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A Zero-One Integer Programming Approach to the Assignments of Lecturers to Courses

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Abstract Assignments of lecturers to courses is one of the problems in education management (or planning) especially in higher institutions such as universities or colleges. This paper investigates the feasibility of determining an optimal assignment of lecturers to courses under various conditions using an operations research method. Beginning with the classic general almost "free entry" or "unrestricted" pure transportation model of linear programming, the problem is extended to include possibilities such as course-sharing and time-tabling clashing. This leads to the formulation of a revised zero-one integer programming model. The results of some empirical tests are also reported.

Keywords Assignment, zero-one integer programming

1 Introduction

Assignment of teachers or lecturers to courses is one of the major problems in education management, especially in institutions of higher learning such as universities or colleges. Every year the decision maker (for example school principal, departmental chairman or the faculty dean) has to decide how should the courses offered be "best allocated" amongst its staff so as to optimise some measure of output criteria as chosen or set by the decision maker or the education authority. The numerous possible permutations highlights the complexities and difficulties face by the decision maker. Thus, this allocation or assignment is normally done manually by the decision maker in consultation with the respective staff.

A number of models and algorithms have been developed to efficiently solve this problem for different objective formulations and constraints [2,3]. In this paper we develop a simplified version of the model by extending the classical transportation-assignment model to encompass constraints that are adhered to the lecturer-courses assignment problems. Although the resulting model is no longer a pure transportation-assignment problem, the structural form of the constraints demand that the solution be zero-one integer as in any assignment problem.

2 Basic Model Formulation

Before we formulate the problem, let us define the variables and parameters relevant to the model. Let

$X(i,j)$ denotes the assignment of lecturer i to teach course j . A value of unity implies that the course j is totally taught by lecturer i , while a value of zero implies otherwise. No intermediate value is acceptable. Thus $X(i,j)$ must assume the value of either zero or unity.

$E(i,j)$ denotes the "effectiveness rating" (or contribution) of assigning lecturer i to teach course j. Various criteria (weighted or unweighted, single or multiple) can be selected and formulated by the decision maker to arrive at a suitable and acceptable measure representing this parameter.

$H(j)$ denotes the weight (or value), for example the number of lecture-hours associated with course j.

$W(i)$ and $V(i)$ be the maximum and minimum work loads (e.g in lecture-hours) respectively to be assigned to lecturer i.These work loads may differ from lecturer to lecturer depending on the additional responsibility (such as administration) held by each lecturer.

The basic lecturer-courses assignment problem involving m lecturers and n courses can thus be stated as a zero-one integer linear programming problem

$$\text{maximise} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} E(i,j)X(i,j) \tag{1}$$

subject to

$$\sum_{j=1}^{n} H(j)X(i,j) \leq W(i), \;\; i = 1,2,...,m \tag{2}$$

$$\sum_{j=1}^{n} H(j)X(i,j) \geq V(i), \;\; i = 1,2,...,m \tag{3}$$

$$\sum_{i=1}^{m} X(i,j) = 1, \;\; j = 1,2,...,n \tag{4}$$

$$X(i,j) = 0 \text{ or } 1, \;\; i = 1,2,...,m; \;\; j = 1,2,...,m. \tag{5}$$

The objective function (1) is the sum of the effective rating of assigning lecturer i to course j. The constraint sets (2) and (3) require that the work load assigned to each lecturer remains within the stipulated range. In most models [2,3], the minimum work load $V(i)$ is taken to equal zero and the inequality constraint set (2) is replaced by an equality constraint set. This is fine for a balanced model, in which case the total lecture-hours of the courses taught meet the total hours of work load, that is

$$\sum_{j=1}^{n} H(j) = \sum_{i=1}^{m} W(i) \tag{6}$$

But in most cases, this might not be true.

Although dummy variables may be introduced to balance the model (as is the normal practice in solving unbalanced transportation-assignment model) complexities and difficulties may be encounter when the model is extended to incorporate constraints that are more representative of the real problem. Next, if the minimum feasible work load, $V(i)$ is set to zero for all $i = 1, 2, ..., m$, while the maximum work load $W(i)$ remains finite, the different measure of effectiveness rating $E(i,j)$ associated with each lecturer-courses assignment $X(i,j)$ will tend to assign those combinations with the highest $E(i,j)$ first, thereby burdening the "*most effective lecturers*" with full maximum work load, while the "*less effective lecturers*" might end up with minimum (or in some cases zero) work load. Constraint sets (2) and (3) overcome this problem by forcing the work load for each lecturer to be within some tolerable range.

Constraint set (4) ensures that each course is assigned to only one lecturer, i.e the total assignment value associated with each course must equal unity. Finally, constraint set (5) imposes the zero-one integer restriction on $X(i,j)$. This basic classical lecturer-courses assignment model comprises of nm zero-one variables with $n + 2m$ constraints.

3 The Extended Model

We now extend the model to include cases such as course-sharing, specialised courses designed exclusively for specific groups of lecturers only, as well as time-tabling clashing.

