0.70, 0.90)$
	d_2	$\widetilde{C}_1^{d_2} = (\mathrm{H}, \mathrm{S}) = (0.70, 0.80, 0.90;\ 0.50, 0.70, 0.90)$
$\vdots$	$\vdots$	$\vdots$
C_4	d_1	$\widetilde{C}_4^{d_1} = (\mathrm{M}, \mathrm{NS}) = (0.30, 0.50, 0.70;\ 0.10, 0.30, 0.50)$
	d_2	$\widetilde{C}_4^{d_2} = (\mathrm{L}, \mathrm{NVS}) = (0.00, 0.10, 0.30;\ 0.00, 0.00, 0.10)$

Table 4: Triangular Z-numbers for Alternative

Alternative (A_i)	**Criteria (C_j)**	**Decision Makers (d_k)**	**Triangular Z-number $\left(\widetilde{A}_{ij}^{d_k}\right)$**
A_1	C_1	d_1	$\widetilde{A}_{11}^{d_1} = (\mathrm{M}, \mathrm{VS}) = (0.30, 0.50, 0.70;\ 0.90, 1.00, 1.00)$
		d_2	$\widetilde{A}_{11}^{d_2} = (\mathrm{VS}, \mathrm{S}) = (0.90, 1.00, 1.00;\ 0.50, 0.70, 0.90)$
	C_2	d_1	$\widetilde{A}_{12}^{d_1} = (\mathrm{H}, \mathrm{VS}) = (0.70, 0.80, 0.90;\ 0.90, 1.00, 1.00)$
		d_2	$\widetilde{A}_{12}^{d_2} = (\mathrm{M}, \mathrm{NVS}) = (0.30, 0.50, 0.70;\ 0.00, 0.00, 0.10)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_3	C_4	d_1	$\widetilde{A}_{34}^{d_1} = (\mathrm{ML}, \mathrm{VS}) = (0.10, 0.30, 0.50;\ 0.90, 1.00, 1.00)$
		d_2	$\widetilde{A}_{34}^{d_2} = (\mathrm{H}, \mathrm{NS}) = (0.70, 0.80, 0.90;\ 0.10, 0.30, 0.50)$

Step 2: Converting Triangular Z-numbers representing Criteria Importance and Rating of Alternative into Regular Fuzzy Numbers

The triangular Z-numbers presented in Table 3 and Table 4 were converted into regular fuzzy numbers. The crisp numbers, weighted Z-numbers, and regular fuzzy numbers for each criterion and alternative can be displayed in Table 5 and Table 6, respectively. Based on Table 5, the example of calculation of the regular fuzzy numbers $\left(\widetilde{Z}_{\widetilde{C}_1}^{d_1\prime}\right)$ for criteria 1 ($\widetilde{C}_1$) by d_1, can be presented in three steps, as follows:

Table 5: The Crisp Number, Weighted Z-numbers, and Regular Fuzzy Numbers for Criteria

Criteria $\left(\widetilde{C}_j\right)$	Decision Makers (d_k)	Crisp Number $\left(\alpha_{\widetilde{B}_k}\right)$	Weighted Z-number $\left(\widetilde{Z}^{d_k}_{\widetilde{C}_j}\right)$	Regular Fuzzy Number $\left(\widetilde{Z}^{d_k\prime}_{\widetilde{C}_j}\right)$
$\widetilde{C}_1$	d_1	0.7000	$\widetilde{Z}^{d_1}_{\widetilde{C}_1} = (0.90, 1.00, 1.00;\ 0.7000)$	$\widetilde{Z}^{d_1\prime}_{\widetilde{C}_1} = (0.7530, 0.8367, 0.8367)$
	d_2	0.7000	$\widetilde{Z}^{d_2}_{\widetilde{C}_1} = (0.70, 0.80, 0.90;\ 0.7000)$	$\widetilde{Z}^{d_2\prime}_{\widetilde{C}_1} = (0.5857, 0.6693, 0.7530)$
⋮	⋮	⋮	⋮	⋮
$\widetilde{C}_4$	d_1	0.3000	$\widetilde{Z}^{d_1}_{\widetilde{C}_4} = (0.30, 0.50, 0.70;\ 0.3000)$	$\widetilde{Z}^{d_1\prime}_{\widetilde{C}_4} = (0.0547, 0.0912, 0.1277)$
	d_2	0.3000	$\widetilde{Z}^{d_2}_{\widetilde{C}_4} = (0.10, 0.30, 0.50;\ 0.3000)$	$\widetilde{Z}^{d_2\prime}_{\widetilde{C}_4} = (0.0000, 0.0182, 0.0547)$

