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Relationship between Fuzzy Edge Connectivity and the Variables in Clinical Waste Incineration Process

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Abstract Structured networks of interacting components are hallmarks of several complex systems and clinical waste incineration process is an example of such a system. Fuzzy graph theory provides important tools to capture various aspects of complexity, imprecision and fuzziness of the network structure of the incineration system as compared to the discrete description of relation of its crisp graph. This paper discusses the use of fuzzy edge connectivity in describing the relation between the variables in the incineration process. It begins with the definition of fuzzy graph that involves five different types of graph fuzziness in which fuzzy edge connectivity constitutes its third type. The fuzzy edge connectivity and the membership values of the fuzzy edge connectivity based on the chemical reactions of the variables of the system are defined and illustrated respectively. Fuzzy graph showing the relation between the variables are also depicted in a diagram to give a better picture of the relation between these variables.

Keywords Fuzzy edge connectivity; incineration process.

1 Introduction

Framework of the model describing the clinical waste incineration process using autocatalytic set has been developed and documented in Sabariah et. al ([1]). Crisp graph explaining the clinical incineration process (Sabariah et al. [2], [1]) has been the basis for this model which were adopted from the model proposed by Jain and Krishna ([3]; [4]; [5]; [6]; [7]; [8]). The implementation of the model to the real process of the clinical waste incineration has shown the inadequacy of the model. The strictly given value of 0 and 1 given to the link strength of the crisp graph could have contributed to such results that inadequately explained the process. Considering the real process of the system, the variability of the link strengths must be adhered. Therefore, c_{ij}, when non-zero, are allowed to take any value in the interval of $(0, 1]$.

In this regard, an extension to this study in looking into the concept of fuzzy graph had been carried out. Blue et. al ([9]; [10]) revealed five interpretations of the fuzziness of graph had led the study to the choice of one of its type to best described the process. This paper discusses on how the links that relate the variables in the clinical incineration process were explained by using Fuzzy Edge Connectivity.

2 Fuzzy Graphs

Blue et al. ([9]; [10]) have given five types of graph fuzziness that they have classified as the five taxonomy of fuzzy graph. Attempt was made to find which type of fuzziness suited the problem. Studying and reviewing all these five types of fuzziness has resulted in the formalization of these five types of graph fuzziness into one definition of fuzzy graph as stated below:

Definition 1: (Fuzzy Graph)
Fuzzy graph is a graph G_F satisfying one of the following types of fuzziness ($G_{F_i}$of the i^{th} type) or any of its combination:

(i) $G_{F_1} = \{ \ G_{1_F}, \ G_{2_F}, \ G_{3_F}, \ \ldots \ , \ G_{n_F} \ \}$ where fuzziness is on each graph G_i.

(ii) $G_{F_2} = \{ \ V, \ E_F \ \}$ where the edge set is fuzzy.

(iii) $G_{F_3} = \{ \ V, \ E(t_F, h_F) \ \}$ where both the vertex and edge sets are crisp, but the edges have fuzzy heads $h(e_i)$ and fuzzy tails $t(e_i)$.

(iv) $G_{F_4} = \{ \ V_F, \ E \ \}$ where the vertex set is fuzzy.

(v) $G_{F_5} = \{ \ V, \ E(w_F) \ \}$ where both the vertex and crisp sets are crisp but the edges have fuzzy weights.

Definition 1 has taken into consideration all the different types of graph fuzziness. In other words, any graph which has any of the five traits could be considered as fuzzy graph. This definition was explored as to formulate a fuzzy graph for the clinical waste incineration process. It was considered that Type 3 to be the most appropriate for the task of improving the model. With this consideration, the following definition and assumption was adopted in obtaining the fuzzy graph representing the clinical incineration process.

2.1 Fuzzy Graph of Type 3

Let $G_C\,(V, E)$ be the crisp graph of a clinical waste incineration process in Malacca, such that,

$$V = \{ \ v_1 \ , \ v_2 \ , v_3 \ , \ v_4 \ , \ v_5 \ , v_6 \ \}$$
$$E = \{ \ e_1 \ , \ e_2 \ , e_3 \ , \ ... \ , \ e_{14} \ \}.$$

Fuzzy graph of this type has the same crisp vertices and crisp edges, but with unknown edge connectivity, that is, the edges have fuzzy heads and tails. This means that the topological set up of the crisp graph and the fuzzy graph (its connectedness) for the clinical waste incineration process are the same. The difference is only on the connectivity of the edges that is prominently shown in the adjacency matrix of the fuzzy graph. In this section, the description of the fuzzy graph of Type 3 in terms of its fuzzy head and fuzzy tail of the edges is given.

