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An Asymmetric Information Problem of Environment Quality

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Abstract The environmental quality standards and preferences vary across different regions. Similarly, regulations on the quality standard of the environment will affect profits of local firms across regions in different ways. Therefore, it would not be efficient to impose a nationwide standard for environmental quality by a central agency since in each region the quality level should allow for specific regional characteristics. Setting a specific standard means assigning property rights both to the polluting firms and to those affected by pollution. Our purpose is to derive some normative implications about property rights: because we are in a world with positive transaction costs, property rights have a central role in determining the ultimate use of resources. How should property rights be designed in order to optimally take into account the asymmetric information structure (the information from both side is not balanced, in one side we get less information than the other side)?. The aim of our paper is to assign exclusive property rights, whether to the one party who creates or to the one party who sustains the externality. Specifically, we argue that it is not optimal to leave the decision about the size of the externality entirely to the bargaining process between the two parties. We prove that a benevolent regulator can increase efficiency by setting a standard, yet allowing for renegotiation between the parties affected, thus making it possible to take into account specific preferences. The importance of the standard is that it defines the threat point of the bargaining game, which in turn affects the efficiency of the system.

Keywords bargaining game, optimization, environmental quality, asymmetric information.

1 Introduction

The government assigns property rights by defining a standard y which determines the minimum environmental quality. We assume that the quality level of the environment can be measured quantitatively with minimum cost. The level y will be enforced, unless the polluter and the local council voluntarily agree to deviate from it. Given the standard y, the polluter and the council are given the right to renegotiate. They can sign a contract specifying a different quality of the environment which takes into account the specific preferences for quality of the local population (represented by the preference of the local council) and the extent to which the profit of the producer is affected by the resulting clean up costs. Exclusive property rights results when the standard takes either extreme value: $y = 0$

implies that the firm has the exclusive right over the environment, while the case where y takes the largest possible value is equivalent to the council owning this right.

Ideally, one would like to design a legal structure such that the ensuing contracts are ex-post Pareto optimal: given the specific preferences of the local council and the profit function of the firm the agreement between both parties always yields the first-best allocation. In general, this is not feasible because both the preferences and the profit function are private information of the respective participants. Thus, the contract has to take into account incentive compatibility constraints. In addition, since each party has to voluntarily agree, the welfare resulting from the contract cannot be lower than the welfare which the parties would get in case the standard y is implemented. In this way, the standard y determines the individual rationality constraints of both parties (the threat point of the game).

In this paper, we analyze how a change in the standard y affects the individual rationality constraints and, thus, have an impact on the outcome (the quality of environment which is obtained from the contract). For illustrative purpose, we consider the following game: the polluting firm offers a contract to the local council which the council either accepts or rejects. In the latter case, the standard y must be met.

As a mean of illustration, we consider an example of one factory the smoke from which has harmful effects on one nearby community. We ignore the problem of aggregating preferences across individuals and assume that the interest of the community can be represented by a local government. We focus on the strategic interaction between the producer and the local government and disregard other potential players, such as other polluters and other nearby communities. The example we consider, suggests a model with two sided private information. Specifically, one would assume that the producer has an informational advantage about her cost for reducing the externality, and also that the preferences of the local government are partially private. For simplicity, we only analyze the case one-sided private information: we assume that the technology used by the producer is common knowledge. In particular, we only discuss a model with two players.

It is shown that, in the above example, the definition of property rights has a predictable impact on the outcome. We will focus the case where the producer "owns" the exclusive right over the environment. In this case, clean environment becomes a good and the firm will be willing to "produce" a higher quality of the environment for a price. However, the asymmetric information structure yields standard inefficiency results; first, for almost all types of local government the polluter under produces quality of the environment except for the local government which values quality of the environment the most, second, the inefficiency is decreasing in types and, third, the local government almost always extracts a positive rent.

2 The Model

The polluting firm produces output q. In the absence of any restrictions concerning environmental quality, the firm would maximize revenue minus cost: $\pi(q) = R(q) - C(q)$. When a minimum environmental quality y has to be obeyed, the firm producing q must incur additional costs $\phi(y, q) > 0$ for all strictly positive pair (y, q) (the costs of keeping environmental quality at y when producing q). A natural assumption is that $\phi(y, q)$ is convex in q and y and that π is concave in q. $\Pi(y)$ denotes the maximum profit of the firm when the quality level y has to be met, that is:

$$\Pi(y) = \max_{q} \pi(q) - \phi(q, y) \tag{1}$$

Proposition 1 Π *is decreasing concave in the quality level* y.

