MATEMATIKA, MJIAM, Volume 40, Number 3, 131–142

Time-Varying Autoregressive Models for Economic Forecasting

[1]Syarifah Inayati, [2]Nur Iriawan* and [3]Irhamah

[1,2,3]Department of Statistics, Faculty of Science and Data Analytics
Institut Teknologi Sepuluh Nopember, Surabaya 60111, Indonesia

[1]Department of Statistics, Faculty of Mathematics and Natural Sciences
Universitas Negeri Yogyakarta, Yogyakarta 55281, Indonesia

*Corresponding author: nur.iriawan@its.ac.id

Article history
Received: 7 November 2024
Received in revised form: 5 December 2024
Accepted: 5 December 2024
Published on line: 31 December 2024

Abstract This research introduces a time-varying autoregressive (TVAR) model, developed to improve the precision of forecasting in economic time series data. The model advances the conventional TVAR framework by incorporating potential mean adjustments and utilizing the Kalman Filter within a Maximum Likelihood Estimation (MLE) framework, with further optimization through the NelderMead method. Applied to the real gross national product (GNP) data of the United States (U.S.), the model effectively captures dynamic patterns and structural changes that traditional models often overlook. The model's performance is rigorously compared with the widely used Markov switching autoregressive (MSAR) model, demonstrating superior results in both training and testing forecasts. The TVAR model consistently achieves lower error metrics, underscoring its robustness and flexibility in capturing dynamic economic trends and providing reliable forecasts. This research emphasizes the TVAR models potential for broader applications in economic policy analysis, strategic planning, and decision-making processes, particularly in understanding and predicting economic growth. The models adaptability and precision make it a valuable tool for economists and policymakers aiming to navigate complex economic fluctuations with greater confidence as well as accuracy.

Keywords Time Series Analysis; Forecasting; Time-Varying Autoregressive; Kalman Filtering; Economic Growth.

Mathematics Subject Classification 62M10, 62M20, 91B84, 93E10.

1 Introduction

Analyzing time series data is essential across many disciplines, particularly in the economic sector, where predicting and comprehending economic indicators is vital. Conventional models like the autoregressive (AR) model usually assume static parameters, restricting their capacity to adjust to data fluctuations. However, these models are restricted by their linear nature, making them unable to capture complex nonlinear patterns often present in economic data [1, 2]. Additionally, economic data are often dynamic, influenced by numerous factors that cause structural changes over time. To address this limitation, time-varying parameter (TVP) models have been developed, allowing model parameters to change over time and providing a more accurate representation of real-world phenomena [3–6].

TVP models are part of a wider group of state-space models, providing the necessary adaptability to accurately represent the shifting and intricate characteristics of economic indicators [7, 8]. These models are particularly effective in handling non-stationarity in underlying processes caused by policy shifts, technological advancements, or other external shocks [9]. By allowing parameters to adapt, TVP models better reflect the true dynamics of the data, leading to more reliable forecasts and insights.

While TVP models are widely applied in economics, the time-varying autoregressive (TVAR) model, a specific variant of TVP models, has also gained prominence in fields such as signal processing and audio analysis [10–12]. The TVAR model incorporates lagged endogenous variables as predictors, making it particularly suitable for capturing the autoregressive nature of time series data. Previous studies have demonstrated the superiority of TVAR models in capturing the dynamic nature of data within signal processing, particularly in applications like sleep EEG analysis and acoustic signal processing. In these contexts, TVAR models have been employed to track time-varying patterns in brain activity, providing insights into both motor imagery and sleep state detection. These models have proven useful in analyzing nonstationary EEG signals, where time-varying coefficients can capture the temporal dynamics of neural oscillations, improving classification accuracy and signal segmentation [11–14].

The adaptability of TVAR models is further demonstrated in their application to sleep EEG studies, where they estimate parameters and identify alterations in signal characteristics via segmentation methods utilizing basis functions [15]. Additionally, TVAR models have proven effective in acoustic signal processing for moving vehicles, where they analyze nonstationary acoustic signatures to provide insights into vehicle activities and types [16]. TVAR models have also been adapted into vector autoregressive frameworks for analyzing time-varying variance in multivariate settings, demonstrating their capability in handling more complex data structures and interdependencies [17]. Furthermore, TVAR approaches have been extended to model non-Gaussian processes, showcasing superior performance in managing complex stochastic behaviors and addressing limitations of Gaussian-based models [18]. However, despite their success in these areas, the application of TVAR models to economic data and forecasting remains largely unexplored, presenting an opportunity for further research and development.

