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Comparative Study and Prediction of Seasonal Adjustment Methods for Economic Time Series

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Abstract Economic time series generally contain seasonal influences. In order to accurately analyze the development and changes of economic phenomena, internationally widely used methods such as X-12-ARIMA and TRAMO/SEATS are applied for seasonal adjustment to remove seasonal effects, thereby providing a theoretical foundation for further modeling, analysis, and forecasting of economic time series. This study takes the total retail sales of consumer goods in China from January 2009 to June 2023 as the research object. By comparing the residuals of two fitted models, it is found that the X-12-ARIMA method performs better. Therefore, the original series is modeled using the ARIMA approach, and data from July 2023 to June 2024 are forecasted, with prediction errors satisfying the national statistical error standards. After seasonal adjustment, seasonal factors are successfully separated from the total retail sales of consumer goods, restoring the underlying structure of the original series. This provides a basis for better understanding macroeconomic characteristics, capturing economic dynamics, and improving the accuracy of economic forecasting.

Keywords X-12-ARIMA; Time series; Seasonal adjustment; Forecast; TRAMO/SEATS.

Mathematics Subject Classification 37M10, 91B02.

1 Introduction

In the process of China's Belt and Road globalization strategy construction, China's economy continues to grow and develop, and China's economy has made remarkable achievements in the world. At the same time, accurately monitoring macroeconomic operations, fully analyzing various factors that affect changes in the economic situation, and effectively exploring the regularity of changes in economic data are important issues of concern to all countries in the world. An economic time series is a chronologically ordered sequence of numerical economic indicators, reflecting the evolution of economic phenomena. Economic time series can reflect the operational status of the economy, the development status and changing process of economic phenomena at different times. Time series analysis methods allow researchers to Study the trends and regularities of the development and changes of phenomena, and conduct analysis, modeling, and prediction.

Due to the influence of long-term trends, cyclical fluctuations, seasonal changes, and random fluctuations on economic sequences, as well as the presence of significant cyclical changes in certain sequences, seasonal effects exist. In order to accurately describe the comparability and regularity of the sequence, it is necessary to adjust the seasonal factors of the sequence separation to better reflect the changes in economic indicators.

Due to the influence of seasonal factors, the time series chart of economic time series generally presents periodic changes, and economic variables cannot be directly calculated for a month-on-month growth rate. Even if the month-on-month growth rate value is calculated according to the formula, it is meaningless due to the lack of comparability. By separating the seasonal effects through seasonal adjustment, the time series chart no longer shows cyclical changes and can reflect the trend regularity of economic variable development. Based on the seasonally adjusted sequence, the calculated month-on-month growth rate can more accurately reflect the changes in economic variables. Through seasonal adjustment, the seasonal factor sequence, trend cycle sequence, and unplanned sequence can be effectively separated, and the seasonally adjusted sequence without seasonal influence can be obtained, reflecting the effects of different influencing factors on economic variables[1]. The seasonal factor sequence can be used to analyze the regularity and degree of impact of cyclical changes in economic phenomena and to organize production and business activities through analysis and prediction based on cyclical patterns.

The X-12-ARIMA method and TRAMO/SEATS method are currently the most commonly used seasonal adjustment methods internationally. Taking the total retail sales of consumer goods in China as the research object, this study compares the effects of the two methods in seasonal adjustment, determines the suitable seasonal adjustment method for this study, and carries out subsequent work processing.

2 Literature Review

There are currently two classic seasonal adjustment theories. One is the X-12 seasonal adjustment method theory based on moving average seasonal adjustment. The other is the theory of TRAMO/SEATS seasonal adjustment method [2].

Poynting (1884) was the first to propose using the moving average method to perform linear trend correction on time series, removing the influence of seasonal factors and obtaining a seasonally adjusted corrected trend series. Then, Macauley (1931) proposed using the moving average ratio method for seasonal adjustment, laying the theoretical foundation for the moving average seasonal adjustment method. Later, Shishkin (1954) applied the X-1 method to adjust time series at the United States Census Bureau seasonally. After several generations of improvement and revision by Box, Hillmer, and Tiao (1978), the seasonal adjustment method gradually developed into the widely used X-11, X-12, X-13 time series seasonal adjustment methods today, with the central idea still based on X-11 theory [3]. The X-12-ARIMA method has improved parameter setting, method selection, trading day setting, missing value, and outlier handling. It has shown good adjustment effects in separating seasonal, trend cycle, and irregular factor influences.

In 1972, Cleveland (1972) proposed using signal extraction methods for seasonal adjustment. He collaborated with Tiao (1976) to decompose the X-11 method into ARIMA, deriving ARIMA decomposition models for trend, season, and unplanned components and laying a theoretical foundation for further improving the theory of seasonal adjustment [4]. Box, Hillmer, and Tiao (1978) systematically used ARIMA models for seasonal adjustment and signal extraction, extracting information from the model as AR process signals representing trends and MA process signals representing seasons [5]. Maravall and Gomez (1997) jointly developed a signal extraction technique based on ARIMA model for seasonal adjustment, and developed a seasonal adjustment software for TRAMO/SEATS method [6]. The TRAMO/SEATS method can flexibly set regression variables and has ideal applications in setting regression factors and adjusting seasonal factors, outliers, and other aspects.

However, Wang Yan (2020) pointed out that time series seasonal adjustment models usually include additive models, multiplicative models, and mixed models [7]. When the sequence has short-term correlation, the ARMA(p,q) model can usually be used to extract information. When the sequence contains seasonal influences and the seasonal influences themselves have correlations, the correlation of seasons can be extracted using the ARMA$(P,Q)_S$ model with periodic step sizes as units. Usually, seasonal adjustments are made to the observation value sequence using a multiplicative model ARIMA$(p,d,q) \times (P,D,Q)_S$. Just like the following model:

$$\nabla^d \nabla_S^D x_t = \frac{\Theta(B)\Theta_S(B)}{\Phi(B)\Phi_S(B)}\varepsilon_t \tag{1}$$

where,

$$\begin{aligned}
\nabla^d &: \ d\text{-order difference,} \\
\nabla_S^D &: \ D\text{-order seasonal difference with period } S, \\
\Theta(B) &= 1 - \theta_1 B - \cdots - \theta_q B^q, \\
\Phi(B) &= 1 - \phi_1 B - \cdots - \phi_p B^p, \\
\Theta_S(B) &= 1 - \theta_1 B^S - \cdots - \theta_Q B^{QS}, \\
\Phi_S(B) &= 1 - \phi_1 B^S - \cdots - \phi_P B^{PS}, \\
\varepsilon_t &: \ \text{random white noise sequence.}
\end{aligned}$$

This article uses a multiplicative model for seasonal adjustments. Zhang Jiaqi (2024) pointed out that a monthly economic time series can be influenced by multiple factors, including trend-cycle, seasonal, and irregular factors. In order to more clearly reveal the movement rules of the economic series, it is necessary to seasonally adjust the original series before analysis to improve its reliability [8].

Moreover, Bai Tongyuan (2021) pointed out that monthly data is significantly affected by time and trends, exhibiting non-stationary characteristics. It is necessary to perform a first-order 12 step differencing process on the data to separate linear increasing trends and seasonal influences. After differencing, the sequence fluctuates up and down within a certain range, eliminating trend. Then, the sequence is subjected to unit root testing and pure randomness testing after differencing, and further model construction is carried out [9]. While, Min Yingying (2020) compared the performance of X-12-ARIMA and TRAMO/SEATS in seasonal adjustment for sales data from Suning Electric. Both methods identified similar seasonal patterns, but their outputs differed in data integrity: TRAMO/SEATS exhibited significant missing values at the endpoints of the adjusted series, with higher volatility in these regions, whereas X-12-ARIMA maintained better data completeness. Based on these observations, the study concluded that X-12-ARIMA provided more reliable separation of seasonal and trend components, justifying its selection for their analysis [10].