Proof:

(i) Denote $K(q, y) = \pi(q) - \phi(q, y)$. Thus, K is a profit of the firm and is concave in q, y. Applying the envelope theorem, we have:

$$\Pi_y(y) = -\phi_y(q^*(y), y) < 0 \tag{2}$$

where q^* denotes the profit maximizing production given the imposed quality level y.

(ii) Proves the first claim that Π is decreasing. To conclude the proof, we define:

$$q_i = Arg \max_{q} K(q, y_i)$$

The concavity of K and the definition of Π imply:

$$\begin{aligned}\Pi(\lambda y_1 + (1-\lambda) y_2) &\geq K(\lambda q_1 + (1-\lambda) q_2, \lambda y_1 + (1-\lambda) y_2) \\ &\geq \lambda K(q_1, y_1) + (1-\lambda) K(q_2, y_2) \\ &\geq \lambda \Pi(y_1) + (1-\lambda) \Pi(y_2)\end{aligned}$$

□

The local council is concerned about environmental quality. Her preferences are represented by function $u(y, a)$, where a is a parameter which characterizes the extent to which good quality is appreciated. The parameter a is assumed private information of the local council. We use a standard assumption for model with asymmetric information: the firm has only information about the density of the characteristic a. For simplicity, we represent the preference of the local council by a specific functional form $u(y, a) = aU(y)$, with U being increasing and concave in y. This restriction implies $u_{ya} > 0$: with stronger preferences for environmental quality the marginal utility increases for each quality level.

The parameter a is private information of the local council. The firm has only information about the density of the characteristic a. Specifically, we assume that $a \in [a_1, a_2]$ and $a \sim f(a)$ with cumulative F(a). For the purpose of this paper the information of the firm, thus, f and F, could be interpreted as subjective. Qualitatively, it would not affect any of the results. We impose the following restrictions on the distribution of a (This is a standard hazard rate assumption. Assumptions of this type are common in asymmetric information problems and are satisfied for the standard distributions):

$$\forall \alpha \in [0, 1], \quad \frac{d}{da}\left[\frac{F(a) - \alpha}{f(a)}\right] > 0 \tag{3}$$

In order to examine the impact of private information and of the environmental quality y on the outcome and, in particular, the efficiency of the contract, we first derive the optimal environmental quality under complete information. The first-best solution is defined for every state as:

$$y^*(a) = \underset{x}{Arg\max}\, \Pi(y) + aU(y) \tag{4}$$

The concavity of the functions Π and U guarantees that the first-best solution is increasing in the parameter a. As the appreciation of the local council for quality increases, the marginal utility curve is shifted upwards and thus optimal quality will increase.

3 The Problem of The Firm

The importance of the standard y under private information is illustrated by analyzing the following game: the firm proposes a contract to the local council. Then, the council either accepts or rejects the proposal. Thus, the firm acts as principal while the local council as the agent. According to the revelation principle, designing an optimal contract is (without loss of generality) equivalent to designing an optimal truth-revealing mechanism (Myerson, [6]). That is, the firm offers an incentive compatible, individually rational contract which induces the local council to reveal her true characteristic a. After revealing a, the quality level $y(a)$ will be realized and the firm makes transfer payments $t(a)$ to the local council. The transfer payments may be negative ($t(a) < 0$). In that case, the firm receives payments from the council. The problem of the firm can be written as a control problem of the form:

$$\max_{y,t} \int_{a_1}^{a_2} \{\Pi(y(a)) - t(a)\} f(a)da$$

$$a \in \underset{r}{Arg\max}\, aU(y(r)) + t(r) \tag{5}$$

$$aU(y(a)) + t(a) \geq aU(y) \tag{6}$$

Equation (5) is the incentive compatibility constraint. It restricts the mechanism in such a way that it will be optimal for the council to reveal her type truthfully. The second constraint, equation (6), guarantees that the council is never worse off by accepting rather than rejecting the mechanism. To solve the firm's problem, it is convenient to reformulate it by eliminating the transfer function. For this purpose, we define by $V(a)$ the payoff from the council's reporting problem, i.e. the maximum payoff for a council with the characteristic a. The incentive compatibility constraint implies the following equality:

$$T(a) = V(a) - aU(y(a)) \tag{7}$$

Because the optimal contract anticipates the maximizing behavior on behalf of the council, the first order condition $aU_x y_a(a) + t_a(a) = 0$ will hold. This implies that the council's maximum payoff varies with her type according to the following equation:

$$\dot{V}(a) = U(y(a)) \tag{8}$$

Using these equations, the problem of the firm can be rewritten in terms of y and V. We follow standard practice. We, initially, substitute the first order condition from the incentive compatibility constraint. We solve the resulting simplified problem. We, then, check out that truthful reporting is, indeed, globally optimal. The simplified optimization problem is:

$$\max_{y,V} \int_{a_1}^{a_2} \{\Pi(y) - aU(y) - V(a)\} f(a) da$$

$$\dot{V}(a) = U(y(a)) \tag{9}$$

$$V(a) \geq aU(y) \tag{10}$$

This is a control problem with a restriction on the state variable. y is the control and V the state variable. We apply a standard sufficiency result from Seierstad and Sydsaeter [8]. The Lagrangian of the above problem is:

$$L(y, V, \lambda, \eta, a) = \{\Pi(y) + aU(y) - V(a)\} f(a) + \lambda U(y) + \eta[V(a) - aU(y)] \tag{11}$$