Table 6: The Crisp number, Weighted Z-numbers, and Regular Fuzzy Number for Alternative

Alternative $\left(\widetilde{A}_{ij}\right)$	Decision makers (d_k)	Crisp Number $\left(\alpha_{\widetilde{B}_k}\right)$	Weighted Z-Numbers $\left(Z^{d_k}_{\widetilde{A}_{ij}}\right)$	Regular Fuzzy Number $\left(\widetilde{Z}^{d_k\prime}_{\widetilde{A}_{ij}}\right)$
$\widetilde{A}_{11}$	d_1	0.9667	$Z^{d_1}_{\widetilde{A}_{11}} = (0.30, 0.50, 0.70; 0.9667)$	$\widetilde{Z}^{d_1\prime}_{\widetilde{A}_{11}} = (0.2950, 0.4916, 0.6882)$
	d_2	0.7000	$Z^{d_2}_{\widetilde{A}_{11}} = (0.90, 1.00, 1.00; 0.7000)$	$\widetilde{Z}^{d_2\prime}_{\widetilde{A}_{11}} = (0.7530, 0.8367, 0.8367)$
⋮	⋮	⋮	⋮	⋮
$\widetilde{A}_{34}$	d_1	0.9667	$Z^{d_1}_{\widetilde{A}_{34}} = (0.90, 1.00, 1.00; 0.9667)$	$\widetilde{Z}^{d_1\prime}_{\widetilde{A}_{34}} = (0.8849, 0.9832, 0.9832)$
	d_2	0.3000	$Z^{d_2}_{\widetilde{A}_{34}} = (0.70, 0.80, 0.90; 0.3000)$	$\widetilde{Z}^{d_1\prime}_{\widetilde{A}_{34}} = (0.3834, 0.4382, 0.4930)$

Step 2 (a): Converting the Expert's Reliability into a Crisp Number

The reliability elements $\left(\widetilde{B}_k\right)$ were converted into crisp numbers representing the weight of reliability, $\alpha_{\widetilde{B}_k}$, using Equation (1). As an illustration, the crisp number, $\alpha_{\widetilde{B}_1}$ associated with the reliability component $\widetilde{B}_1$ by the decision maker, d_1 is calculated as follows:

$$\alpha_{\widetilde{B}_1} = \left(b_1^{d_1} + b_2^{d_1} + b_3^{d_1}\right)/3 = (0.50 + 0.70 + 0.90)/3 = 0.70.$$

Step 2 (b): Forming the Weighted Z-Numbers

By using Equation (2), the weighted Z-number $\left(\widetilde{Z}^{d_1}_{\widetilde{C}_1}\right)$ is presented in the form of:

$$\widetilde{Z}^{d_1}_{\widetilde{C}_1} = \left(a_1^{d_1}, a_2^{d_1}, a_3^{d_1};\ \alpha_{\widetilde{B}_S}\right) = (0.90, 1.00, 1.00;\ 0.70).$$

Step 2 (c): Generating the Regular Fuzzy Numbers

Using Equations (3), (4) and (5), the weighted Z-number $\left(\widetilde{Z}^{d_1}_{\widetilde{C}_1}\right)$ are converted into regular fuzzy numbers $\left(\widetilde{Z}^{d_1\prime}_{\widetilde{C}_1}\right)$, such that:

$$\widetilde{Z}^{d_1\prime}_{\widetilde{C}_1} = \left(\sqrt{\alpha_{\widetilde{B}_S}}\ \left(a_1^{d_1}\right),\ \sqrt{\alpha_{\widetilde{B}_S}}\ \left(a_2^{d_1}\right),\ \sqrt{\alpha_{\widetilde{B}_S}}\ \left(a_3^{d_1}\right)\right) = (0.7530, 0.8367, 0.8367).$$

The regular fuzzy numbers with respect to the alternative are also obtained in the same manner.

Step 3: Aggregating the Fuzzy Decision Matrix

Based on Table 5 and Table 6, the regular Z-numbers given by the decision-makers for each criterion and alternative were aggregated using Equations (6) and (7). The aggregated fuzzy decision matrix for each criterion and alternative is presented in Table 7 and Table 8, respectively.