In summary, to accurately analyze the characteristics of economic time series, it is not only necessary to examine the stationarity of the series and whether there is autocorrelation in the seasons but also to decompose it according to the constituent elements of the time series, separate the seasonal components, trend cycle components, and irregular components, in order to obtain a seasonally adjusted sequence that reflects the essential regularity of the economic series. The present work first started with the preprocessing of time series data and conducted research on stationarity. Secondly, two seasonal adjustment methods should be applied to adjust the total retail sales of consumer goods in Chinese society. Finally, the seasonal adjustment method that is more suitable for this case is compared, which facilitates the selection of appropriate methods for seasonal adjustment of similar phenomena for research.

3 Preprocessing and Stationarity Testing of Economic Time Series

This article selects the total retail sales of consumer goods in China from January 2009 to June 2023 as the research object, and the specific data refers to the monthly data of the National Bureau of Statistics of China [11].

3.1 Methods for Handling Missing Data

Since 2012, the monthly data of the China's total retail sales of consumer goods is no longer separately calculated for January and February, but released in the cumulative amount from January to February, to eliminate the impact of uncertainties during the Chinese Spring Festival holiday. Due to the lack of data on the total retail sales of consumer goods in January and February of each year after 2012, it is necessary to supplement the missing data for seasonal adjustments in the future. Since the Chinese Spring Festival occurs irregularly in January or February every year, it has different degrees of impact on the statistics of total retail sales of consumer goods in China. There are many methods to fill in missing data in time series, such as interpolation, regression prediction, RAMO/SEATS method, seasonal index method, etc. Due to the particularity of the data in this case, there is no data for January and February of each year, but the cumulative total is available. In the case of two consecutive missing data, the general method is not applicable.

The European Statistical System (ESS) states in [12] that when only quarterly or cumulative data are available and there is no clear evidence of sub-period fluctuations (such as monthly variation), the equal-split method can be considered a conservative and transparent approach. It is therefore suitable for preliminary analysis or situations involving missing data. Therefore, in order to simplify data processing in this study, the cumulative amount is evenly distributed to supplement the total retail sales in January and February. This method simplifies the estimation and calculation process of missing values, ensures data accuracy, and improves data comparability.

3.2 Stability Test of Time Series

For the convenience of defining variables, this present study defines the variable sequence name of the total retail sales of consumer goods in China as Y and analyzes monthly data from January 2009 to June 2023 as the research object.

3.2.1 Timing diagram verification

Figure 1 shows that the total retail sales of consumer goods in China have maintained a strong upward trend from 2009 to 2023, with a clear linear growth trend, and the variable shows a relatively regular periodic fluctuation with an annual cycle. Obviously, this sequence is non-stationary. From 2020 to 2023, due to the impact of COVID-19, variables fluctuated greatly, and the growth rate slowed down during the gradual recovery of the national economy in the first half of 2023, showing a downward trend.

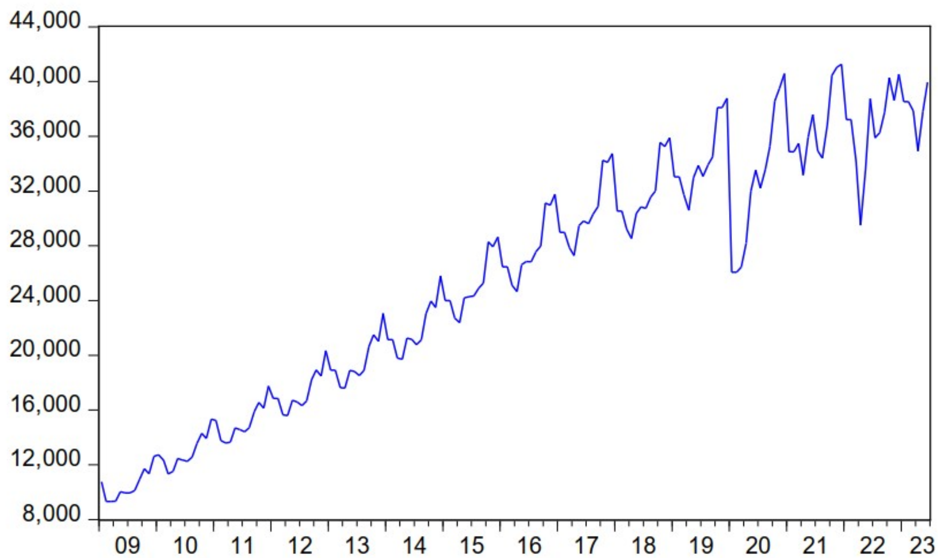

Figure 1: Time Series Chart of Total Retail Sales of Consumer Goods (y) in China from 2020 to 2023 (x)

3.2.2 Unit root test

Traditional time series analysis methods rely on the assumption of stationarity to establish models. If the data is stationary, it indicates that its statistical patterns are consistent over time, which helps the model to more accurately capture data patterns, reduce noise interference caused by non-stationarity, and improve prediction accuracy. If the data is non-stationary, it cannot be directly modeled. It is necessary to preprocess the data for stationarity, such as differencing and de-seasonalization, and then establish a corresponding time series model. Therefore, stationarity testing is of great significance for the construction and prediction of models.

Based on the revised data of total retail sales of consumer goods, the unit root test was conducted, and Table 1 was obtained.

Table 1: ADF Test Results of Total Retail Sales of Consumer Goods

Significance Level	Test Critical Values	T-Statistic	P-value	Conclusion
1%	-2.579404	2.487279	0.9970	Non-stationary
5%	-1.942818			
10%	-1.615392			

According to the Augmented Dickey–Fuller (ADF) test results, since the significance test statistic $p = 0.9970 > 0.1$, the null hypothesis H_0 is accepted, and the original sequence Y is non-stationary. Moreover, because the test statistic $T = 2.487279$ is greater than the critical values at the 1%, 5%, and 10% levels, the null hypothesis is accepted. Hence, the original sequence is a non-stationary sequence with unit roots, which shows a clear trend in the time series plot.

Through the time series chart and unit root test, it can be shown that the total retail sales of Chinese consumers are a non-stationary time series. Therefore, this present study will not proceed with other stationarity tests.

3.3 Differential Analysis of Time Series Trends

Figure 1 also shows that the total retail sales of consumer goods in China exhibit an approximately linear increasing trend. For sequences with significant linear trends, we usually perform first-order differencing on the sequence to examine the information extraction effect of differencing on the trend term of the sequence. Through differential operation, linear trends in time series can be removed, which plays a role in separating long trends in seasonal adjustment work, making non-stationary series stationary. Regression analysis of stationary series can obtain more accurate results. If the sequence contains a curve trend, a stationary sequence can be obtained by separating the influence of the curve trend through second-order or higher-order differencing. If a differential operation with a step size as the period can be adopted for sequences containing fixed periodic changes, better periodic information can usually be obtained.

3.3.1 Analysis of first-order differenced sequence timing diagram

The difference sequence DY obtained through first-order differencing in this article is shown in Figure 2. From the difference time series in Figure 2, it can be seen that the sequence DY becomes more stable than the sequence Y after first-order differencing and overall fluctuates around a certain mean. Whether the sequence DY reaches true stationarity, we still needs further stationarity testing [13]. Firstly, ADF test was performed on the first-order difference sequence DY, and the results are shown in Table 2.

According to the ADF test results in Table 2, $p < 0.1$, the null hypothesis H_0 is rejected and the DY sequence is stationary. Because the test statistic $T = -5.175326$ is less than the critical values at the 1%, 5%, and 10% significance levels, the null hypothesis is rejected. The sequence DY is stationary without unit roots, consistent with the time series diagram.