We propose a modified TVAR model that integrates potential mean adjustments and employs the Kalman filter for parameter estimation. This modification enables real-time updates of parameter values as new data is received. The TVAR model merges the flexibility of time-varying parameters with an autoregressive structure, improving its capacity to respond to shifts in underlying data dynamics. This recursive estimation approach ensures that the model stays relevant and precise over time, making it highly effective for forecasting purposes.

In this research, we applied the TVAR model to U.S. real GNP data to demonstrate its effectiveness in economic forecasting. The TVAR model's performance is evaluated against the widely adopted Markov switching autoregressive (MSAR) model [19–21]. The MSAR model was first introduced by Hamilton [19] to handle time series data exhibiting nonlinear structural changes, such as economic data undergoing recessions or policy shifts. This model is highly popular due to its ability to capture regime shifts or different states in the data, such as transitions from high to low economic growth. Although MSAR has many advantages, such as the ability to detect structural changes, it has limitations in accommodating dynamic changes in the data over time, particularly when model parameters also change over time.

Some of the results from applying the MSAR model to the same dataset in this study are drawn from our previous research [21]. In that study, we compared the MSAR model with the Markov switching models with time-varying parameters (MSAR-TVP), finding that the MSAR-TVP model outperformed the MSAR model. However, in this study, our focus is on developing and estimating parameters for the base TVAR model and applying it for forecasting, which contributes to the body of knowledge in time-series modeling. The TVAR model we have developed is particularly suited for data that exhibits changes over time. Its effectiveness is demonstrated by comparing it with the MSAR model. This comparison aims to highlight the advantages of incorporating time-varying parameters in economic forecasting. Model accuracy is assessed using metrics such as mean absolute percentage error (MAPE) and mean absolute error (MAE). This research contributes to the expanding literature on dynamic modeling methodologies and provides important insights for economists and policy advisors.

The structure of this paper is as follows: Section 2 provides a detailed explanation of the materials and methods, Section 3 focuses on the discussion of the results and their implications, and Section 4 presents the conclusions along with recommendations for future research.

2 Materials and Methods

2.1 Time-Varying Autoregressive Model

Time-varying parameter (TVP) models, a subset of state-space models, are designed to account for the dynamic nature of economic data, allowing parameters to evolve over time [3,22]. In the context of autoregressive models, this flexibility helps capture structural changes and trends that static models might miss. A specific variant, the time-varying autoregressive (TVAR) model, extends this concept by incorporating an endogenous lag variable, effectively capturing time-dependent dynamics in the data [4,10–12].

The TVAR model developed in this study incorporates mean adjustments to effectively handle non-stationary data, with the aim of enhancing forecasting precision The model operates on an AR(p) process, integrating parameter estimation through the Kalman filter within the MLE framework, and is further optimized using the NelderMead method. This structure allows the model to adapt its coefficients dynamically over time, capturing the evolving data patterns.

The TVAR model is formulated as follows:

$$(y_t - \mu) = \beta_{t,1}(y_{t-1} - \mu) + ... + \beta_{t,p}(y_{t-p} - \mu) + \varepsilon_t, \tag{1}$$

$$(\beta_{t,k} - \delta_k) = \phi_k(\beta_{t-1,k} - \delta_k) + \nu_{t,k}, \quad k = 1, 2, ..., p. \tag{2}$$

In these equations, y_t is the dependent variable at time t, y_{t-k} is the explanatory variable with the endogenous lag of y_t, μ is the mean of the observed data, $\beta_{t,k}$ is the unknown time-varying parameter, ϕ_k is the autoregressive coefficient, δ_k is the mean of autoregressive process, and ε_t and $\nu_{t,k}$ are the error terms, which are assumed to follow $\varepsilon_t \sim$ i.i.d $N(0, \sigma^2)$ and $\nu_{t,k} \sim$ i.i.d. $N(0, \sigma_k^2)$. Equation (1) is known as the measurement equation, while Equation (2) is referred to as the parameter transition equation.

For simplicity, this research examines the TVAR model with an AR order of $p = 1$, denoted as the TVAR(1) model, expressed as

$$y_t = \mu^* + y_{t-1}\beta_{t,1} + \varepsilon_t, \tag{3}$$

$$\beta_{t,1} = \delta_1^* + \phi_1\beta_{t-1,1} + \nu_{t,1}, \tag{4}$$

where $\mu^* = (1 - \beta_{t,1})\,\mu$ and $\delta_1^* = (1 - \phi_1)\,\delta_1$.

This approach demonstrates the TVAR model's capability to adapt to changing economic conditions by adjusting its parameters in real-time, providing more accurate and reliable forecasts.