- *Course-sharing :* Let course $j = s$ be the course that must be shared between two lecturers. Split the course into two smaller courses, say s_1 and s_2 with lecture hours $H(s_1)$ and $H(s_2)$ respectively, such that

$$H(s_1) + H(s_2) = H(s). \tag{7}$$

Constraint sets (2) and (3) then become (for all $i = 1, 2, ..., m$.)

$$\sum_{j=1}^{s-1} H(j)X(i,j) + \{H(s_1)X(i,s_1) + H(s_2)X(i,s_2)\} + \sum_{j=s+1}^{n} H(j)X(i,j) \leq W(i). \tag{2a}$$

and

$$\sum_{j=1}^{s-1} H(j)X(i,j) + \{H(s_1)X(i,s_1) + H(s_2)X(i,s_2)\} + \sum_{j=s+1}^{n} H(j)X(i,j) \geq V(i). \tag{3a}$$

followed by an additional constraint set

$$X(i,s_1) + X(i,s_2) \leq 1, \; i = 1, 2, ..., m, \tag{8}$$

for all shared courses $s \in j = 1, 2, ..., n$. This additional constraint set ensures that no lecturer is assigned both parts of the shared course. A lecturer is either assigned to one part of the shared course or not assigned at all.

- *Specialised Courses :* Some courses (for example electives or modules) are exclusively reserved for a specific group of lecturers (e.g Applied Mathematics, Pure Mathematics or Statistics). Suppose there are k groups of lecturers, each with a finite number of courses. Thus a member of each group can only be assigned courses exclusive to the group and also courses that are available to all groups, but not courses exclusive to other groups. This gives rise to *block* or *partitioned* variables, some of which are easily identified as zeros as depicted in Table 1.

This partitioning of variables greatly reduces the number of decision variables adhered to the problem, thereby reducing the computational time.

Constraint sets (2) and (3) can thus be splitted further into a more simplified form according to groups. For the maximum work load restriction we have

$$\sum_{j \notin G2, G3 \ldots Gk} H(j)X(i,j) \leq W(i),\ \forall i \in G1 \tag{2a.1}$$

$$\sum_{j \notin G1, G3 \ldots Gk} H(j)X(i,j) \leq W(i),\ \forall i \in G2 \tag{2a.2}$$

$$\vdots$$

$$\sum_{j \notin G1, G2, \ldots G(k-1)} H(j)X(i,j) \leq W(i),\ \forall i \in Gk \tag{2a.k}$$

while for the minimum work load restriction we have

$$\sum_{j \notin G2, G3 \ldots Gk} H(j)X(i,j) \geq V(i),\ \forall i \in G1 \tag{3a.1}$$

$$\sum_{j \notin G1, G3 \ldots Gk} H(j)X(i,j) \geq V(i),\ \forall i \in G2 \tag{3a.2}$$

$$\vdots$$

$$\sum_{j \notin G1, G2, \ldots G(k-1)} H(j)X(i,j) \geq V(i),\ \forall i \in Gk \tag{3a.k}$$

Similarly, we can also split the n equalities of the constraint set (4) accordingly, such that

$$\sum_{i \in G1} X(i,j) = 1, \ \forall j \in G1 \tag{4a.1}$$

$$\sum_{i \in G2} X(i,j) = 1, \ \forall j \in G2 \tag{4a.2}$$

$$\vdots$$

$$\sum_{i \in Gk} X(i,j) = 1, \ \forall j \in Gk \tag{4a.k}$$

and

$$\sum_{i=1}^{m} X(i,j) = 1, \ \forall j \in (\text{common courses}). \tag{4.0}$$

This again reduces the number of probable non-zero decision variables, thereby contributing to the reduction of computational time.

- *Time-table Clashing :* Let us assume that the timetabling of courses are already fixed and known. Then no lecturer can teach (i.e should be assigned) two or more courses belonging to the same time slot. Thus for each time slot, say $l = 1, 2, ..., L$, we must have

$$\sum_{j \in l} X(i,j) \leq 1, \ \forall \ i = 1, 2, ..., m; \ l = 1, 2, ..., L. \tag{9}$$

This constitute mL inequality constraints. A lecturer is either assigned exactly one course in time slot $l(= 1, 2, ..., L)$ or not assigned at all.

By combining the objective function (1) with the constraint sets (2a.1) - (2a.k), (3a.1) - (3a.k), (4a.1) - (4a.k), (4.0), (8) and (9), we can formally state the extended zero-one integer programming lecturer-courses assignment problem (for m lecturers of k groups and n courses) as

$$\text{maximise} \sum_{i=1}^{m} \sum_{j=1}^{n} E(i,j)X(i,j) \tag{1.0}$$

subject to (the maximum work load restrictions):

$$\sum_{j \notin G2, G3...Gk} H(j)X(i,j) \leq W(i), \ \forall i \in G1 \tag{2.1}$$

$$\sum_{j \notin G1, G3...Gk} H(j)X(i,j) \leq W(i), \ \forall i \in G2 \tag{2.2}$$

$$\vdots$$

$$\sum_{j \notin G1, G2, \ldots G(k-1)} H(j) X(i,j) \leq W(i),\ \forall i \in Gk \tag{2.k}$$