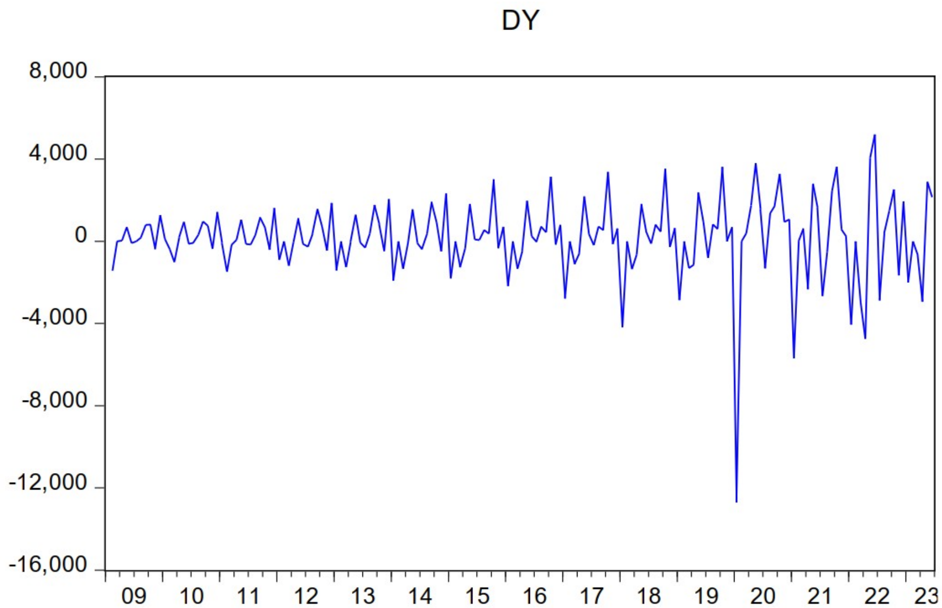

Figure 2: First order Difference Time Series of Total Retail Sales of Consumer Goods

Table 2: ADF Test Results on the First-Order Difference Sequence DY

Significance Level	**Test Critical Values**	**T-Statistic**	**P-value**	**Conclusion**
1%	-3.471454	-5.175326	0.0000	Stationary
5%	-2.879494			
10%	-2.576422			

3.3.2 Descriptive statistics and histogram analysis of first-order difference sequences

The results in Figure 3 show that the series Y of total retail sales of consumer goods in China has undergone first-order differencing. After separating the linear trend, the series DY is a stationary sequence with a mean of 168.7561. Since the mean is not 0, the series DY is a non-white-noise sequence, and there is still a lot of information in the sequence that has not been separated. It is precisely because economic variables are also influenced by factors such as cyclic fluctuations, seasonal variations, and random fluctuations that the first-order differenced sequence DY has a non-zero mean and exhibits periodic fluctuations on the time series chart.

4 Research on Seasonal Adjustment of Economic Time Series

Due to the joint influence of long-term trends (T), cyclical fluctuations (C), seasonal variations (S), and random fluctuations (I) on economic time series, it is usually assumed that the series exhibits different characteristics when all or part of these four factors are involved in the processing of time series. The interaction between the four factors mainly includes the additive model ($X_t = T_t + C_t + S_t + I_t$), the multiplicative model ($X_t = T_t \cdot C_t \cdot S_t \cdot I_t$), and the mixed

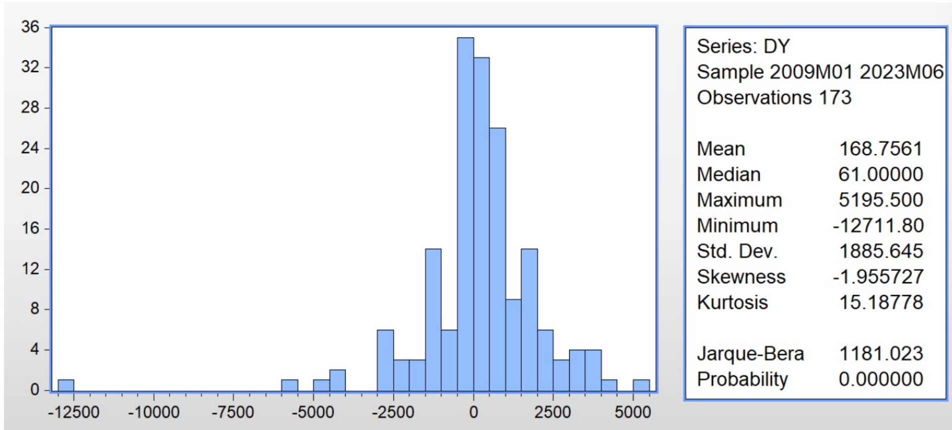

Figure 3: Histogram of the First-Order Difference Sequence

additive–multiplicative model [14].

The information of sequence DY after first-order differencing, which separates linear trends, still includes the effects of other factors. From Figure 2, it can be observed that the periodic seasonal variation trend of sequence DY after first-order differencing is very obvious. Seasonal adjustment is the process of studying economic time series by removing the influence of seasonal factors, revealing the essential characteristics or basic trends of the time series, and decomposing the factors that affect the changes in the series into components to analyze the results of each factor affecting economic changes.

4.1 Application X-12-ARIMA Seasonal Adjustment

By applying X-12-ARIMA seasonal adjustment to the original sequence Y, the sequence can be decomposed according to influencing factors. Components such as the trend cycle component (Y_TC), seasonal component (Y_SF), irregular component (Y_IR), and seasonally adjusted sequence (Y_SA) can be separated. Based on experience, a multiplicative model is used in this case, as shown in Figure 4.

From Figure 4, it can be seen that: (1) Analysis of the Impact of Irregular Changes: There will be varying degrees of fluctuation at the beginning of each year, especially from 2020 to 2023, due to the impact of COVID-19, economic growth and residents' income and expenditure have experienced a rare decline in many years, which has had different degrees of impact on China's society, production, life and other aspects, resulting in significant changes in the total retail sales of consumer goods in China during special periods. (2) Seasonal factor analysis: From 2009 to 2023, the sequence exhibits regular periodic fluctuations, which are significantly influenced by seasons. Seasonal factors always fluctuate around 1, with each year as a cycle length. (3) Trend cycle analysis: There is a significant linear trend between 2009 and 2023; although there have been some fluctuations after 2020, the overall trend is still upward, and the trend remains strong. (4) Seasonally adjusted sequence analysis: After seasonal adjustment, the Y_SA sequence is a sequence that removes seasonal influences from the original sequence. From the graph, it shows an overall linear upward trend, but there have been abnormal fluctuations since 2020, indicating a significant impact of COVID-19 on the Chinese economy. At the same time, China has introduced many relevant policies to control the epidemic, providing effective

