MATEMATIKA, 2009, Volume 25, Number 2, 107–111

Dual Solutions of the Extended Blasius Problem

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Abstract This paper considers the extended problem of the boundary layer flow over a flat plate or Blasius problem by considering a uniform free stream parallel to a static or moving flat plate. The plate is assumed to move in the same or opposite directions to the free stream. The governing partial differential equations are first transformed into an ordinary differential equation, before being solved numerically. Dual solutions are found to exist when the plate and the free stream move in the opposite directions.

Keywords Blasius problem; dual solutions; moving plate; numerical solution.

1 Introduction

The classical boundary layer flow past a static flat plate or Blasius problem has attracted considerable interest of many researchers since introduced by Blasius [1]. Recently, Wang [2] employed the Adomian decomposition method (ADM) to solve numerically the classical Blasius equation. The numerical results reported by Wang [2] were then improved by Hashim [3] using the ADM-Padè approach. Quite recently, Cortell [4] considered the extended Blasius equation to a nonlinear equation, namely $af''' + ff'' = 0$, where a is a constant. The numerical results were obtained using a Runge-Kutta algorithm for high-order initial value problems for some values of a within the range $1 \leq a \leq 2$. The same problem was then reconsidered by Fang et al. [5], and they showed that the solution for an arbitrary value of a could be obtained from the classical Blasius equation's solution with an appropriate transformation. They also found that the equation could be generalized to other parameter domains and even for a being a function of the independent similarity variable.

In this study, we extend the classical Blasius problem [1] by considering a flat plate moving in the same or opposite directions to a parallel free stream, all with constant velocities. This work is different from those mentioned above, where the authors considered the boundary layer flow over a static flat plate. We introduce a new parameter, namely the velocity ratio parameter $\lambda = U_w/U_\infty$, where U_w and U_∞ are the plate velocity and the free stream velocity, respectively. When $\lambda = 0$, the present problem reduces to those considered by Blasius. By introducing the velocity ratio parameter λ, we are able to analyze the case when both the flat plate and the free stream are in moving situations. Dual solutions are found to exist when the plate and the free stream move in the opposite directions ($\lambda < 0$), which was not found for the static plate case ($\lambda = 0$).

2 Mathematical Formulation

The continuity and momentum equations for steady boundary layer flow past a flat plate are given by (see White [6])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where u and v are the velocity components in the x and y directions, respectively, and ν is the kinematic viscosity. We consider the flow is subjected to the boundary conditions

$$u = U_w, \quad v = 0 \quad \text{at} \quad y = 0; \quad u \to U_\infty \quad \text{as} \quad y \to \infty, \tag{3}$$

where U_w and U_∞ are constants and correspond to the plate velocity and the free stream velocity, respectively.

The continuity equation (1) is satisfied by introducing a stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The momentum equation (2) can be transformed into the corresponding nonlinear ordinary differential equation by the transformation

$$\eta = (U_\infty/\nu\, x)^{1/2}\, y, \quad \psi = (U_\infty \nu\, x)^{1/2}\, f(\eta), \tag{4}$$

where η is the independent similarity variable. The transformed nonlinear ordinary differential equation is

$$2f''' + ff'' = 0, \tag{5}$$

and the boundary conditions (3) now become

$$f(0) = 0, \quad f'(0) = \lambda, \quad f'(\infty) \to 1, \tag{6}$$

where primes denote differentiation with respect to η and $\lambda = U_w/U_\infty$ is the velocity ratio of the plate to the free stream. We notice that when $\lambda = 0$, the problem reduces to the classical Blasius problem, while $\lambda > 0$ and $\lambda < 0$ correspond to the case when the plate and the free stream move in the same or in the opposite directions, respectively.

The physical quantities of interest are the skin friction coefficient C_f and the displacement thickness δ^*, which can be shown to have the following form (see White [6]):

$$\frac{1}{2}C_f Re_x^{1/2} = f''(0), \quad \frac{\delta^* Re_x^{1/2}}{x} = \int_0^\delta (1 - f')\,d\eta, \tag{7}$$

where $Re_x = U_\infty x/\nu$ is the local Reynolds number and δ is the boundary layer thickness.