2.2 Kalman Filtering in TVAR Model

To estimate the time-varying parameter $\beta_{t,1}$ in the TVAR(1) model as defined in Equations (3)-(4), we employ the Kalman filter under the assumption that hyperparameters $\mu^*, \delta_1^*, \phi_1, \sigma^2$ and σ_1^2 are known. If these hyperparameters are not known, they can be estimated using the MLE method before applying the Kalman filter. The Kalman filter is a recursive algorithm that optimally estimates the unobserved state, here the time-varying parameter $\beta_{t,1}$ by utilizing the available information up to time t, minimizing the mean squared error.

Let $Y_t = \{y_t, y_{t-1}, ..., y_2, y_1\}$ represent the set of observation data up to time t, and $Y_{t-1} = \{y_{t-1}, y_{t-2}, ..., y_2, y_1\}$ represent the set up to time $t-1$. Key notations used include $\beta_{t|t-1} = E\left[\beta_t|Y_{t-1}\right]$, with $w_{t|t-1} = E\left[\left(\beta_t - \beta_{t|t-1}\right)^2\right]$ representing the variance based on this information. Similarly, $\beta_{t|t} = E\left[\beta_t|Y_t\right]$ is the estimate of β_t based on information up to time t, with $w_{t|t} = E\left[\left(\beta_t - \beta_{t|t}\right)^2\right]$ representing the corresponding variance. The prediction of y_t based on information up to time $t-1$ is $y_{t|t-1} = E\left[y_t|y_{t-1}\right]$ and the prediction error, which provides new information about β_t is given by $\eta_{t|t-1} = y_t - y_{t|t-1}$. The conditional variance of the prediction error is denoted as $f_{t|t-1} = E\left[\eta_{t|t-1}^2\right]$. Finally, when considering the entire sample, the estimate of β_t based on information up to time T is $\beta_{t|T} = E\left[\beta_t|Y_T\right]$ with $w_{t|T} = E\left[\left(\beta_t - \beta_{t|T}\right)^2\right]$, representing the variance based on this full information set.

The Kalman filter then iteratively predicts and updates these estimates as new data become available. The prediction and updating equations are as follows:
Prediction:

$$\beta_{t|t-1} = \delta_1^* + \phi_1\beta_{t-1|t-1}, \tag{5}$$

$$w_{t|t-1} = \phi_1^2 w_{t-1|t-1} + \sigma_1^2, \tag{6}$$

$$\eta_{t|t-1} = y_t - \left(\mu^* - y_{t-1}\right)\beta_{t|t-1}, \tag{7}$$

$$f_{t|t-1} = y_{t-1}^2 w_{t|t-1} + \sigma^2. \tag{8}$$

Updating:

$$\beta_{t|t} = \beta_{t|t-1} + w_{t|t-1}y_{t-1}\left[f_{(t|t-1)}\right]\eta_{(t|t-1)}, \tag{9}$$

$$w_{t|t} = \left(1 - \frac{w_{t|t-1}y_{t-1}^2}{f_{t|t-1}}\right)w_{t|t-1}. \tag{10}$$

After applying the Kalman filter, the next step in the TVAR(1) model estimation involves the integration of filtering techniques with a focus on optimizing the model's parameters. Initially, the values for $\beta_{0|0}$ and $w_{0|0}$ are set to initialize the Kalman filter process. As the filter iterates, it recursively computes β_t at each time step t, updating predictions with the arrival of new data.

The log-likelihood function, which measures the fit of the model to the observed data, is then approximated by calculating the conditional density $f(y_t|Y_{t-1})$ at each time step. This recursive process of updating and refining estimates is crucial for maintaining the accuracy and reliability of the model. The log-likelihood function is then approximated by:

$$l(\boldsymbol{\theta}) = \ln L(\boldsymbol{\theta}) = \sum_{t-1}^{T} \ln f\left(y_t|Y_{(t-1)}\right), \tag{11}$$

where $f(y_t|Y_{t-1})$ is the density of y_t given the past observations up to time $t-1$. The recursive likelihood function for the TVAR model is given by:

$$l(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t-1}^{T}\left[\ln\left((2\pi)\left|f_{t-1}\right) + \eta_t^2 f_{t-1}^{-1}\right]. \tag{12}$$

To optimize the model parameters, the NelderMead method, which is a nonlinear optimization technique, is employed [23, 24]. This method iteratively adjusts the parameter vector $\boldsymbol{\theta}$ to maximize the likelihood function, which serves as the objective criterion. Convergence criteria, including the maximum number of iterations and changes in function value, are set to ensure that the optimization process yields the best possible estimate for the model's parameters. This ensures that the model's predictions are not only precise but also adaptable to new data, reflecting the dynamic nature of the TVAR framework.