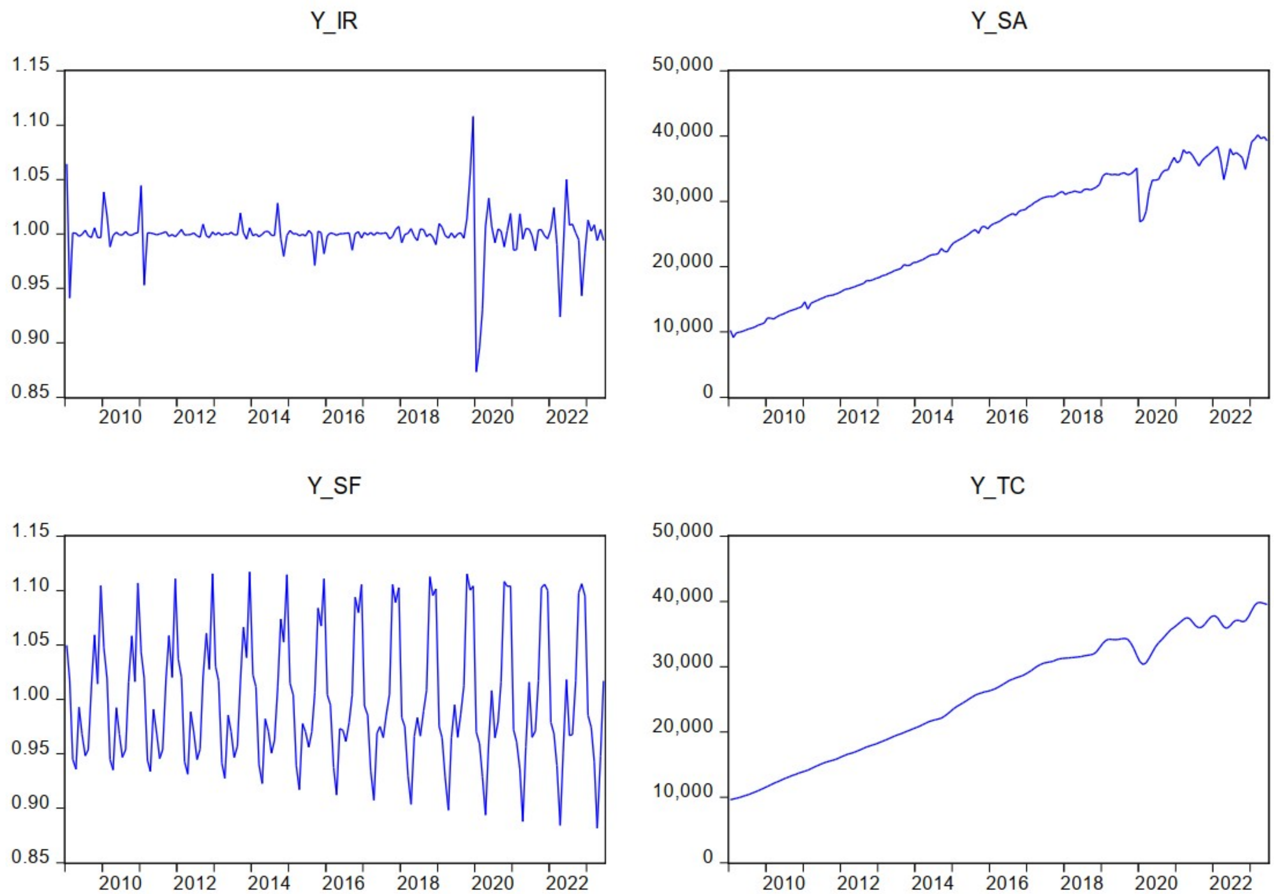

Figure 4: Seasonal Adjustment Effect Chart of Total Retail Sales of Consumer Goods in China (2009–2023) by X-12-ARIMA

guarantee mechanisms for stable economic development.

4.1.1 Stability test of X-12-ARIMA seasonally adjusted sequence Y_SA

By using the X-12-ARIMA seasonal adjustment method, the total retail sales of consumer goods in China were adjusted to obtain the adjusted series Y_SA. Next, the series Y_SA will be used as the research object to investigate how the adjustment effect of the adjusted series Y_SA will affect the subsequent modeling work of time series seasonal adjustment.

(1) ADF test of adjusted sequence Y_SA

From Figure 4, we can observe that the adjusted sequence Y_SA shows a clear linear increasing trend. From the perspective of a time series diagram, the sequence Y_SA belongs to a non-stationary sequence. According to the sequence Y_SA, the unit root test is shown in Table 3.

Table 3: ADF Test Results of Adjusted Series of Total Retail Sales of Consumer Goods (Y_SA)

Significance Level	Test Critical Values	T-Statistic	P-value	Conclusion
1%	-3.468295	-0.900043	0.7864	Non-Stationary
5%	-2.878113			
10%	-2.575684			

According to the ADF test results analysis, the significance level indicates that $p > 0.1$ for the sequence Y_SA, and the test statistic $T = -0.900043$ is greater than the critical values at the 1%, 5%, and 10% significance levels. Therefore, the null hypothesis is accepted, indicating that the original sequence is non-stationary with a unit root. The test results are consistent with the analysis of the time series graph.

(2) First-order differential analysis of adjusted sequence Y_SA

Due to the clear linear trend shown in the adjusted sequence Y_SA time chart, it belongs to a non-stationary sequence. Perform differential operation to separate linear trends for non-stationary sequences. The timing diagram of the first-order difference of sequence Y and the first-order difference of sequence Y_SA is shown in Figure 5.

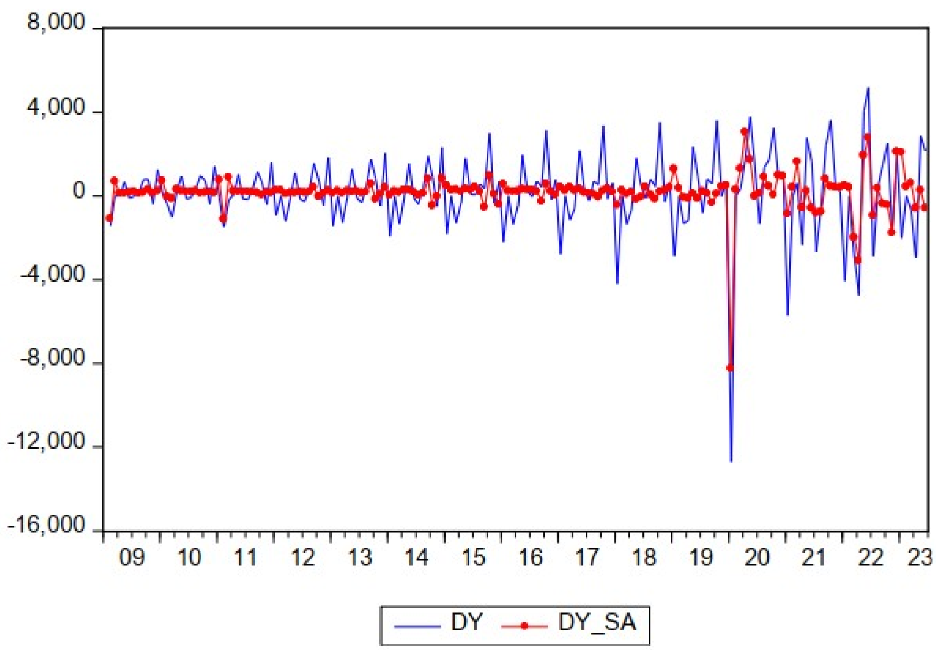

Figure 5: DY and DY_SA Effect Chart of Total Retail Sales of Consumer Goods in China (2009-2023) by X-12-ARIMA

From the time series diagrams of the first-order difference (DY) of sequence Y and the first-order difference (DY_SA) of seasonally adjusted Y_SA, it can be seen that DY has a larger fluctuation amplitude in its graph, while DY_SA has a smaller fluctuation amplitude. Therefore, DY_SA appears more stable. Indicating that the first-order difference sequence after seasonal adjustment using ARIMA is more operable. The DY_SA sequence time chart shows weak periodic fluctuations from 2009 to 2019, indicating that the seasonal adjustment effect of the DY_SA sequence is good, effectively eliminating seasonal fluctuations and making the sequence more stable, in line with theoretical expectations. Seasonal adjustment can only handle predictable seasonal factors, but cannot eliminate the influence of irregular factors (such as epidemics, natural disasters, and other sudden events). China was hit by the COVID-19 in 2020-2023, which led to the non-seasonal abnormal fluctuations in the total retail sales of consumer goods. The drastic changes in DY_SA during this period in the figure reflect this unpredictable external shock, which is consistent with the description [15].

4.1.2 First-order differencing stationarity test of adjusted sequence Y_SA

According to the ADF test results in Table 4, the significance test result for the sequence DY_SA indicates that $p < 0.1$, and the test statistic $T = -10.04061$ is less than the critical values at the 1%, 5%, and 10% significance levels. Therefore, the null hypothesis is rejected, and the sequence DY_SA is stationary without a unit root. The test results are consistent with the time-series analysis [16].

Table 4: ADF Test Results of the Adjusted First-Order Difference Sequence (DY_SA)

Significance Level	**Test Critical Values**	**T-Statistic**	**P-value**	**Conclusion**
1%	-3.468980	-10.04061	0.0000	Stationary
5%	-2.878413			
10%	-2.575844			

After seasonal adjustment and differencing operations, the economic time series can be transformed into a stationary series, which can further be used for modeling. The model that passes parameter testing can then be analyzed and used for forecasting.

4.2 Application of TRAMO/SEATS Seasonal Adjustment

Using TRAMO/SEATS seasonal adjustment to perform a seasonal adjustment on the original sequence Y according to the multiplication model. For the convenience of comparison, the sequence based on the TRAMO/SEATS method for seasonal adjustment will be named TS. Therefore, the sequence can be decomposed according to influencing factors, and information such as trend component (TS_TRD), seasonal component (TS_SF), irregular component (TS_SR), season, and adjusted sequence (TS_SA) can be separated [17], as shown in Figure 6.