Specifically, the aggregated regular fuzzy for $\widetilde{C}_1$ is calculated as follows:

$$\widetilde{Z}'_{\widetilde{C}_1} = \left(\widetilde{Z}^{d_1\prime}_{\widetilde{C}_1} + \widetilde{Z}^{d_2\prime}_{\widetilde{C}_1}\right)/q = \left(\widetilde{a}^{d_1}_1 + \widetilde{a}^{d_2}_1, \widetilde{a}^{d_1}_2 + \widetilde{a}^{d_2}_2, \widetilde{a}^{d_1}_3 + \widetilde{a}^{d_2}_3\right)/2 = (0.6693, 0.7530, 0.7948).$$

Table 7: The Aggregated Fuzzy Decision Matrix for Criteria

Criteria $\left(\widetilde{C}_j\right)$	**Decision makers** (d_k)	**Regular Fuzzy Number** $\left(\widetilde{Z}^{d_k\prime}_{\widetilde{C}_j}\right)$	**Aggregated Fuzzy Number** $\left(\widetilde{Z}'_{\widetilde{C}_j}\right)$
$\widetilde{C}_1$	d_1	$\widetilde{Z}^{d_1\prime}_{\widetilde{C}_1} = (0.7530, 0.8367, 0.8367)$	$\widetilde{Z}'_{\widetilde{C}_1} = (0.6693, 0.7530, 0.7948)$
	d_2	$\widetilde{Z}^{d_2\prime}_{\widetilde{C}_1} = (0.5857, 0.6693, 0.7530)$	
⋮	⋮	⋮	⋮
$\widetilde{C}_4$	d_1	$\widetilde{Z}^{d_1\prime}_{\widetilde{C}_4} = (0.0547, 0.0912, 0.1277)$	$\widetilde{Z}'_{\widetilde{C}_4} = (0.0274, 0.0547, 0.0912)$
	d_2	$\widetilde{Z}^{d_2\prime}_{\widetilde{C}_4} = (0.0000, 0.0182, 0.0547)$	

Table 8: The Aggregated Fuzzy Decision Matrix for Alternative

Alternative $\left(\widetilde{A}_{ij}\right)$	**Decision makers** (d_k)	**Regular Fuzzy Number** $\left(\widetilde{Z}^{d_k\prime}_{\widetilde{A}_{ij}}\right)$	**Aggregated Fuzzy Number** $\left(\widetilde{Z}'_{\widetilde{A}_{ij}}\right)$
$\widetilde{A}_{11}$	d_1	$\widetilde{Z}^{d_1\prime}_{\widetilde{A}_{11}} = (0.2950, 0.4916, 0.6882)$	$\widetilde{Z}'_{\widetilde{A}_{11}} = (0.5240, 0.6641, 0.7624)$
	d_2	$\widetilde{Z}^{d_2\prime}_{\widetilde{A}_{11}} = (0.7530, 0.8367, 0.8367)$	
⋮	⋮	⋮	⋮
$\widetilde{A}_{34}$	d_1	$\widetilde{Z}^{d_1\prime}_{\widetilde{A}_{34}} = (0.8849, 0.9832, 0.9832)$	$\widetilde{Z}'_{\widetilde{A}_{34}} = (0.2409, 0.3666, 0.4923)$
	d_2	$\widetilde{Z}^{d_2\prime}_{\widetilde{A}_{34}} = (0.3834, 0.4382, 0.4930)$	

Step 4: Calculating the Centroid of the Regular Fuzzy Numbers for Criteria and Alternatives

Table 9 and Table 10 display the regular fuzzy numbers and the centroid of the regular fuzzy numbers for the criteria and the alternatives, respectively. As an illustration, the centroid of the regular fuzzy numbers for $\widetilde{C}_1$, $C_{Z'_{\widetilde{C}_1}} = \left(x\left(Z'_{\widetilde{C}_1}\right), y\left(Z'_{\widetilde{C}_1}\right)\right) = (\gamma_1, \delta_1)$ is calculated using Equation (9), where:

$$\gamma_1 = (\widetilde{a}_1 + \widetilde{a}_2 + \widetilde{a}_3)/3 = (0.6693 + 0.7530 + 0.7948)/3 = 0.7390,$$
$$\delta_1 = \frac{1}{3} = 0.3333.$$

We have $C_{Z'_{\widetilde{C}_1}} = (\gamma_1, \delta_1) = (0.7390, 0.3333)$.