(the minimum work load restrictions):

$$\sum_{j \notin G2, G3 \ldots Gk} H(j) X(i,j) \geq V(i),\ \forall i \in G1 \tag{3.1}$$

$$\sum_{j \notin G1, G3 \ldots Gk} H(j) X(i,j) \geq V(i),\ \forall i \in G2 \tag{3.2}$$

$$\vdots$$

$$\sum_{j \notin G1, G2, \ldots G(k-1)} H(j) X(i,j) \geq V(i),\ \forall i \in Gk \tag{3.k}$$

(exactly one lecturer for each course) :

$$\sum_{i \in G1} X(i,j) = 1,\ \forall j \in G1 \tag{4.1}$$

$$\sum_{i \in G2} X(i,j) = 1,\ \forall j \in G2 \tag{4.2}$$

$$\vdots$$

$$\sum_{i \in Gk} X(i,j) = 1,\ \forall j \in Gk \tag{4.k}$$

$$\sum_{i=1}^{m} X(i,j) = 1,\ \forall j \in (\text{common courses}). \tag{4.0}$$

(shared-courses restriction) :

$$X(i, s_1) + X(i, s_2) \leq 1,\ \ i = 1, 2, ..., m,\ \forall s \in j = 1, 2, ..., n. \tag{5.0}$$

(time slot restrictions) :

$$\sum_{j \in l} X(i,j) \leq 1,\ \ i = 1, 2, ..., m;\ \ l = 1, 2, ..., L. \tag{6.0}$$

with $X(i,j) = 0, 1$.

4 An Illustrative Example

To illustrate the applicability of the model, let us consider a small hypothetical academic department with the following characteristics.

- The department comprises of exactly fifteen lecturers including one chairman and one deputy chairman.

- The lecturers are categorised into three specialised groups - A, B and C (five lecturers in each group) - and are denoted by LA1, ..., LA5 in group A, LB1, ..., LB5 in group B and LC1, ..., LC5 in group C.

- *First Year Courses.* The department offers three shared-courses of 30 lecture-hours each which are equivalent to six single courses of 15 lecture-hours each, and are assignable to any lecturer regardless of his/her specialisation. The courses are (SJ11 + SJ12) ; (SJ13 + SJ14) and (SJ15 + SJ16). However, no lecturer is to be assigned to teach both parts of the shared-courses.

- *Second Year Courses :* A total of nine courses are offered, three from each group.

  Group A : SJ21, SJ22 and SJ23,
  Group B : SJ24, SJ25 and SJ26,
  Group C : SJ27, SJ28 and SJ29.

  Each course is allocated 30 lecture-hours and assignable only to those in the group.

- *Third Year Courses :* Each group offers three unshared single 30 lecture-hours courses and one shared-course (an equivalent to two courses of 15 lecture-hours each). The courses are denoted according to their groupings.

  Group A : SJA31, SJA32, SJA33 and (SJA34 + SJA35),
  Group B : SJB31, SJB32, SJB33 and (SJB34 + SJB35),
  Group C : SJC31, SJC32, SJC33 and (SJC34 + SJC35).

- Without loss of generality, let lecturer LA1 be the departmental chairman, assisted by lecturer LB1, each to be assigned of not more than 50% and 75% of the normal individual work load respectively. Also assumed that no first year courses is to be assigned to both of them.

- Other lecturers are subjected to a maximum and minimum work load of 60 lecture-hours and 30 lecture-hours respectively.

- The measure or index of the *effectiveness rating,* $E(i,j)$ and the time-tabling of courses offered are known and given in the appendix. Observe that of the fifteen time-slots, only the first six must meet the time-slot constraints. Each slot from the remaining nine time-slots comprises of courses from different groups and hence will not be assigned to the same lecturer.

5 Results and Discussion

The problem of the illustrative example is solved using the Linear Interactive and Discrete Optimizer (LINDO) package. The nature of the problem, in general produces multiple optimal solutions. One of these is presented below (Table 2)

Since the model is not a balanced model (the total maximum work load exceeds the total courses lecture-hours), not all lecturers are assigned the full work load. Only five "*good*" lecturers are assigned the full 60 hours of teaching load. They are LA3, LB3, LB4, LB5 and LC1. One lecturer, LC3 is assigned the minimum teaching load of 30 hours (equivalent to that of the chairman). Majority of the lecturers however are allocated a work load of 45 lecture-hours. This unbalanced allocation of teaching load can be overcome by either reducing the number of staff (under the assumption that the department is overstaff) or introducing new courses so as to fully utilise the expertise of the existing staff. The second alternative seems more appealing. The maximum total effectiveness rating generated by the optimal assignment solution is 268.0.

6 Conclusion

In this paper we present an extended revised version of the classical lecturer-courses assignment problems to include cases of shared courses, specialised courses restricted to specific groups and the problem of time-tabling clashing. The applicability of zero-one integer programming model formulated is successfully illustrated by a hypothetical example. The block or partitioned structure of the set of decision variables adhered to the problem suggests the possibility of an alternative solution approach such as the Dantzig-Wolfe Decomposition principle.

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Appendix