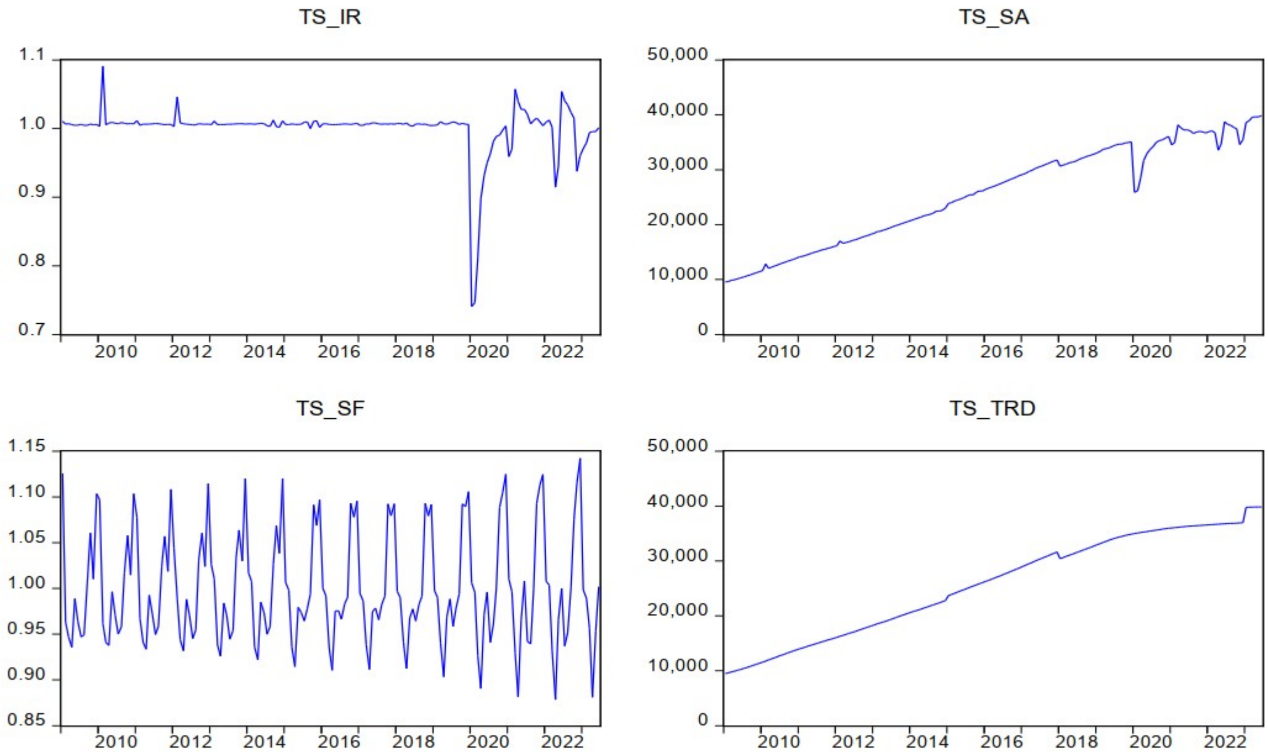

Figure 6: Seasonal Adjustment Effect Chart of Total Retail Sales of Consumer Goods in China (2009–2023) by TRAMO/SEATS

After seasonal adjustment using TRAMO/SEATS, the total retail sales of consumer goods in China can be observed in Figure 6 and are analyzed as follows.

(1) Analysis of the impact of irregular changes: The graphs showed significant fluctuations in 2010 and 2012, which were due to the good operation of the Chinese economy and the rapid growth of various economic indicators during this period. From 2020 to 2023, there will be significant fluctuations in the chart, and due to the impact of COVID-19, the total retail sales of consumer goods in China will experience significant changes during this special period.

(2) Seasonal factor analysis: From 2009 to 2023, the sequence was significantly affected by seasons, with seasonal factors fluctuating around 1, with each year as a cycle length.

(3) Trend cycle analysis: There is a significant linear trend between 2009 and 2023, although there have been some fluctuations after 2020, the overall trend is still upward and the trend remains strong.

(4) Seasonal adjusted sequence analysis: After excluding seasonal influences, the linear trend is obvious. After 2020, due to the impact of COVID-19, the Chinese national economy was severely affected, resulting in significant fluctuations in the TS_SA sequence in the later stage.

4.2.1 Stability test of TS_SA in TRAMO/SEATS seasonal adjusted sequence

According to the TS_SA data of the TRAMO/SEATS seasonally adjusted sequence, the following results were obtained through unit root testing, as shown in Table 5.

Table 5: ADF Test Results of Adjusted Series of Total Retail Sales of Consumer Goods (TS_SA)

Significance Level	Test Critical Values	T-Statistic	P-value	Conclusion
1%	-2.578555	2.248619	0.9943	Non-Stationary
5%	-1.942699			
10%	-1.615467			

The ADF test results showed that the significance test statistic indicates $p = 0.9943 > 0.1$ and the test statistic $T = 2.248619$ were greater than the critical values at the 1%, 5%, and 10% significance levels. Therefore, the null hypothesis H_0 was accepted, and the adjusted sequence TS_SA was found to be non-stationary, which is consistent with the time series chart.

4.2.2 First-order differential analysis of adjusted sequence TS_SA

Due to the instability of the adjusted sequence TS_SA, the time series plot shows a clear linear trend. For such sequences, differencing can be performed to remove the linear trend. First-order differencing is applied to the adjusted sequence TS_SA and the original sequence Y, and the resulting differenced time series is shown in Figure 7.

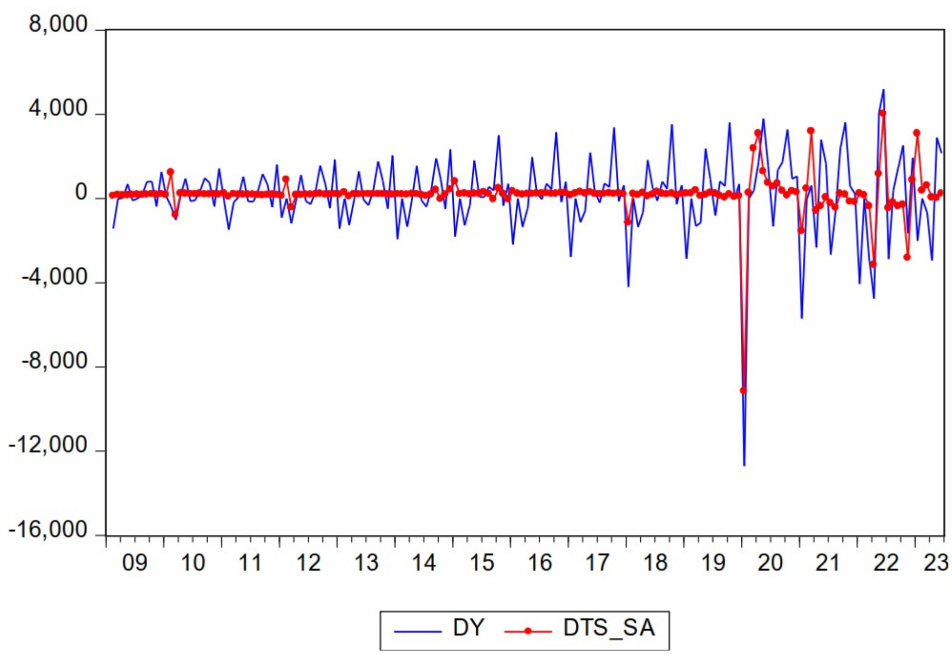

Figure 7: DY and DTS_SA Effect Chart of Total Retail Sales of Consumer Goods in China (2009-2023) by TRAMO/SEATS

From the time series diagrams of the first-order difference (DY) of sequence Y and the first-order difference (DTS_SA) of TS_SA after seasonal adjustment, it can be seen that DTS_SA performs more smoothly and has smaller wave amplitudes compared to DY. For sequences with linear trends, the influence of linear trends can be eliminated to achieve stationarity and weaken the influence of seasonal and irregular factors after differencing. Therefore, from 2009 to 2019, there were weak fluctuations, and the adjustment effect was good. The significant changes in the TS_SA sequence from 2020 to 2023 are due to the COVID-19 pandemic in China.