The centroid of regular fuzzy numbers with respect to the rest of the criteria and alternatives is obtained in the same manner.

Table 9: The Regular Fuzzy Number and the Centroid of the Regular Fuzzy Numbers for Criteria

Criteria $(\widetilde{C}_j)$	**Regular Fuzzy Number** $\left(Z'_{\widetilde{C}_j}\right)$	**Centroid of Regular Fuzzy Numbers** $\left(C_{Z'_{\widetilde{C}_j}}\right)$
$\widetilde{C}_1$	$Z'_{\widetilde{C}_1} = (0.6693, 0.7530, 0.7948)$	$C_{Z'_{\widetilde{C}_1}} = (0.7390, 0.3333)$
$\widetilde{C}_2$	$Z'_{\widetilde{C}_2} = (0.6364, 0.7071, 0.7071)$	$C_{Z'_{\widetilde{C}_2}} = (0.6835, 0.3333)$
$\widetilde{C}_3$	$Z'_{\widetilde{C}_3} = (0.6364, 0.7071, 0.7071)$	$C_{Z'_{\widetilde{C}_3}} = (0.3183, 0.3333)$
$\widetilde{C}_4$	$Z'_{\widetilde{C}_4} = (0.0274, 0.0547, 0.0912)$	$C_{Z'_{\widetilde{C}_4}} = (0.0578, 0.3333)$
		$\sum_{j=1}^{4} \gamma_j = 1.7987, \quad \sum_{j=1}^{4} \delta_j = 1.3333$

Table 10: The Aggregated Fuzzy Numbers and the Centroid of Regular Fuzzy Numbers for Alternative

Alternative $\left(\widetilde{A}_{ij}\right)$	**Aggregated Fuzzy Number** $\left(\widetilde{Z}'_{\widetilde{A}_{ij}}\right)$	**Centroid Fuzzy Weighted Average** $\left(C_{Z'_{\widetilde{A}_{ij}}}\right)$
$\widetilde{A}_{11}$	$\widetilde{Z}'_{\widetilde{A}_{11}} = (0.5240, 0.6641, 0.7624)$	$(\alpha_{11}, \beta_{11}) = (0.6502, 0.3333)$
$\vdots$	$\vdots$	$\vdots$
$\widetilde{A}_{34}$	$\widetilde{Z}'_{\widetilde{A}_{34}} = (0.2409, 0.3666, 0.4923)$	$(\alpha_{34}, \beta_{34}) = (0.3666, 0.3333)$

Step 5: Calculating the Centroid of Fuzzy Weighted Average for the Alternative with respect to the Criteria

Based on Table 9 and Table 10, the centroid of the fuzzy weighted average for the alternatives is calculated using Equations (11) and (12). As an illustration, the centroid fuzzy weighted average for Alternative 1, $\widetilde{\theta}_1$, is calculated as follows:

$$\sum_{i,j=1}^{m=3,n=4} [\gamma_j \alpha_{ij} - \delta_j \beta_{ij}] = (\gamma_1 \alpha_{11} + \gamma_2 \alpha_{12} + \ldots + \gamma_4 \alpha_{14}) - (\delta_1 \beta_{11} + \delta_2 \beta_{12} + \ldots + \delta_4 \beta_{14}) = 0.5460,$$

$$\sum_{j=1}^{4} \gamma_j = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 = 1.7987.$$

$$\sum_{i,j=1}^{m=3,n=4} [\delta_j \alpha_{ij} + \beta_{ij} \gamma_j] = (\delta_1 \alpha_{11} + \delta_2 \alpha_{12} + \cdots + \delta_4 \alpha_{13}) + (\beta_{11} \gamma_1 + \beta_{12} \gamma_2 + \cdots + \beta_{13} \gamma_4) = 1.3495,$$

$$\sum_{j=1}^{4} \delta_j = \delta_1 + \delta_2 + \delta_3 + \delta_4 = 1.3333.$$

Thus, μ_1 was calculated using Equation (11) as follows:

$$\mu_1 = \sum_{i,j=1}^{m=3,n=4} [\gamma_j \alpha_{ij} - \delta_j \beta_{ij}] \sum_{j=1}^{n} \gamma_j + \sum_{i,j=1}^{m=3,n=4} [\delta_j \alpha_{ij} + \beta_{ij} \gamma_j] \sum_{j=1}^{n} \delta_j = 2.7814.$$