Applying the above mentioned theorem yields the following set of conditions:

$$\{\Pi_y + aU_y\} f(a) + \lambda U_y = 0 \tag{12}$$

$$-f(a) + \eta = -\lambda \tag{13}$$

$$\dot{V}(a) = U(y(a)) \tag{14}$$

$$\eta \geq 0 \tag{15}$$

$$\eta[V - aU(y)] = 0 \tag{16}$$

(12) is a standard Euler equation. (13) is the co-state equation. (14) is the law of motion, i.e. constraint (9). Inequality (15) imposes a non-negativity restriction on the Lagrange multiplier. The justification is the same as in the Kuhn-Tucker theorem. Finally, (16) is the complementary slackness condition. We have omitted the boundary conditions, as they depend on the size of y.

If the firm owns the right to pollute, the environment quality should be $y = 0$. More generally, we also identify with this case any quality y such that clean environment is acceptable, i.e. such that the solution of the bargaining process yields for every type a quality of the environment larger than the y. $M_- = [y_-(a), t_-(a)]$ denotes the optimal mechanism for this case. Thus, analytically, we consider in this section the case where $y_-(\dot{a}) > y$ for all a, where $y_-(\dot{a})$ is a quality level of environment.

Proposition 2 *Define $y_-(a)$ as the implicit solution of the following equation:*

$$\Pi_y + aU_y + \frac{F(a) - 1}{f(a)} U_y = 0 \tag{17}$$

If $y < y_-(a_1)$, then the solution to the firms problem is given by (17) for the optimal environmental quality and by the following equations:

$$v(a) = \int_{a_1}^{a} U(y_-(\tilde{a}))d\tilde{a} + a_1 U(y) \tag{18}$$

$$t_-(a) = v(a) - aU(y_-(a)) \leq 0 \tag{19}$$

if $y_-(a)$ is increasing in a; $y_-(a) < y^*(a)$ for all $a < a_2$ and $y_-(a_2) = y^*(a_2)$.

Proof: If for all types $y_-(a) > y$, then according to equations (9) and (10) the slope of the profit function is steeper than the slope of the constraint:

$$\dot{V}(a) = U(y_-(a)) > U(y) \tag{20}$$

The form tries to keep $V(a)$ at the lowest feasible level. Since the function $V(a)$ is always steeper than the function $aU(y)$, the individual rationality constraint $V(a) \geq aU(y)$ will be binding at a_1 (see Figure 1), that is:

$$V(a_1) = a_1 U(y) \tag{21}$$

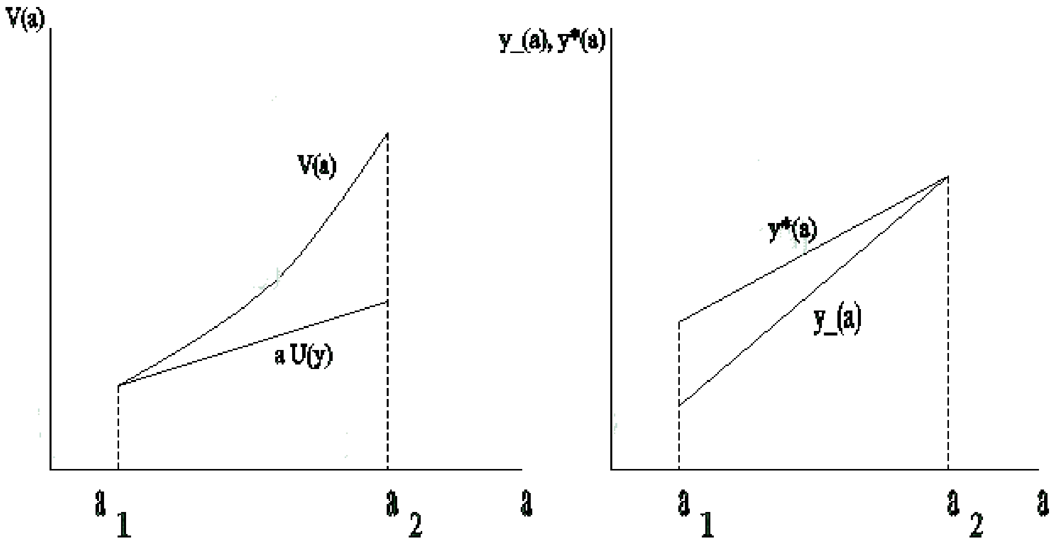

Figure 1 Figure 2

The complementary, (16) requires $\eta = 0$. The co-state equation, (13), then implies $\lambda = F(a) + \text{constant}$. Since $V(a_2)$ is free, the boundary condition implies $\lambda(a_2) = 0$; therefore $\lambda = F(a) - 1$. Substituting this result into the law of motion yields the definition of $y_-(a)$ given by equation (17). The slope of $y_-(a)$ follows by implicit differentiation:

$$\frac{dy_-}{da} = -\frac{U_y + \frac{d}{da}\left[\frac{F(a)-1}{f(a)}\right]U_y}{\Pi_{yy} + aU_{yy} + \left(\frac{F(a)-1}{f(a)}\right)U_{yy}} > 0 \tag{22}$$

$a+\frac{[F(a)-1]}{f(a)} > 0$ follows from equation (17), because $\Pi_y < 0$ and $U_y > 0$. Rewriting equation (17) proves by the concavity of $\Pi + aU$ that $y_-(a) \leq y^*(a)$ for all a.

$$\Pi_y(y) + aU_y(y) = \left[\frac{1-F(a)}{f(a)}\right]U_y(y) = 0 \tag{23}$$

Figure 2 compares the outcome of the contract with the first-best solution. The figure also shows that the case discussed here is relevant if the standard is so low that $y_-(a_1) \geq y$. The definition of $V(a)$, (18), follows from (14), and the observation that $V(a_1) = a_1U(y)$. Equality (19) follows immediately from (7). We now prove that the transfers are always negative. From the incentive compatibility constraint we know that

$$t_a(a) = -aU_y y_a(a) < 0 \tag{24}$$

Since U and y are increasing, t is decreasing in a. Therefore it is sufficient to show that $t(a_1) \leq 0$ to prove the claim. But $t(a_1) = a_1[U(y) - U(y_-(a_1))] \leq 0$.

Finally, to conclude the proof we show that truthful reporting is, indeed, globally incentive compatible for the mechanism M_-. Define $\tilde{U}(a,r) = aU(y(r)) + t(r)$, that is $\tilde{U}(a,r)$ denotes the utility of the local council when her type is a and she reports r. Thus the reporting problem of the local government is:

$$V(a) = \max_r \tilde{U}(a,r)$$

By construction, $\tilde{U}_r(a,a) = 0$, $\forall a$. Totally differentiating yields

$$\tilde{U}_{ra}(a,a) + \tilde{U}_{rr}(a,a) = 0 \tag{25}$$

Thus, $\tilde{U}_{ra} = U_y y_a > 0$ for all pairs guarantees global incentive compatibility. □

Behind the results lies a straightforward intuition: With a low enough standard, the council is willing to pay in order to increase the quality of the environment for all values of a. In that case, the firm is selling part of its right to pollute, receiving in return, compensation payments ($t(a) \leq 0$). It behaves like a discriminating monopolist facing a council with unknown demand.

The firm wants to minimize the informational rent extracted by the council. If the firm offered to sell to each type its efficient amount charging the council's reservation price, a council with high valuation would have an incentive to disguise as a low value type in order to catch a rent. Thus, to give an incentive for high value types to choose the contract designed for them, the firm has to bribe them by paying an information rent. This rent can be minimized by distorting the allocation (selling less than the efficient amount) for low types, thus making it less attractive to pretend being a low-type council.

Only the council with highest valuation buys the efficient amount of the property right; she gets the highest information rent. In contrast, only the council with lowest valuation get no rent at all. When the firm is the owner of the right to pollute, the council as a buyer has an incentive to understate her true valuation of the good. This tendency results in an inefficiently low quality of environment for almost all types.

4 Concluding Remarks

In this paper, for simplicity, we have analyzed the case of one-sided private information. The results can be generalized to the case of two sided asymmetric information when there is no correlation and one of the parties has complete bargaining power. Then this party acts as the principal.

The optimal contract between the two parties depends on the distribution of relative market power; when the standard is low the firm is in a strong position, whereas the council is in a strong position, when the standard is high. From these results, we concluded that assigning exclusive property rights is never optimal because ex-ante inefficiency is the highest when the threat point is a corner solution. This result will remain valid, even if there is two-sided asymmetric information because it results from the distortion of the allocation in the direction of the threat point.

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