Define $h(e_i)$ to be the fuzzy head of the edge $e_i \in E$ and $t(e_i)$ to be the fuzzy tail of the edge $e_i \in E$. This is shown in Figure 1.

The values of $t(e_i)$ and $h(e_i)$ are defined based on the system to be as follows:

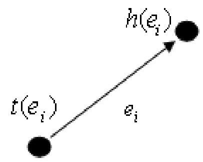

Figure 1: Fuzzy head and tail of the i^{th} edge

- The fuzzy tail,
$$t(e_i) = 1, \quad \forall\, e_i \in\ E,$$
This is due to the fact that each variable was taken as a whole before it evolved to other variables.
- The fuzzy head,
$$h: \quad E \longrightarrow [0,1]\,,$$
The value of fuzzy head is based on the reaction taken place and the strength of connection to the other variables in the system.

The idea leads to the development of the following definition of fuzzy edge connectivity and employ specifically for the clinical waste incineration process.

Definition 2: (Fuzzy edge connectivity)
Fuzzy edge connectivity C for $e_i \in E$ denoted as $C(e_i)$ is a tuple of

$$(\ t(e_i),\ h(e_i)\ )$$

Thus, the set of all ordered pairs of fuzzy edge connectivity is given by

$$C = \{\ (t(e_i), h(e_i))\ :\ e_i \in E\ \}\,.$$

In relation to the system, the value known as the membership value of the fuzzy edge connectivity has to be determined. Next, the membership value for fuzzy edge connectivity C of $e_i \in E$ is defined as follows:

Definition 3: (Membership value for fuzzy edge connectivity)
Membership value for fuzzy edge connectivity for each edge i, denoted as $\mu(e_i)$is defined as

$$\mu(e_i) = \min\ \{\ t(e_i),\ h(e_i)\ \}$$

In this case, since the $t(e_i) = 1$, the membership value for the fuzzy connectivity for each e_i is taken to be $\mu(e_i) = h(e_i)$.

To obtain these membership values of fuzzy edge connectivity, the chemical reaction equations of the formation of the variables or the relation between them are considered. Simply, it was taken that the fuzzy tail value to be 1, assuming that each variable was considered wholly before it evolved to other variables. In this case, the membership value of fuzzy edge connectivity taken to be the minimum between the head and the tail of each edge would simply equal to the value of the heads. Some properties of $\mu(e_i)$are given below.

Properties of $\mu(e_i)$

(i) $0 < \mu(e_i) \leq 1$

(ii) If e_i is a loop, then $\mu(e_i) = 1$.

Proposition 1: Fuzzy graph for the clinical waste incineration process can therefore be denoted as

$$G_F = \{\ V,\ E(\mu(e_i))\ \},$$

where $\mu(e_i)$is the membership value for fuzzy edge connectivity of G_F as defined in Definition 3.

In this notation, the number of nodes and edges of fuzzy graph is expected to be the same as the one in crisp graph. The existence of $\mu(e_i)$ in the bracket indicates that the edges contain fuzzy connectivity. These values of fuzzy edge connectivity are revealed in the adjacency matrix of fuzzy graph defined as the followings:

$$C_{F_{ij}} = \begin{cases} 0 & \text{for } i = j \quad \text{and} \quad e_i \notin E \\ \mu(e_i) & \text{for } i \neq j. \end{cases} \tag{1}$$

In this paper, fuzzy graph of Type 3 is referred to simply as as fuzzy graph.

2.2 Clinical Waste Incineration Process as a Fuzzy Graph

Physically, fuzzy graph for the clinical incineration process is the replication of the crisp graph G_c as in Figure 2. This is due to the Type 3 chosen in defining the fuzzy graph for the process. The difference is on the algebraic characteristic of the graphs that is represented by the membership value of fuzzy connectivity of the edges. These values are determined based on the Definition 2 and the fuzzy values of $t(e_i)$ and $h(e_i)$. All the edges of the fuzzy graph representing the process are the same as the edges of the crisp graph G_c illustrated in Figure 2. The reasoning for the value of $\mu(e_i)$ for each of these edges is discussed next.

2.3 Membership Value of Fuzzy Edge Connectivity ($\mu(e_i)$) for G_F

There are fifteen edges presented in Figure 2 that correspond to fifteen values of $\mu(e_i)$ for the graph. The following discussion reveals on how these values are determined.

$\mu(e_1)$: It was assumed that there is methane gas emitted from waste due to biological process prior to the incineration process. This gas is expected to increase the concentration of fuel but at a very minute quantity.

Thus we let $\mu(e_1) = 0.00001$.