4.2.3 First-order differential stationarity test of adjusted sequence TS_SA

According to the ADF test results, the significance level indicates that $p < 0.1$, and the test statistic $T = -11.53785$ is less than the critical values at the 1%, 5%, and 10% significance levels. Therefore, the null hypothesis is rejected, and the sequence DTS_SA is stationary without a unit root. The test results are consistent with the time series analysis. The inspection results are shown in Table 6.

Table 6: ADF Test Results of Adjusted First-order Difference Sequence (DTS_SA)

Significance Level	**Test Critical Values**	**T-Statistic**	**P-value**	**Conclusion**
1%	-3.468749	-11.53785	0.0000	Stationary
5%	-2.878311			
10%	-2.575791			

After seasonal adjustment and differencing of economic time series, a stable time series can be obtained, which can further carry out modeling work and lay the foundation for subsequent statistical analysis and prediction of time series.

4.3 Analysis of the Effects of Two Methods

Through the use of X-12-ARIMA and TRAMO/SEATS methods to study the seasonal adjustment of the total retail sales of consumer goods in China, it can be found that both methods handle the separation of seasonal factors very well and can effectively decompose the sequence into trend components, seasonal components, and irregular components, thereby obtaining an adjusted sequence that reflects the removal of seasonal influences. The comparative suitability and effectiveness of these two methods remain unresolved and warrant further research.

4.3.1 Seasonal adjusted sequence comparison

After seasonal adjustment of the total retail sales of consumer goods in China using two methods, the sequences Y_SA and TS_SA were obtained. From Figure 8, it can be seen that there is no essential difference between the two methods in eliminating seasonal effects in the adjusted sequence charts obtained by seasonal adjustment, and both methods achieve good results. The two curves are basically consistent, and it is impossible to determine which method is more suitable for this case from the sequence chart.

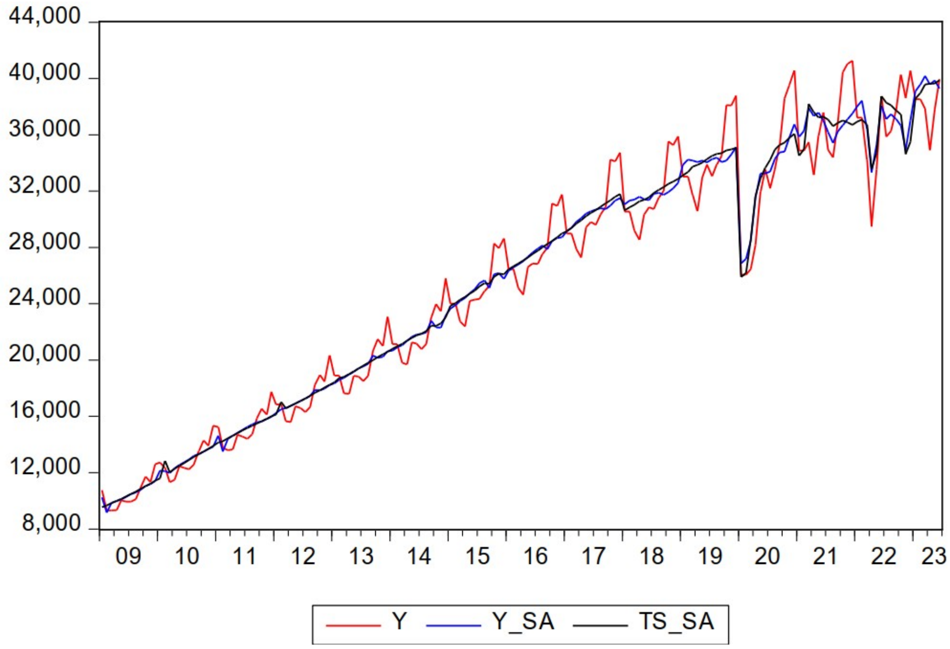

Figure 8: Original Sequence and Seasonally Adjusted Sequence Time Chart

4.3.2 Analysis of deviation between observational values and factor decomposition fitting values

The evaluation standard for measuring the quality of econometric models is whether they can correctly reflect the real laws of economic phenomena. From the perspective of the model, it is whether the fitting effect meets the significance requirement. The minimum sum of squares or mean square deviation of the model fitting residual can determine which method has better fitting effect. Due to the lack of prediction in this case, residuals cannot be used to determine the effectiveness of the study. However, the evaluation can be based on the decomposition effect of time series factors. This article takes the difference (decomposition error) between the observed values and the fitted values of the trend values, seasonal factors, and irregular factors after sequence decomposition as the analysis object to determine the effect of seasonal adjustment.

For X-12-ARIMA method, the decomposition error is given by

$$\sum (Y_i - Y_{TC,i} \cdot Y_{SF,i} \cdot Y_{IR,i}) = 0, \tag{2}$$

$$\sum (Y_i - Y_{TC,i} \cdot Y_{SF,i} \cdot Y_{IR,i})^2 = 0. \tag{3}$$

where, Y_i denotes the observation values of the original sequence, $Y_{TC,i}$ denotes the trend values under the ARIMA method, $Y_{SF,i}$ denotes the seasonal factors under the ARIMA method, and $Y_{IR,i}$ denotes the irregular factors under the ARIMA method. This indicates that the fitting effect is very good, indicating that the X-12-ARIMA method has completely decomposed the time series and there were no errors in fitting the original time series (this result is surprising, but it is true).

Meanwhile, for TRAMO/SEATS method, the decomposition error is given by

$$\sum (TS_i - TS_{TRD,i} \cdot TS_{SF,i} \cdot TS_{IR,i}) = -802.48062 \tag{4}$$

$$\sum (TS_i - TS_{TRD,i} \cdot TS_{SF,i} \cdot TS_{IR,i})^2 = 4174.079301 \tag{5}$$

where, TS_i denotes the observation values of the original sequence (same as Y_i), $TS_{TRD,i}$ denotes the trend values under the TRAMO/SEATS method (same as $Y_{TC,i}$), $TS_{SF,i}$ denotes the seasonal factors under the TRAMO/SEATS method (same as $Y_{SF,i}$), and $TS_{IR,i}$ denotes the irregular factors under the TRAMO/SEATS method (same as $Y_{IR,i}$). The result indicates the presence of fitting errors in the decomposition process, suggesting that the model does not fully capture all components of the time series.

Therefore, by comparing two methods for analyzing the deviation between observed values and factor decomposition fitting values after seasonal adjustment, the X-12-ARIMA method has a better effect on seasonal adjustment, providing the basic conditions for the next steps of model fitting, parameter estimation, statistical prediction, and other work [18].

5 Application of X-12-ARIMA for Seasonal Adjustment

Based on the above analysis, the X-12-ARIMA method is considered more suitable for seasonal adjustment and hence, the multiplicative seasonal model ARIMA$(p, d, q)\times(P, D, Q)_S$ is adopted for the subsequent time series analysis.

5.1 The First-Order 12 Steps Differencing Stationarity Test

Based on the characteristics of economic time series, which exhibit trends and are influenced by monthly seasonal factors, the first-order 12-step differenced series of China's total retail sales of consumer goods, Y, is subjected to the ADF stationarity test. Since the test statistic $T = -9.522322$ is less than the critical values at the 1%, 5%, and 10% significance levels, the null hypothesis is rejected. Therefore, the original series Y becomes stationary after first-order 12-step differencing.

5.2 Pure Randomness Test of Difference Sequence

According to the results of the stationarity test, the total retail sales of consumer goods series becomes stationary after first-order differencing and seasonal differencing, denoted by $\nabla\nabla_{12}Y_t$. However, not all stationary sequences are suitable for modeling. Only sequences that exhibit strong correlation with their historical values, and thus have meaningful implications for future development, are worth analyzing to extract useful information and to construct predictive models.