2.3 Model Performance Evaluation

The mean absolute percentage error (MAPE) and mean absolute error (MAE) are critical metrics for evaluating the accuracy of forecasting models [25]. MAPE expresses accuracy as a percentage, making it easily understandable for non-technical audiences, while MAE indicates the average error magnitude without regard to direction, reflecting how much predicted values deviate from actual observations. Lower values of MAPE and MAE generally signify better model accuracy. MAPE is particularly valued for its clarity and comprehensive error representation [26]. For reference, typical ranges of MAPE values for industrial and business data, and their corresponding interpretations, useful for evaluating forecasting model performance, are detailed in [27].

The formulas for MAPE and MAE are defined as follows:

$$\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{y_t - \hat{y}_t}{y_t}\right|, \tag{13}$$

$$\text{MAE} = \frac{1}{T}\sum_{t=1}^{T}\left|y_t - \hat{y}_t\right|, \tag{14}$$

where y_t is the actual value and $\hat{y}_t$ is the predicted value.

3 Results and Discussion

This section describes the implementation and performance evaluation of the TVAR model using quarterly U.S. real GNP data, following historical insights from Hamilton's MSAR model [19]. The training data, denoted by , spans from Q3 1952 to Q4 1984, while the testing period covers Q1 1985 to Q4 1986. These datasets, denominated in billions of chained 1982 dollars, were sourced from the *Business Conditions Digest* issues of February 1986 and March 1990. Specifically, series 50, pages 102 and 80, represent the training and testing datasets. The data can be accessed at: https://fraser.stlouisfed.org/title/business-conditions-digest-43?browse=1980s#7474 (accessed on 1 January 2024). A comparative analysis was carried out to evaluate the forecasting performance between the TVAR model and the MSAR model.

Before initiating the analysis, the fundamental properties of the dataset were investigated. The Tersvirta test results indicated nonlinearity, with a p-value of 0.0306, while the Chow test identified significant structural changes with a p-value below 2.2×10^{-16}, assuming a significance level of $\alpha = 0.05$. The TVAR model, which is well-suited for handling structural changes in data, was applied to the transformed and differenced series. This data transformation involved scaling by 100 times the log-difference of quarterly real GNP, represented as $Y_t = 100 \times \Delta \ln (Z_t)$, which captures the economic growth rate.

Following this, the TVAR model was applied to Y_t under an AR(1) process, formulated as an TVAR(1) model. Parameter estimation was performed using MLE, integrating the Kalman filter, and refined through numerical optimization with the NelderMead method. The resulting equations are as follows:

$$\hat{y}_t = \hat{\mu}^* + y_{t-1}\hat{\beta}_{t,1}, \tag{15}$$

$$\hat{\beta}_{t,1} = \hat{\delta}_1^* + 0.5527\hat{\beta}_{t-1,1}. \tag{16}$$

The estimation process provides the time-varying parameter $\hat{\beta}_{t,1}$, which is depicted in Figure 1. Additionally, the estimates for other parameters across various time intervals, including $\hat{\mu}^*$ and $\hat{\delta}_1^*$, are presented in Figure 2.