$\mu(e_2)$: The calculation for $e_2,\ e_3,\ e_4$ and e_5 are based on the approximate of total chemical composition of typical clinical waste and the product of combustion discussed by Green (1992) in Table 5.3 of his book.

Total product of combustion (POC) in lb/t ≈ 3279.

Thus, the products of combustion namely CO, CO_2, O_2 and H_20 and other pollutants can be seen through the ratio of these products with respect to its total value.

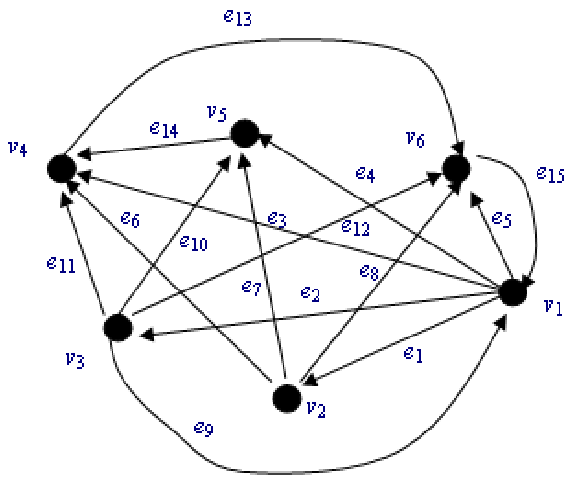

Figure 2: Crisp Graph G_c for the clinical waste incineration process

Hence, the ratio was taken to be the membership value in due consideration that these proportion of the resulted gases symbolizes the connectivity between them and the variable v_1 (Waste). These values are taken to be $\mu(e_i)$ for $e_2,\ e_3,\ e_4$ and e_5.

Value of O_2 in POC = 512 Thus $\mu(e_2) = 512 / 3279 = 0.15615$

$\mu(e_3)$: Value of CO_2 in POC = 1693.

$\mu(e_3) = 1693/3279 = 0.51632.$

$\mu(e_4)$: The contribution of waste to CO is only when there is insufficient of O_2. Therefore, the connectivity between the nodes are minimal and hence we take $\mu(e_4) = 0.00001$.

$\mu(e_5)$: Since we take $\mu(e_4) = 0.00001$, therefore

the value of CO expected in POC = 0.3279.

Value of H_2O and other Pollutants in POC = 1073.96721.

Therefore $\mu(e_5) = 1073.96721/3279 = 0.32752.$

$\mu(e_6)$: In any burning of fuel, products of its combustion are CO_2, H_2O and a trace of CO. This explained the existence of edges $e_6,\ e_7$ and e_8. Based on the material balance chart of the incinerator, the value of H_2O and CO_2 emitted after the secondary chamber were taken to calculate the ratio which signifies the contribution of fuel towards the formation of these gases. Since these ratios symbolizes the connectivity between these variables and Fuel, they were taken to be the membership values $\mu(e_i)$ for $e_6,\ e_7$ and e_8.

Values of H_2O + Values of CO_2 = 236.8 +503.3 = 740.1.

In this case, $\mu(e_6) = 503.3 / 740.1 = 0.68004.$

$\mu(e_7)$: A trace of CO means that the contribution of fuel to CO is minute.

Hence, we let $\mu(e_8) = 0.00001$.

$\mu(e_8)$: The computation for $\mu(e_8)$is similar to $\mu(e_6)$, whereby we have:

$\mu(e_8) = 236.8/740.1 = 0.3199.5$.

$\mu(e_9)$: From Green (1992), 11.5% of waste constitutes of O_2. Therefore

11.5% of 250 kg of waste contains = 28.75 kg of O_2.

Total O_2 at the beginning of process = 440.35 kg.

Thus, contribution of O_2 to waste = 28.75 / 440.35

$$\therefore\ \mu(e_9) = 0.06529$$

$\mu(e_{10})$: From the Handbook of Incineration system, by Bruner [11] states that at 20% of excess air supplied, the amount of CO formed per pound of stoichiometric air is 1.972 $\times\ 10^{-5}$. Thus, the contribution of O_2 to CO is 0.00002.

In other words, $\mu(e_{10}) = 0.00002$.

$\mu(e_{11})$: The balance of the amount of O_2 at this point is 0.9347. The membership value for e_6 and e_7 is used as the ratio to find $\mu(e_{11})$and $\mu(e_{12})$respectively.

Here, $\mu(e_{11}) = 0.68004 \times 0.9347 = 0.63563$.

$\mu(e_{12})$: $\mu(e_{12}) = 0.31995 \times 0.9347 = 0.29906$.