In order to determine whether the differenced series has analytical value, it is necessary to conduct a randomness test on the sequence $\nabla_{12}Y_t$. If $\nabla_{12}Y_t$ is verified to be a purely random sequence, this indicates that sufficient information has been extracted, and the sequence $\nabla\nabla_{12}Y_t$ can be directly modeled. On the other hand, if $\nabla_{12}Y_t$ does not satisfy the conditions of a purely random sequence, this implies that the extracted information is insufficient. In such a case, further transformations are required to extract additional effective information from $\nabla\nabla_{12}Y_t$ until the sequence satisfies the requirements for randomness.

The Box–Pierce test was performed on the first-order seasonal differenced sequence $\nabla_{12}Y_t$, and the results are shown in Table 7. By considering lags of 6 and 12 periods, the corresponding p-values are all less than 0.05. Therefore, the null hypothesis can be rejected at the 95% confidence level, indicating that the sequence $\nabla\nabla_{12}Y_t$ is not a white noise sequence. This suggests that the available information has not been fully extracted and that further correlation modeling is required.

Table 7: Pure Randomness Test Results for $\nabla\nabla_{12}Y_t$

Lag	X-squared	P-value
6	14.879	0.02122
12	39.148	9.948e-05

5.3 Analysis of Autocorrelation and Partial Correlation Diagrams

Figure 9 shows the autocorrelation and partial autocorrelation of the differenced sequence $\nabla\nabla_{12}Y_t$. From the autocorrelation plot of the sequence after first-order seasonal differencing, it can be observed that the autocorrelation coefficient exceeds the confidence bounds at lag 1. Although it falls within the confidence bounds at lag 2, it exceeds the bounds again at subsequent lags. This indicates the presence of both short-term autocorrelation and a clear tailing effect.

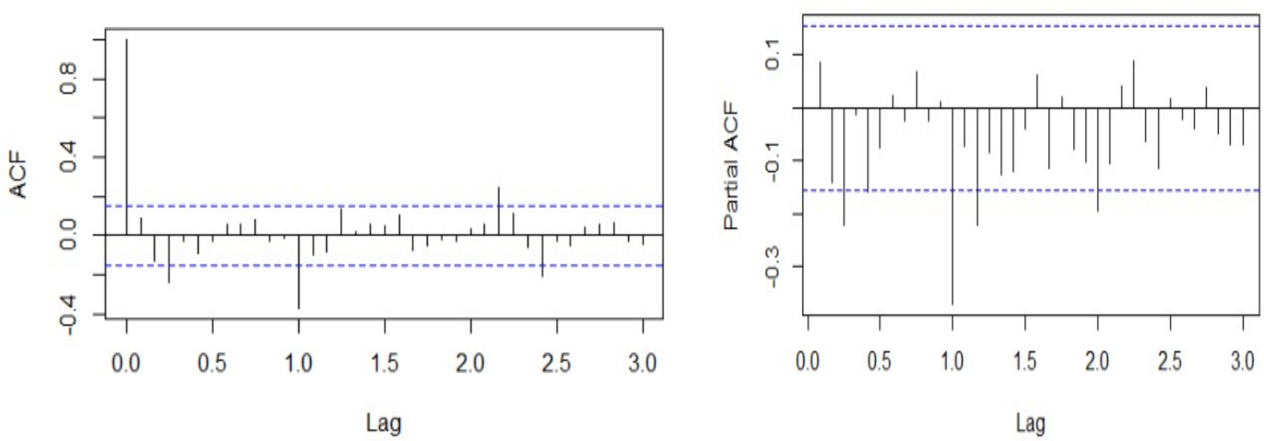

Figure 9: Autocorrelation (Left) and Partial Correlation (Right) of Difference Sequence $\nabla\nabla_{12}Y_t$

Based on the partial autocorrelation plot, the partial autocorrelation coefficients at lags 1 and 2 both exceed the confidence bounds, indicating a tailing pattern within the seasonal cycle. Overall, the differenced sequence exhibits tailing characteristics, which are consistent with the properties of an ARMA(p, q) model. Considering the partial autocorrelation truncation and the autocorrelation tailing within the seasonal period, the multiplicative seasonal model is proposed as ARIMA$(1, 1, 1) \times (0, 1, 1)_{12}$ or ARIMA$(2, 1, 1) \times (0, 1, 1)_{12}$.

5.4 Series Modeling of Total Retail Sales of Consumer Goods in Society

The results of the pure randomness test for the residual sequences of the fitted models ARIMA$(1,1,1)\times(0,1,1)_{12}$ and ARIMA$(2,1,1)\times(0,1,1)_{12}$ are shown in Table 8.

Table 8: Results of Pure Randomness Test for Residual Sequences of Two Fitted Models

Lag	**ARIMA**$(1,1,1)\times(0,1,1)_{12}$		**ARIMA**$(2,1,1)\times(0,1,1)_{12}$	
	X-squared	**P-value**	**X-squared**	**P-value**
6	13.05	0.04225	2.3096	0.8891
12	16.915	0.1528	6.805	0.8702

After fitting the ARIMA$(1,1,1)\times(0,1,1)_{12}$ model to the total retail sales of consumer goods, the residuals fail the pure randomness test at lag 6, as the corresponding p-value is less than 0.05, indicating rejection of the null hypothesis at this lag. However, at lag 12, the p-value is greater than 0.05, so the null hypothesis cannot be rejected at this lag. Overall, the presence of significant autocorrelation at lag 6 suggests that the residual sequence is not fully white noise, indicating that the model does not completely extract the information from the original series. Hence, this model is not considered fully adequate.

In contrast, after fitting the ARIMA$(2,1,1)\times(0,1,1)_{12}$ model, the residuals pass the pure randomness test at both lag 6 and lag 12, as the p-values are greater than 0.05. Thus, the null hypothesis of pure randomness is accepted, and the residual sequence can be regarded as white noise. This indicates that the model successfully captures the information in the original series. Therefore, it is concluded that this model is appropriate for the given data.

By fitting the ARIMA$(2,1,1)\times(0,1,1)_{12}$ model using the ARIMA modeling method, the estimated parameters ϕ_1, ϕ_2, θ_1, and Θ_1, which describe the relationship between past values (ϕ), past errors (θ), and seasonal effects (Θ), are obtained as shown in Table 9.

Table 9: Model Fitting Parameter Results

	ϕ_1	ϕ_2	θ_1	Θ_1
Coefficients	0.8721	-0.2228	-0.9504	-0.4696
s.e.	0.0824	0.0788	0.0351	0.0814

The estimated error variance is $\sigma^2 = 1{,}220{,}585$, with log-likelihood -1359.02 and Akaike Information Criterion (AIC) value of 2728.04. Significance tests show that all parameters ϕ_1, ϕ_2, θ_1, and Θ_1 are significantly different from zero, indicating that the model is statistically valid. Therefore, the fitted multiplicative seasonal ARIMA model is expressed as

$$\nabla\nabla_{12}x_t = \frac{(1-\theta_1 B)(1-\Theta_1 B^{12})}{(1-\phi_1 B-\phi_2 B^2)}\varepsilon_t, \tag{6}$$

where B is the backshift operator, ∇ denotes first-order differencing, and ∇_{12} denotes seasonal differencing with period 12. Finally, the residual sequence passes the pure randomness test, indicating that no significant autocorrelation remains. This confirms that the fitted model is statistically adequate for the data.