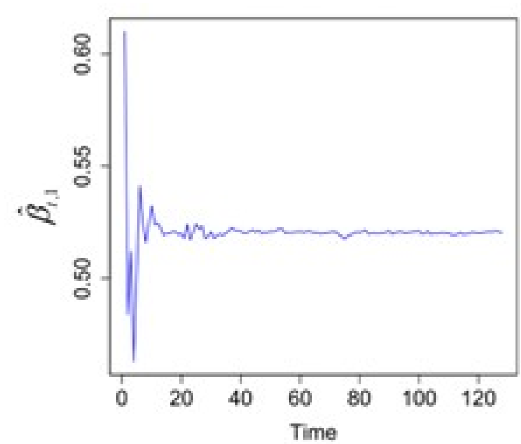
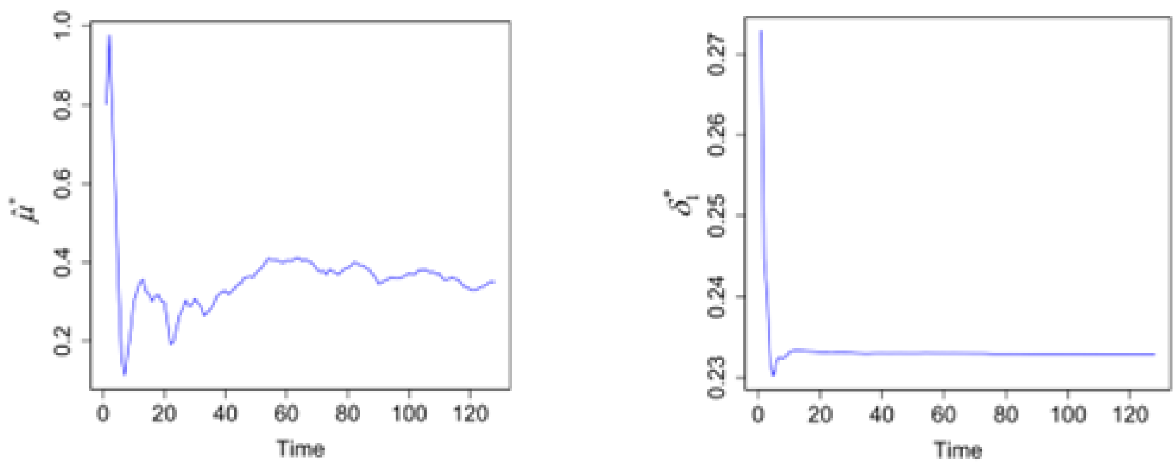


Figure 1: A Time-Varying Parameter $\hat{\beta}_{t,1}$ of the TVAR Model



(a) (b)

Figure 2: Parameter Estimates at Various Time Points: (a) $\hat{\mu}^*$, (b) $\hat{\delta}_1^*$

Figure 1 shows the time-varying parameter $\hat{\beta}_{t,1}$ which represents the autoregressive coefficient in the TVAR model. This parameter fluctuates over time, reflecting the model's ability to adjust to the dynamic nature of the data. Meanwhile, Figure 2 displays the fluctuations of parameters $\hat{\mu}^*$ and $\hat{\delta}_1^*$ over time. Parameter $\hat{\mu}^*$ exhibits significant variation in the early time periods, followed by gradual stabilization, while $\hat{\delta}_1^*$ demonstrates a more stable pattern after an initial shift. These figures illustrate how the parameters in the TVAR model adapt to changes in the data, highlighting the model's capability to capture structural shifts effectively.

In this research, the TVAR model's performance on U.S. real GNP data was compared to that of the MSAR model, which applies an AR(1) process across its two regimes (denoted as MS(2)-AR(1)). The MSAR model estimation employed the MLE method, utilizing the Hamilton filter, and optimization was achieved through the NelderMead method. A comprehensive discussion of the MSAR model's application to U.S. real GNP data can be found in our earlier work [21].

Table 1: Comparison Between TVAR and MSAR Models for U.S. Real GNP (1952-1986)

Model	Training		Testing	
	Maple (%)	MAE	MAPE (%)	MAE
TVAR	2.2168	49.7792	2.4261	87.9299
MSAR	3.3042	62.4868	4.3318	159.2326

Based on the analysis shown in Table 1, the TVAR model outperforms the MSAR model in both training and testing evaluations. During the training period, the TVAR model achieves a lower MAPE of 2.22% and an MAE of 49.78, highlighting its capability to capture underlying patterns and structural shifts in the data effectively.

In the testing period, the TVAR model continues to excel, with a MAPE of 2.43% and an MAE of 87.60, outperforming the MSAR model, which has a higher MAPE of 4.03% and an MAE of 147.71. These results indicate that the TVAR model provides more accurate predictions, making it better suited for both training and testing forecasting of U.S. real GNP data.

The diagnostic tests shown in Table 2 indicate that the residuals from both the TVAR and MSAR models follow a normal distribution and exhibit white noise characteristics. The p-values from the Kolmogorov-Smirnov (KS) test [28] and the Durbin-Watson (DW) test [29, 30] all surpass the significance level of $\alpha = 0.05$, suggesting that the residuals meet the necessary assumptions for valid statistical analysis.

Table 2: Residual Diagnostics of TVAR and MSAR Models for U.S. Real GNP (19521984)

Model	Regime	KS Test ($p-$Value)	DW Test ($p-$Value)
TVAR	–	0.6268	0.9739
MSAR	Regime 1	0.4025	0.1099
	Regime 2	0.1699	0.5465

As shown in Table 2, the MSAR model divides the data into two regimes, labeled Regime 1 and Regime 2, representing periods of economic expansion and recession, respectively. Each regime is modeled independently with its own parameters to account for structural shifts in the data. In contrast, the TVAR model does not rely on regimes; instead, it continuously adjusts its parameters over time, capturing changes without the need for predefined structural breaks as seen in the MSAR model.

