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The Basic Model of the Production and Shipment Policy for the Single-vendor Single-buyer when Demand Rate is Linearly Decreasing with Time

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Abstract A basic model for a supply chain in which a vendor supplies a product to a buyer is considered. The vendor manufactures the product at a finite rate and periodically ships the output to the buyer. The buyer then consumes the product at a linearly decreasing time-varying rate. Costs are attached to manufacturing batch set up, the delivery of a shipment and stockholding at the vendor and buyer. The objective is to determine the shipment policy which minimises the total cost, assuming the vendor and buyer collaborate and find a way of sharing the consequent benefits. How the optimal shipment policy may be derived when the shipments size and shipments interval are identical is shown. These procedures are illustrated with numerical examples.

Keywords Production, inventory, single-vendor single-buyer, time-varying demand.

1 Introduction

Much attention has been paid in recent years to manage supply chains. Over the years mathematical models have been developed to describe the behaviour of such integrated systems and to determine optimal control policies. Because such systems are complex much of this research has concentrated on deterministic models with fixed demand rate. The hope and expectation is that these models provide some degree of qualitative insight into the behaviour of more complex real-world problems, which generally involves levels of uncertainty.

Goyal [2] was probably one of the first papers to investigate the integrated single-supplier single-customer problem. Bannerjee [1] considered the vendor manufacturing for stock at a finite rate and delivering the whole batch to the buyer as a single shipment - a 'lot for lot' model. Goyal [3] demonstrated how lower-cost policies generally result from allowing a production batch to be split and delivered as a number of shipments. Lu [6] set out the optimal production and shipment policy when the shipment sizes are all equal. Goyal [4] demonstrated how lower cost policies sometimes result when successive shipment sizes increase by a ratio which is equal to production rate divided by the demand rate. Hill [5] derived the form of the optimal policy if shipment sizes may vary. This consists of a number of shipments which increase by the ratio used in Goyal [4] followed by a number of equal-sized shipments.

A common assumption that has been made is that the demand rate at the buyer is fixed over an infinite horizon. The basic model considered here consists of a single vendor who manufactures for stock in the last batch and then transfers the stock to a single customer as a number of shipments. The buyer has to satisfy from stock a linearly decreasing and continuous demand process for a finite horizon. There are costs associated with batch set-up, delivering a shipment, and holding stock at either the vendor or the buyer. The objective is to determine the shipment policy which minimises the total system cost.

At time 0 the vendor is about to manufacture the last batch at rate P. The size of batch Q will be exactly what is required to meet all remaining demand. At time 0 the buyer holds a quantity x in stock; x could be the amount at the beginning of the previous production cycle based on a fixed shipment size policy or based on a variable shipment size policy or it could just be an arbitrary amount of stock which we are considering in this paper.

The problem is to find the optimal number of shipments and the sizes of those shipments. The shipments size could be all the same size but not evenly spaced in time or they could be evenly spaced in time but not equal in size or they could be arbitrarily spaced in time or arbitrarily size. In this paper we consider the first two cases.

In Section 2 we develop the mathematical formulation of the models. In Section 3 we look at numerical examples and draw some conclusions in Section 4.

2 Mathematical Formulation

In this section a general cost model will be developed.

2.1 Definitions and Assumptions

To develop the model, the following terminology is used:

- The demand rate of finished product at time t is $f(t) = a - bt$ for $t \in (0, H)$. H is the time horizon.
- The finite production rate is P units per unit time and $P > f(t)$.
- There is a fixed production set-up cost of A_1.
- There is a fixed order/shipment cost of A_2.
- There is a carrying inventory cost for vendor of h_1 per unit per unit time for finished product.
- There is a carrying inventory cost for buyer of h_2 per unit per unit time.
- n is the number of shipments.
- q_i is the size of the ith shipment in a batch production run.
- x is the initial stock in the system when the production of the last batch starts.
- C is the total cost for the system.