3 Results and Discussion

Equation (5) subject to the boundary conditions (6) has been solved numerically using the Keller-box method, which is described in the books by Na [7] and Cebeci and Bradshaw [8]. The step size $\Delta\eta$ in η, and the position of the edge of the boundary layer η_∞ have to be adjusted for different values of λ, to maintain the accuracy. In this study, the values of $\Delta\eta$ between 0.001 and 0.1 were used, in order that the numerical values obtained are independent of $\Delta\eta$ chosen, at least to four decimal places. However, a uniform grid of $\Delta\eta = 0.01$ was found to be satisfactory for a convergence criterion of 10^{-5} which gives accuracy to four decimal places. On the other hand, the boundary layer thickness η_∞ between 5 and 50 was chosen where the infinity boundary condition is achieved. For some values of λ, there is a possibility that two values of η_∞ is obtained, which gives two different velocity profiles, and in consequence produces two different values of the surface shear stress (dual solutions).

Figure 1 shows the skin friction coefficient in terms of $f''(0)$ as a function of λ. The values of $f''(0)$ are positives when $\lambda < 1$, zero when $\lambda = 1$, and negatives when $\lambda > 1$. The change of sign is due to the directions of the plate and the free stream. Physically positive value of $f''(0)$ means that the fluid exerts a drag force on the plate and negative value means the opposite. The zero skin friction when $\lambda = 1$ in this case does not mean separation, but corresponds to the parallel flow $U_w = U_\infty$. Further, for this case ($\lambda = 1$), the solution of equation (5) subject to the boundary conditions (6) is given by $f(\eta) = \eta$, which implies $f''(\eta) = 0$ for any values of η. Figure 1 also shows that when the plate and the free stream move in the same direction ($\lambda > 0$), the skin friction coefficient $f''(0)$ decreases as λ increases, while dual solutions are found to exist when they move in the opposite directions. The solution can be obtained up to a critical value of λ (say λ_c), beyond which the boundary layer separates from the surface, thus no solution is obtained. The absolute value of the skin friction coefficient $f''(0)$, which influence the heat transfer rate at the surface decreases as $\lambda \to 1$. Thus, the velocity ratio parameter λ can be used to control the magnitude of the skin friction. On the other hand, Figure 2 presents the displacement thickness δ^* of the flow, which represents the distance a streamline just outside the boundary layer is displaced away from the wall compared to the inviscid solution.

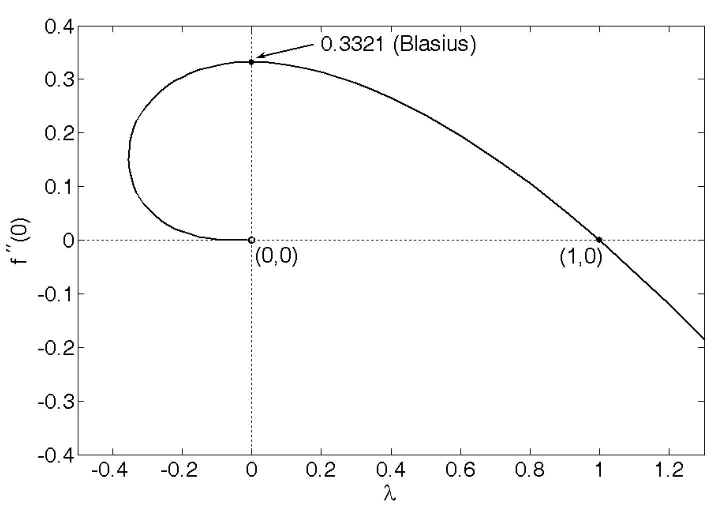

Figure 1: Skin Friction Coefficient $f''(0)$ as a Function of λ

It is evident from Figure 1 and Figure 2 that dual solutions exist when the plate and the free stream move in the opposite directions ($\lambda < 0$). These figures show that there exists