In summary, the TVAR model demonstrates superior performance over the MSAR model when analyzing U.S. real GNP data, applicable for both training and testing periods. The TVAR model's strength lies in its ability to better capture and adapt to dynamic structural changes, making it a robust tool for economic time series forecasting. In contrast, while the MSAR model shows strong initial performance, the TVAR model consistently surpasses it in longer-term predictions, emphasizing its robustness and flexibility in modeling complex economic phenomena.

Figure 3 illustrates the comparative training and testing performance of the TVAR and MSAR models on U.S. real GNP data. The TVAR model aligns more closely with actual data during the training period, displaying high accuracy in tracking real GNP trends. It also demonstrates a superior ability to capture dynamic patterns and structural changes in the testing period, evidenced by its lower MAPE and better alignment with the test data compared to the MSAR model.

The TVAR model was utilized to predict outcomes on an expanded dataset, encompassing training data from Q1 1947 to Q4 2017 and testing data from Q1 2018 to Q4 2019. The dataset which is obtained from the Federal Reserve Economic Data (FRED) and available at https://fred.stlouisfed.org/series/GNPC96 (accessed on 30 June 2024), enabled an evaluation of the model's forecasting capability over a longer time span. The TVAR model exhibited strong performance with a MAPE of 5.06% and an MAE of 605.09 for the training data, indicating a high level of forecasting accuracy. For the testing data covering eight quarters, the model recorded a MAPE of 18.02% and an MAE of 3,738.56, which still falls within the range of good forecasting accuracy. This performance is depicted in Figure 4, illustrating the TVAR model's training and testing predictions.

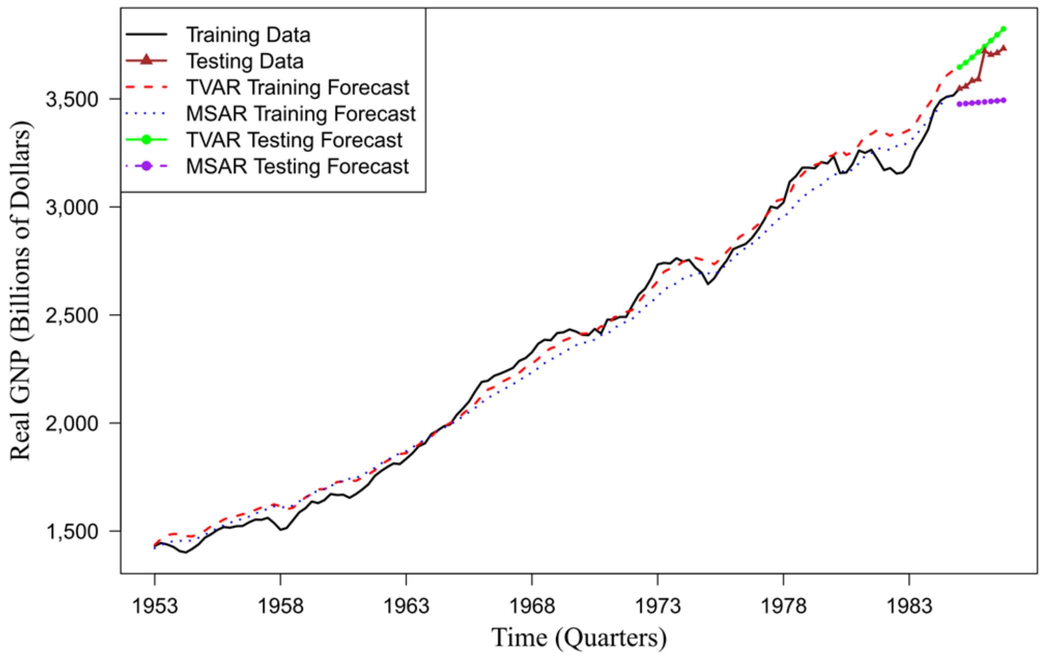


Figure 3: Comparison of TVAR and MSAR Model Performance on U.S. Real GNP (1952-1986)