$\mu(e_{13})$: CO_2 react with water vapor to produce carbonic acid which in this case has been classified asv_6. Since CO_2 is a stable gas, we take that this connectivity is very minimal.

Thus $\mu(e_{13}) = 0.00001$.

$\mu(e_{14})$: From the equation, $2CO + O_2 \rightarrow 2CO_2$, most of CO will result in CO_2 since enough air is in the chamber for the reaction to arise.

$$\therefore\ \mu(e_{14}) = 0.99999.$$

$\mu(e_{15})$: Waste contains approximately 15% of water ([12]), that is

15% of 250 kg of waste = 37.5 kg.

H_2O+ other gases emitted = 242.33 kg.

Total H_2O+ other gases = 279.83kg.

Therefore, $\mu(e_{15}) = 37.5\ /\ 279.83 = 0.13401$.

With the membership values and Equation 1, we have the following adjacency matrix representingG_F

$$C_{F_{ij}} = \begin{pmatrix} 0 & 0 & 0.06529 & 0 & 0 & 0.13401 \\ 0.00001 & 0 & 0 & 0 & 0 & 0 \\ 0.15615 & 0 & 0 & 0 & 0 & 0 \\ 0.51632 & 0.68004 & 0.63563 & 0 & 0.99999 & 0 \\ 0.00001 & 0.00001 & 0.00002 & 0 & 0 & 0 \\ 0.32752 & 0.31995 & 0.29906 & 0.00001 & 0 & 0 \end{pmatrix} \qquad (2)$$

Since all the entries are the membership values $\mu(e_i) \in [0, 1]$, therefore the adjacency matrix for G_F is totally different from the adjacency matrix of the crisp graph G_c. The entries of C for G_c are discrete of values 0 and 1 whereas those for $G_F$are of values in the interval of $[0, 1]$. However the links and vertices of G_F are the same as those of G_c. The comparison between the crisp graph and fuzzy graph of Type 3 in representing the clinical waste incineration process can be seen in Figure 3.

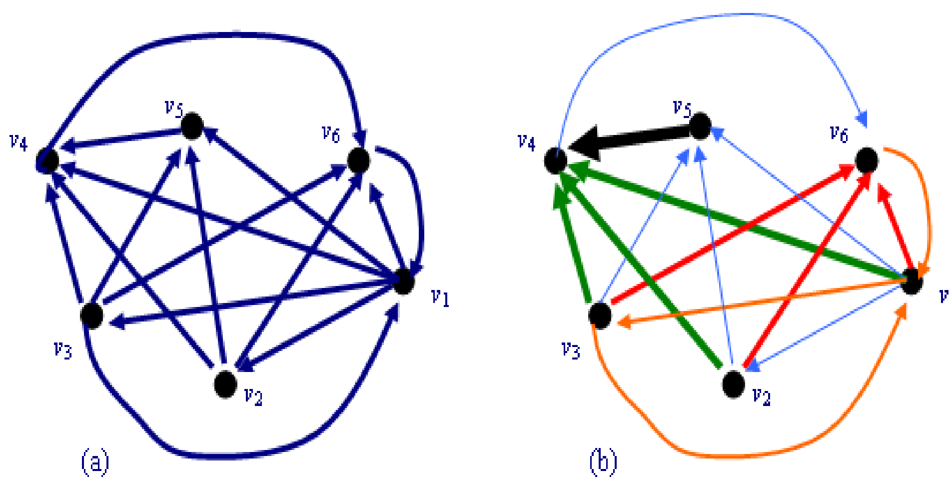

Figure 3: (a) Crisp Graph G_d and
(b) Fuzzy Graph of Type 3 G_F for the clinical waste incineration process

The same color and thickness of each link in the crisp graph shown in Figure 3(a) reveals that the connectivity between the vertices in the graph is considered the same. In this case, the value 1 is assigned when there is a link between the vertices and 0 when there is none. As compared to fuzzy graph shown in Figure 3(b), the greater the value of connectivity between the vertices, the thicker is the link between them. The different color signifies the different range of membership value for the fuzzy edge connectivity. Here, five distinct colors were used to differentiate the intensity of the connectivity between the vertices.

3 Conclusion

This paper have investigated and explored the realm of fuzzy graph (in particular the fuzzy graph of Type 3) in its relation to the clinical waste incineration process. Fuzzy edge connectivity and its membership value for each relation of the variables involved in the process have been defined. These results have served as a breakthrough of the relation between fuzzy graph and autocatalytic sets. Representing the system as fuzzy graph of Type 3 has initiated the modeling of the system in a different mode and in fact this would initiate a new model proves to be more realistic compared to the one using crisp graph.

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