2.2 Total Time-weighted on System Stock

In Figure 1 the stock level, y_1 at time t in (t_p, H) is $\int_t^H f(t)dt (= \int_0^H f(t)dt - \int_0^t f(t)dt)$ and y_2 in $(0, t_p)$ is $Pt + x - \int_0^t f(t)dt$ where t_p is the production up time. We also have $Pt_p + x = \int_0^H f(t)dt$ and so $t_p = \frac{1}{P}(D - x)$. The total time-weighted system stock (see Omar & Smith [7]) , TSS, is

$$TSS = \int_0^{t_p} y_2(t)dt + \int_{t_p}^{H} y_1(t)dt = \frac{1}{6}bH^3 - \frac{1}{2}bH^2 t_p + x t_p + \frac{1}{2}P{t_p}^2. \quad (1)$$

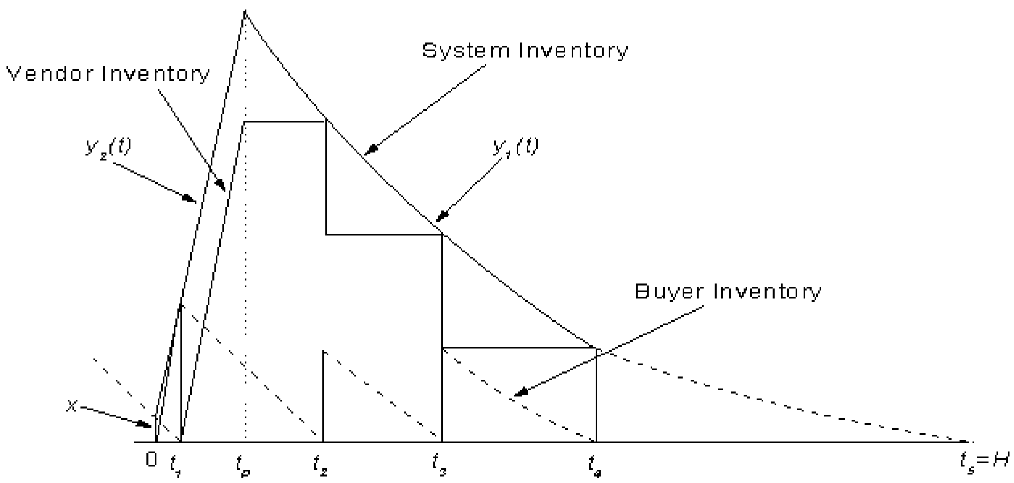

Figure 1: Plot of inventory against time

2.3 Total Time-weighted Buyer Stock

The size of the $i-$shipment is $q_i = \int_{t_i}^{t_{i+1}} f(t)dt$. Then the general total buyer stock from the $i-$shipment is

$$\begin{aligned}\int_{t_i}^{t_{i+1}} [a(t_{i+1} - t) - \frac{b}{2}(t_{i+1}^2 - t^2)]dt \quad &= \quad a t_{i+1}(t_{i+1} - t_i) - \frac{a}{2}(t_{i+1}^2 - t_i^2) \\ &\quad - \frac{b}{2} t_{i+1}^2 (t_{i+1} - t_i) + \frac{b}{6}(t_{i+1}^3 - t_i^3).\end{aligned}$$

It follows that the total buyer stock from $n-$shipments, TBS, is

$$TBS = a\sum_{i=1}^{n-1}(t_{i+1}^2 - t_i t_{i+1}) - \frac{a}{2}(H^2 - t_1^2) - \frac{b}{2}\sum_{i=1}^{n-1}(t_{i+1}^3 - t_i t_{i+1}^2) + \frac{b}{6}(H^3 - t_1^3). \quad (2)$$

Case 1: All the shipments size are the same after the first shipment

If x is a stock level at the buyer when the production is about to start, then $t_1 = \frac{1}{b}(a - \sqrt{a^2 - 2bx})$. In this case we assume that the first shipment size is pt_1 which is delivered at time t_1 to consume until time t_2 where

$$\begin{aligned} a(t_2 - t_1) - \frac{b}{2}(t_2^2 - t_1^2) &= q_1, \\ t_2 &= \frac{a - \sqrt{a^2 - 2b(at_1 - \frac{1}{2}bt_1^2 + q_1)}}{b}. \end{aligned} \quad (3)$$