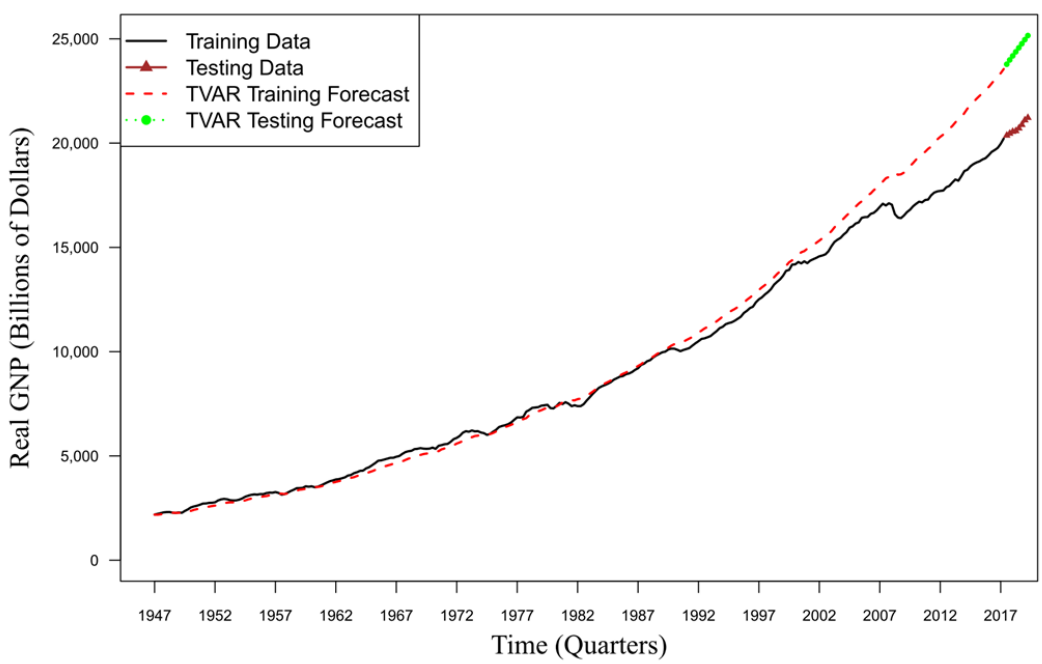


Figure 4: Comparison of Actual and Forecasted U.S. Real GNP using the TVAR Model (1947-2019)

Despite the increased error in the testing forecast, the TVAR model successfully met the assumptions of normality and white noise, validating its accuracy. However, the model faced challenges in maintaining prediction precision over extended testing periods, indicating some limitations in longer-term forecasts.

4 Conclusion

This study introduces a modified time-varying autoregressive (TVAR) model designed to improve forecasting accuracy for economic time series data, specifically U.S. real GNP. The model demonstrates superior performance compared to the MSAR model, with lower MAPE and MAE values during both the training and testing phases. However, limitations exist, particularly in maintaining accuracy over extended periods, as the model relies on first-order autoregressive processes, which may hinder its effectiveness for long-term forecasts. Additionally, the models performance may decline with non-Gaussian data. Future research could address these issues by incorporating higher-order autoregressive processes, exploring non-Gaussian error distributions, and employing more advanced parameter estimation methods such as Bayesian Markov chain Monte Carlo (MCMC) with Gibbs sampling. Overall, the TVAR model proves to be a valuable tool for economic analysis and forecasting, offering insights for policymakers and economists, while its flexibility makes it a strong candidate for future development in time series forecasting.

Acknowledgments

I would like to extend my sincere gratitude to the Center for Higher Education Funding (BPPT) under the Ministry of Education, Culture, Research, and Technology and the Indonesia Endowment Fund for Education (LPDP) under the Ministry of Finance of the Republic of Indonesia for their generous scholarship and financial support, which made this research possible.

References

[1] Tzagkarakis, G., Dionysopoulos, T. and Achim, A. Recurrence quantification analysis of denoised index returns via alpha-stable modeling of wavelet coefficients: Detecting switching volatility regimes. *Studies in Nonlinear Dynamics & Econometrics*. 2016. 20(1): 75–96.

[2] Meyler, A., Kenny, G. and Quinn, T. Forecasting Irish inflation using ARIMA models. *Central Bank of Ireland Technical Paper 3/RT/98*. 1998.

[3] Tanizaki, H. The time-varying parameter model revisited. *Kobe University Economic Review*. 1999. 45: 41–57.

[4] Kim, C. J. and Nelson, C. R. *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications.* Cambridge, MA, USA: The MIT Press. 2000.

[5] Blasques, F., Harvey, A. C., Koopman, S. J. and Lucas, A. Time-varying parameters in econometrics: The editors foreword. *Journal of Econometrics.* 2023. 237(2): 105439.