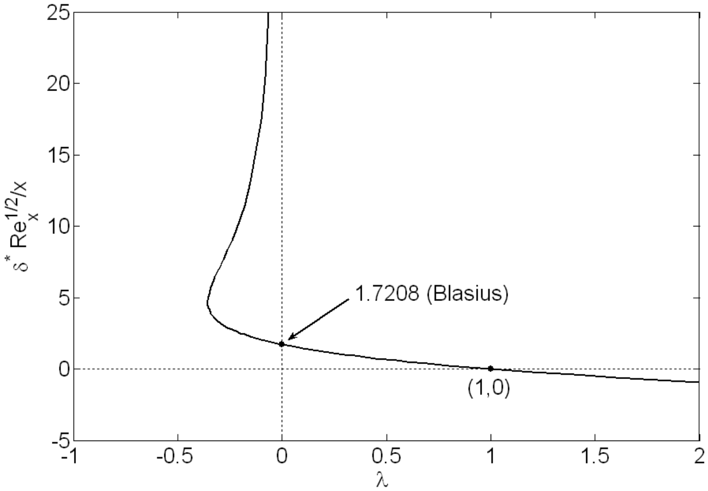

Figure 2: Displacement Thickness $\delta^* \sqrt{Re_x}/x$ as a Function of λ

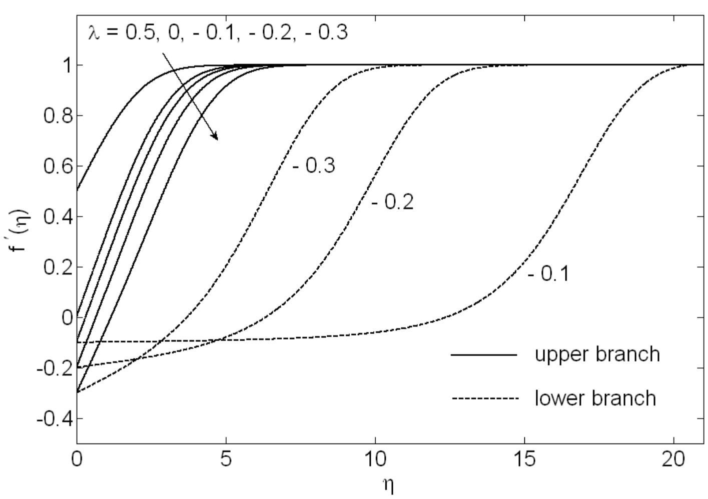

Figure 3: Velocity Profiles $f'(\eta)$ for Different Values of λ

a critical value of λ (i.e. λ_c), for which the upper branch solution meets the lower branch solution. The solution is unique when $\lambda \geq 0$ as well as $\lambda = \lambda_c$, dual when $\lambda_c < \lambda < 0$ and no solution when $\lambda < \lambda_c$. Based on our computation, $\lambda_c = -0.3541$. The selections of velocity profiles for different values of λ are presented in Figure 3. It is observed that the lower branch profiles also satisfy the far field boundary condition $f'(\infty) \to 1$ asymptotically, thus support the validity of the present results, besides supporting the dual nature of the solution to the boundary value problem (5)-(6) presented in Figure 1 and Figure 2. Which solution actually occurs depends on the flow stability, which is not investigated in this paper.

4 Conclusions

We extended the classical Blasius problem by considering a uniform free stream parallel to a static or moving flat plate. The transformed equation is solved numerically by a finite-difference method. When the plate and the free stream move in the same direction ($\lambda > 0$), the skin friction decreases as λ increases, while dual solutions are found to exist when they move in the opposite directions.

Acknowledgement

This work is supported by a research grant (Science Fund 06-01-02-SF0610) from Ministry of Science, Technology and Innovation (MOSTI), Malaysia.

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