We also have,

$$\sum_{i,j=1}^{m=3,n=4} [\delta_j \alpha_{ij} + \gamma_j \beta_{ij}] = (\delta_1 \alpha_{11} + \delta_2 \alpha_{12} + \ldots + \delta_4 \alpha_{14}) + (\gamma_1 \beta_{11} + \gamma_2 \beta_{12} + \ldots + \gamma_4 \beta_{14}) = 1.3495,$$

$$\sum_{i,j=1}^{m=3,n=4} [\gamma_j \alpha_{ij} - \delta_j \beta_{ij}] = (\gamma_1 \alpha_{11} + \gamma_2 \alpha_{12} + \ldots + \gamma_4 \alpha_{14}) - (\delta_1 \beta_{11} + \delta_2 \beta_{12} + \ldots + \delta_4 \beta_{14}) = 0.5460.$$

Thus, ν_1 is calculated using Equation (12) as follows:

$$\nu_1 = \sum_{i,j=1}^{m=3,n=4} [\delta_j \alpha_{ij} + \gamma_j \beta_{ij}] \sum_{j=1}^{n} \gamma_j - \sum_{i,j=1}^{m=3,n=4} [\gamma_j \alpha_{ij} - \delta_j \beta_{ij}] \sum_{j=1}^{n} \delta_j = 1.6993.$$

Based on the calculated μ_1 and ν_1, the centroid fuzzy weighted average for Alternative 1, $\tilde{\theta}_1$ is obtained as:

$$\tilde{\theta}_1 = (\mu_1, \nu_1) = \left(\frac{2.7814}{5.0130}, \frac{1.6993}{5.0130} \right) = (0.5548, 0.3390).$$

Table 11 displays the centroid fuzzy weighted average for each alternative.

Table 11: The Centroid Fuzzy Weighted Average for Alternative

Alternative $\left(\widetilde{A}_{ij}\right)$	$\gamma_j\alpha_{ij}$	$\delta_j\beta_{ij}$	$\delta_j\alpha_{ij}$	$\beta_{ij}\gamma_j$	μ_i	ν_i	**Centroid Fuzzy Weighted Average**, $\widetilde{\theta}_i = (\mu_i, \nu_i)$
$\widetilde{A}_{11}$	$\gamma_1\alpha_{11} =$ 0.4805	$\delta_1\beta_{11} =$ 0.1111	$\delta_1\alpha_{11} =$ 0.2167	$\beta_{11}\gamma_1 =$ 0.2463			
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\mu_1 =$ 2.7814	$\nu_1 =$ 1.6993	$\widetilde{\theta}_1 =$ $(0.5548, 0.3390)$
$\widetilde{A}_{14}$	$\gamma_4\alpha_{14} =$ 0.0354	$\delta_4\beta_{14} =$ 0.1111	$\delta_4\alpha_{14} =$ 0.2041	$\beta_{14}\gamma_4 =$ 0.0193			
Total	**0.9904**	**0.4444**	**0.7499**	**0.5996**			
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\widetilde{A}_{31}$	$\gamma_1\alpha_{31} =$ 0.4019	$\delta_1\beta_{31} =$ 0.1111	$\delta_1\alpha_{31} =$ 0.1813	$\beta_{31}\gamma_1 =$ 0.2463			
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\mu_3 =$ 4.0614	$\nu_3 =$ 2.2745	$\widetilde{\theta}_3 =$ $(0.8102, 0.4537)$
$\widetilde{A}_{34}$	$\gamma_4\alpha_{34} =$ 0.0212	$\delta_4\beta_{44} =$ 0.1111	$\delta_4\alpha_{34} =$ 0.1222	$\beta_{34}\gamma_4 =$ 0.0193			
Total	**1.2967**	**0.4444**	**0.8862**	**0.5996**			

Step 6: Ranking the Alternatives

Based on the calculated centroid-weighted average for each alternative $(\widetilde{\theta}_i)$ in Table 11, the score of the alternative (S_{A_i}) was calculated using Equation (13). As an illustration, the score of alternative 1, S_{A_i}, was calculated as follows:

$$S_{A_1} = \sqrt{\mu_1^2 + \nu_1^2} = 0.6502.$$

The rank of the alternatives was determined based on the score of the alternative (S_{A_i}), as shown in Table 12. The alternative with the highest rank represents the best alternative to be implemented.