[6] Amir-Ahmadi, P., Matthes, C. and Wang, M. C. Choosing prior hyperparameters: With applications to time-varying parameter models. *Journal of Business & Economic Statistics.* 2020. 38(1): 124–136.

[7] Kim, C. J. Dynamic Linear Models with Markov-Switching. *Journal of Econometrics.* 1994. 60(1-2): 1–22.

[8] He, Z. Time-dependent shrinkage of time-varying parameter regression models. *Econometric Review.* 2024. 43(1): 1–29.

[9] Li, X. and Yuan, J. DeepTVAR: Deep learning for a time-varying VAR model with extension to integrated VAR. *International Journal of Forecasting*. 2024. 40(3): 1123–1133.

[10] Casas, I. and Fernandez-Casal, R. tvReg: Time-varying coefficient linear regression for single and multi-equations in R. *SSRN Electronic Journal*. 2019.

[11] Rajan, J. J. and Rayner, P. J. W. Parameter estimation of time-varying autoregressive models using the Gibbs sampler. *Electronics Letters.* 1995. 31(13): 1035–1036.

[12] Rajan, J. J., Rayner, P. J. W. and Godsill, S. J. Bayesian approach to parameter estimation and interpolation of time-varying autoregressive processes using the Gibbs sampler. *IEE Proceedings - Vision, Image & Signal Processing*. 1997. 144(4): 249–256.

[13] Liu, Z., Wang, L., Xu, S. and Lu, K. A multiwavelet-based sparse time-varying autoregressive modeling for motor imagery EEG classification. *Computers in Biology and Medicine.* 2023. 155: 106196.

[14] Gutirrez, D. and Salazar-Varas, R. EEG signal classification using time-varying autoregressive models and common spatial patterns. In *Proceedings of the 33rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBS)*, August 30September 3. Boston, MA, USA: IEEE. 2011. 6585–6588

[15] Amir, N. and Gath, I. Segmentation of EEG during sleep using time-varying autoregressive modeling. *Biological Cybernetics*. 1989. 61: 447–455.

[16] Eom, K. B. Analysis of acoustic signatures from moving vehicles using time-varying autoregressive models. *Multidimensional Systems and Signal Processing*. 1999. 10: 357–378.

[17] Patilea, V. and Rassi, H. Adaptive estimation of vector autoregressive models with time-varying variance: Application to testing linear causality in mean. *Journal of Statistical Planning and Inference.* 2012. 142(11): 2891–2912.

[18] Genaa, D., Kuruolu, E. E. and Ertzn, A. Modeling non-Gaussian time-varying vector autoregressive processes by particle filtering. *Multidimensional Systems and Signal Processing.* 2009. 21: 73–85.

[19] Hamilton, J. D. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica.* 1989. 57(2): 357–384.

[20] Doornik, J. A. A Markov-switching model with component structure for US GNP. *Economics Letters.* 2013. 118(2): 265268.

[21] Inayati, S., Iriawan, N. and Irhamah. A Markov switching autoregressive model with time varying parameters. *Forecasting.* 2024. 6(3): 568–590.

[22] Kalman, R. E. A new approach to linear filtering and prediction problems. *Journal of Basic Engineering.* 1960. 82(1): 35–45.

[23] Nelder, J. A. and Mead, R. A simplex method for function minimization. *Computer Journal.* 1965. 7(4): 308–313.

[24] R Core Team and Contributors Worldwide. *The R Stats Package, Version 4.3.1.* Available online: https://www.rdocumentation.org/packages/stats/versions/3.6.2/topics/optim (accessed on 25 June 2023).

[25] Vega, R., Flores, L. and Greiner, R. SIMLR: Machine learning inside the SIR model for COVID-19 forecasting. *Forecasting.* 2022. 4(1): 72–94.

[26] Moreno, J. J. M., Pol, A. P., Abad, A. S. and Blasco, B. C. Using the R-MAPE index as a resistant measure of forecast accuracy. *Psicothema.* 2013. 25(4): 500–506.

[27] Lewis, C. D. *Industrial and Business Forecasting Methods: A Practical Guide to Exponential Smoothing and Curve Fitting.* Penang, Malaysia: Heinemann. 1982.

[28] Daniel, W. W. *Applied Nonparametwaaric Statistics,* 2nd Edition. Boston, Massachusetts: PWS-Kent. 2000.

[29] Durbin, J. and Watson, G. S. Testing for serial correlation in least squares regression, I. *Biometrika.* 1950. 37(3/4): 409–428.

[30] Durbin, J. and Watson, G. S. Testing for serial correlation in least squares regression, II. *Biometrika.* 1951. 38(1/2): 159–177.