The total demand in the planning horizon is $D = aH - bH/2$. However, the production size, Q, for this policy is $D - x$. Then the total remaining demand, Q_r, after the first shipment is $Q - q_1$. It follows

$$a(t_{i+1} - t_i) - \frac{b}{2}(t_{i+1}^2 - t_i^2) = \frac{Q_r}{n-1}, \qquad i = 2, 3, ..., n.$$

Then the remaining shipment times are

$$t_{i+1} = \frac{a - \sqrt{a^2 - 2b(at_i - \frac{1}{2}bt_i^2 + (Q_r/n-1))}}{b}, \qquad i = 2, 3, ..., n. \quad (4)$$

In equation (4), we assume $P(t_{i+1} - t_i) \geq Q_r/n - 1$, that is, we have enough time to produce the shipment size of $Q_r/n - 1$. Subtituting all values of t in equation (2) will give the total time-weighted buyer stock for this policy. Then a general total cost for the system is

$$C = A_1 + nA_2 + h_1 TSS + (h_2 - h_1)TBS. \quad (5)$$

Case 2: All the time intervals between shipments are the same after the first shipment

In this case the values of t_1, q_1 and t_2 are the same as in the Case 1. The intervals of shipments are identical after t_2, then the remaining shipment times for this policy are

$$t_{i+2} = t_2 + \frac{iT_r}{n-1}, \qquad i = 1, 2, ..., n-1, \quad (6)$$

where $T_r = H - t_2$.

Similarly, subtituting (6) in equation (2) will give the total time-weighted buyer stock for this policy and a general total cost for the system is given by (5).

3 Numerical Illustration

To demonstrate the effectiveness of these models, we present numerical examples. For these examples, $f(t) = 100 - bt$ for $t \in (0, 5)$. The other parameters are $A_1 = 400, A_2 = 25, h_1 = 4, h_2 = 5$ and $P = 1000$. The initial stock level, x, varies from 2 to 15.

Table 1 gives the optimal result for the Cases 1 and 2 when $x = 15$. The minimum cost for the Case 1 is 2215.35 when $n = 3$. The shipments size for this policy are 152.32, 41.34 and 41.34. Similarly, the minimum cost for the Case 2 is 2220.45 with the shipment sizes 152.32, 62.01 and 20.67. Table 2 gives the minimum total cost for both cases when x takes values from 2 to 12. Based on our numerical results, the model of Case 1 is always superior to the model of Case 2.

Table 1: Comparison between models of Case 1 and Case 2 when $x = 15$

	Model for Case 1		Model for Case 2	
n	TBS	Cost	TBS	Cost
2	218.95	2225.17	218.95	2225.17
3	184.14	2215.35*	189.24	2220.45*
4	170.92	2227.13	174.93	2231.14
5	163.87	2245.08	166.95	2248.16
6	159.46	2265.67	161.90	2268.11
7	156.43	2287.65	158.42	2289.63
8	154.22	2310.44	155.88	2312.10

Table 2: Comparison between models of Case 1 and Case 2 for different values of x

	Model for Case 1		Model for Case 2	
x	n	Cost	n	Cost
2	5	2181.34	5	2195.46
4	5	2174.06	5	2186.17
6	4	2170.55	5	2182.41
8	4	2170.44	4	2181.37
10	4	2176.97	4	2185.71
12	3	2187.91	3	2196.44

4 Conclusion

In this paper we have presented the basic models for a single-vendor who manufactures stock in the last batch and then transfers to a single-buyer when demand rate is linearly decreasing with time. We proposed two models with the first shipment size is dependent on the initial stock level. In our limited numerical examples, the model for Case 1 is always superior to Case 2.

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