Table 12: The Score of the Alternatives

Alternative	**Score of the Alternative** (S_{A_i})	**Ranking**
A_1	$S_{A_1} = 0.6502$	2
A_2	$S_{A_2} = 0.6128$	3
A_3	$S_{A_3} = 0.9286$	1

Based on the scores presented in Table 12, the ranking of alternatives is $A_3 > A_1 > A_2$, with Barriers (A_3) identified as the best option, followed by Elevation (A_1) and Wet Flood Proofing (A_2).

5 Discussion

The results of flood mitigation measures demonstrate that the proposed methodology effectively supports decision-making by providing a reliable and computationally feasible means to assess and rank alternatives under uncertainty. Unlike conventional fuzzy MCDM method, this method incorporates both the performance of the alternatives and confidence level of expert resulting in decisions that are more transparent and justifiable.

To further validate the robustness and applicability of the proposed methodology, a comparative analysis with existing approaches is conducted. Table 13 presents a detailed comparison between the proposed methodology and the established approach introduced by Alam *et al.* [17]. While both methodologies employ Z-numbers within FMCDM, they differ in their representation and processing of Z-numbers.

Table 13: A Comparative Analysis

Aspect	Proposed Methodology	Alam *et al.* [17]
Methodology	Triangular Z-numbers-based MCDM using the centroid method	IZNs
Z-number Shape	Triangular	Trapezoidal
Component Handling	Convert reliability to crisp and combine with restriction components	Calculates IMC for non-membership, converts IZN to IFN
Defuzzification Method	Standard centroid method (applied directly on fuzzy decision matrix and weights)	IMC approach (involving sub-centroids, IMC for non-membership, IMC index, and IFN transformation)
Aggregation Approach	Fuzzy weighted average	Arithmetic averaging (for multiple decision makers and criteria)
Interpretability	High (uses standard fuzzy and centroid operations)	Moderate (requires non-membership logic, IMC index calculation)
Computational Step and Complexity	Low to moderate with 11 steps/sub-steps (six steps with five sub-steps)	High with 14 stages/sub-steps (three phases with eleven sub-steps)
Computational Load	Linear with the number of criteria and alternatives	Higher due to centroid partitioning and additional IFN conversion steps
Score of the alternative	$A_1 = 0.6502,\ A_2 = 0.6128,\ A_3 = 0.9286$	$A_1 = 0.7318,\ A_2 = 0.4601,\ A_3 = 0.7748$
Ranking of the alternative	$A_3 > A_1 > A_2$	$A_3 > A_1 > A_2$

The proposed methodology adopts classical Z-numbers in the form of triangular representations and utilizes the standard centroid method for defuzzification. In contrast, Alam *et al.* [17] utilized Intuitionistic Z-Numbers (IZNs), which involve a more complex representation that includes non-membership degrees and require transformation into regular Intuitionistic Fuzzy Numbers (IFNs) through the Intuitive Multiple Centroid Method (IMC) approach.

Despite these theoretical differences, both methods yield the same ranking of alternatives when applied to identical criteria and linguistic evaluations. This consistency indicates that the proposed methodology retains robustness and reliability while significantly improving ease of use and efficiency. This, in turn, makes it more suitable for practical applications involving expert judgement under uncertainty.

6 Conclusion

Although Z-numbers, developed by Zadeh, have been widely adopted and extended in various methodological frameworks, they continue to face criticisms and limitations. Previous studies have highlighted that the interpretation and application of Z-numbers lead to computational complexity and potential loss of information, particularly due to probabilistic representation of the reliability component. These challenges become more pronounced in a large-scale decision-making context that requires numerous and extensive mathematical operations. Despite these limitations, Z-numbers remain a valuable tool for modeling uncertainty and imprecision.

In this study, we proposed a methodology that integrates triangular Z-numbers with the centroid method for defuzzification. The key contribution of this methodology lies in its ability to simplify the interpretation and computation of Z-numbers while still preserving information during the defuzzification process and maintaining their capacity to model uncertainty effectively.

The comparative analysis confirms that the proposed methodology produces stable and consistent rankings of the alternatives, even when compared to more complex methods such as IZNs. This demonstrates that our method provides a robust yet computationally efficient solution for decision-making under uncertainty. Ultimately, this makes it particularly useful for practical applications and large-scale problems where simplicity and reliability are crucial.

In future work, the methodology will be extended to incorporate other shapes of Z-numbers, such as trapezoidal Z-numbers, to further demonstrate its flexibility and consistency across various types of fuzzy representation.

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