5.5 Model Prediction Application

According to the fitted model in Equation (6), the total retail sales of consumer goods in China from July 2023 to June 2024 are forecasted, and the forecast series Y_{forecast} is obtained. Prediction intervals at the 80% and 95% confidence levels are also constructed, and the results are shown in Table 10. In the forecast table, "Lo" and "Hi" represent the lower and upper bounds of the prediction intervals, respectively, where "Lo" indicates the minimum expected value and "Hi" indicates the maximum expected value within a given confidence level (for 80% or 95%). These intervals describe the range in which the actual observed values are expected to fall with a certain level of confidence. The error percentage is calculated as the relative difference between the forecasted value and the actual value, expressed as

$$\text{Error } (\%) = \frac{Y_{\text{forecast}} - Y_{\text{true}}}{Y_{\text{true}}} \times 100\%.$$

Table 10: Comparison and Statistics of Total Retail Sales of Consumer Goods from July 2023 to June 2024

Month	Y_{true}	Y_{forecast}	Lo 80%	Hi 80%	Lo 95%	Hi 95%	Error (%)
07,2023	36760.70	37303.09	35887.23	38718.95	35137.72	39468.46	1.48
08,2023	37932.70	37676.01	35750.52	39601.50	34731.23	40620.79	-0.68
09,2023	39826.00	39440.62	37318.26	41562.99	36194.75	42686.50	-0.97
10,2023	43333.00	42505.16	40310.68	44699.64	39149.00	45861.32	-1.91
11,2023	42505.00	41918.06	39694.60	44141.52	38517.58	45318.54	-1.38
12,2023	43550.20	43215.08	40976.71	45453.44	39791.79	46638.36	-0.77
01,2024	40653.45	39634.14	37385.44	41882.84	36195.05	43073.23	-2.51
02,2024	40653.45	39637.12	37379.50	41894.74	36184.39	43089.85	-2.50
03,2024	39019.90	38566.13	36299.93	40832.34	35100.28	42031.99	-1.16
04,2024	35699.10	35600.66	33325.89	37875.44	32121.69	39079.63	-0.28
05,2024	39211.00	38786.61	36503.20	41070.02	35294.44	42278.78	-1.08
06,2024	40731.60	41529.37	39237.29	43821.44	38023.94	45034.80	1.96

By predicting the data of the sequence from July 2023 to June 2024 and drawing the prediction effect graph, as shown in Figure 10, the predicted area in the graph is represented by the solid line indicating the predicted value of the sequence, the dark shaded sequence is the 80% confidence interval, and the light shaded sequence is the 95% confidence interval. According to the prediction effect chart, the fitting model has a good fitting effect on the data from July 2023 to June 2024, reflecting the trend and seasonal characteristics of the total retail sales of consumer goods in China, and continuing the development trend of the original sequence. However, this prediction has the following limitations. (1) Data distribution assumption: The model is based on the assumption that residuals follow a normal distribution and white noise, but the actual total retail sales of consumer goods data may have a fat tail due to various factors, and the probability of special events may be underestimated; (2) The impact of abnormal values caused by the COVID-19 epidemic: The abnormal fluctuations caused by the COVID-19 epidemic from 2020 to 2022 are significant. Although seasonally adjusted, their structural effects may exist for a certain period of time. The series of stimulus consumption policies introduced by China after the epidemic will lead to an increase in long-term prediction errors. Due to the existence of these situations, it may affect the accuracy of predictions for the next 12 months.

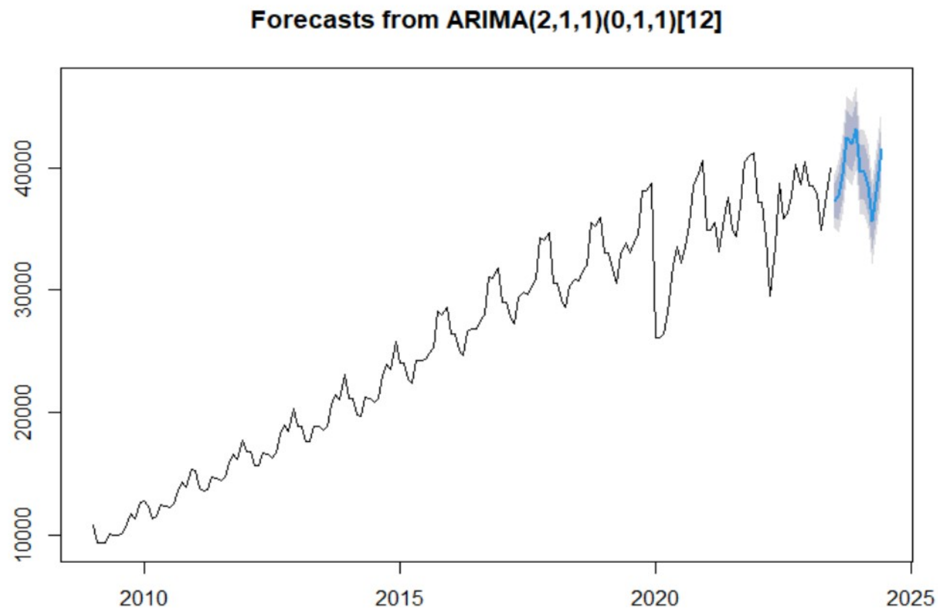

Figure 10: Prediction of the Total Retail Sales of Consumer Goods in China

In addition, using the actual values in Column 2, the forecasted values in Column 3, and the prediction interval bounds in Columns 4 to 6 of Table 10, the graph in Figure 11 is constructed to illustrate the comparison between the actual values, predicted values, and the upper and lower prediction intervals. From the results, it can be seen that the prediction performance is very good, with only small fluctuations around the true values. Moreover, both the 95% and 80% prediction intervals contain the actual values, indicating that the forecasting results are highly reliable. This further confirms that it is feasible to forecast the total retail sales of consumer goods in China using Model (6).

The requirement for error control in Figure 11 is that it should remain within 5%, indicating that the statistical results are considered acceptable and the data reliable. Based on the prediction error percentages, all monthly errors are controlled within 5%, with the largest absolute errors occurring in January and February, reaching 2.51% and 2.50%, respectively. The errors in the remaining months are all within 2%. The root mean square error percentage (RMSE%) is 1.57%, indicating a high level of prediction accuracy. Overall, the forecasting performance is satisfactory, further confirming that the fitted model is effective and can be used to predict the total retail sales of consumer goods in China, thereby providing a theoretical basis for formulating macroeconomic policies.

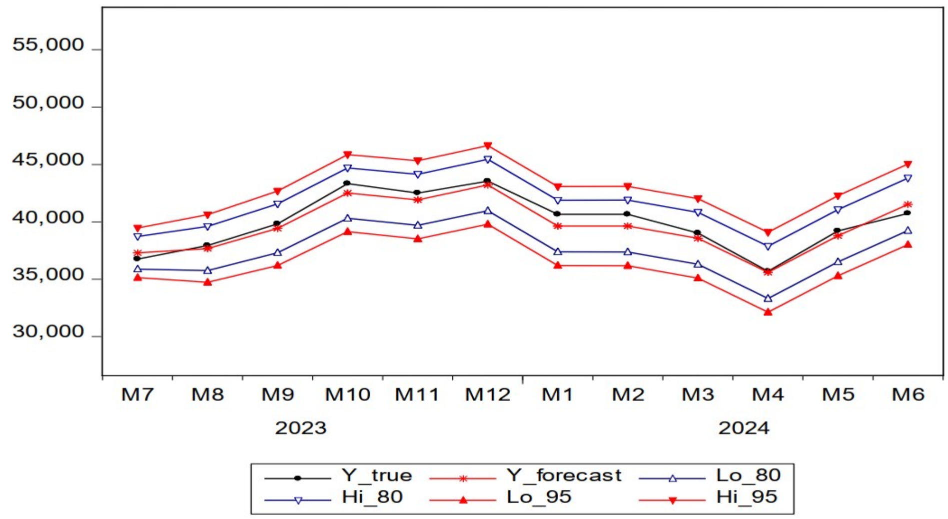

Figure 11: Comparison Chart of Predicted and Actual Retail Sales of Consumer Goods in China

6 Summary

This article takes the total retail sales of consumer goods in China from 2009 to 2023 as the research object. Based on the use of unit root method and time series chart method to study the stationarity of the series, X-12-ARIMA and TRAMO/SEATS methods are applied to seasonally adjust the economic time series. Both methods perform well in the process of seasonal adjustment, correctly separating trend cycle factors, seasonal factors, and irregular factors, effectively achieving sequential seasonal adjustment. By comparing two methods, the X-12-ARIMA method has a greater advantage in the seasonal adjustment process of the total retail sales of consumer goods in China. Based on this method, the model fitting has a very ideal effect, and the prediction results have a reasonable range of error compared to the true values. After seasonal adjustment, the sequence can accurately reflect the reality of economic phenomena, restore the trends and characteristics of the economic sequence itself, and lay the foundation for correctly controlling macroeconomic characteristics, mastering economic operation laws, and further accurately predicting the economic situation.

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