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On the Jaccard Index Similarity Measure in Ranking Fuzzy Numbers

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Abstract Ranking of fuzzy numbers plays an important role in practical use and has become a prerequisite procedure for decision-making problem in fuzzy environment. Various techniques of ranking fuzzy numbers have been developed and one of them is based on the similarity measure technique. Jaccard index similarity measure has been introduced in ranking the fuzzy numbers where fuzzy maximum, fuzzy minimum, fuzzy evidence and fuzzy total evidence are used in determining the ranking. However, the study of Jaccard index similarity measure only focuses on the triangular fuzzy numbers and so far has not been utilized to other shapes of fuzzy numbers. Sometimes in some cases, it cannot discriminate the ranking between two different fuzzy numbers effectively. This paper extends the Jaccard index similarity measure in ranking all shapes of fuzzy numbers such as trapezoidal and general forms of fuzzy numbers. We have shown that when the degree of optimism concept is applied in determining the fuzzy total evidence, the ranking results has improved.

Keywords Jaccard index similarity measure; fuzzy maximum; fuzzy minimum; fuzzy total evidence; ranking fuzzy numbers.

1 Introduction

In fuzzy environment, the ranking of fuzzy numbers plays an important role in practical use and has become a prerequisite procedure for decision-making problem. Various techniques of ranking fuzzy numbers have been developed such as distance index by Cheng [1], signed distance by Yao and Wu [2] and Abbasbandy and Asady [3], area index by Chu and Tsao [4], index based on standard deviation by Chen and Chen [5], score value by Chen and Chen [6], distance minimization by Asady and Zendehnam [7] and centroid index by Wang and Lee [8]. These methods range from the trivial to the complex, including one fuzzy number attribute to many fuzzy numbers attributes. The similarity measure concept using Jaccard index was also proposed in ranking the fuzzy numbers. This method, which was proposed by Setnes and Cross [9], evaluates the agreement between each pair of fuzzy numbers in similarity manner. The fuzzy maximum, fuzzy minimum, fuzzy evidence and fuzzy total evidence were used in determining the ranking of fuzzy numbers.

However, the Jaccard index similarity measure proposed by Setnes and Cross [9] focuses on the triangular fuzzy numbers only. It cannot discriminate the ranking between two different fuzzy numbers effectively especially for cases when two fuzzy numbers have the same core and one is symmetrical included in the other [10]. This paper extends the Jaccard index similarity measure in ranking all shapes of fuzzy numbers such as trapezoidal and general forms of fuzzy numbers. We propose the degree of optimism concept in calculating the total fuzzy evidence instead of using the mean aggregation. The degree of optimism concept can solve the ranking issue of two non-identical fuzzy numbers with the same core. Besides, instead of using the binary relation and column vector to order n fuzzy numbers, this paper proposes the ordinal scaling approach which has simple calculation and eliminates some steps in the procedure.

2 Preliminaries

In this section, we briefly review the definition of fuzzy numbers, fuzzy maximum and fuzzy minimum.

2.1 Fuzzy Number

A fuzzy number is a fuzzy subset in the universe discourse that is both convex and normal. The membership function of a fuzzy number $\tilde{A}$ can be defined as

$$f_{\tilde{A}}(x) = \begin{cases} f^L_{\tilde{A}}(x) & , a \leq x \leq b \\ 1 & , b \leq x \leq c \\ f^R_{\tilde{A}}(x) & , c \leq x \leq d \\ 0 & , \texttt{otherwise} \end{cases}$$

where $f^L_{\tilde{A}}$ is the left membership function that is increasing and $f^L_{\tilde{A}} : [a,b] \to [0,1]$. $f^R_{\tilde{A}}$ is the right membership function that is decreasing and $f^R_{\tilde{A}} : [c,d] \to [0,1]$. If $f^L_{\tilde{A}}$ and $f^R_{\tilde{A}}$ are linear and continuous, then $\tilde{A}$ is a trapezoidal fuzzy number denoted by (a,b,c,d). Triangular fuzzy numbers which are special cases of trapezoidal fuzzy numbers with $b{=}c$ are denoted as (a,b,d).

The α-cut of $\tilde{A}$ is defined as $A_\alpha = \{x \in \Re | f_{\tilde{A}}(x) \geq \alpha\}$.

2.2 Fuzzy Minimum and Fuzzy Maximum

For two fuzzy numbers, A and B, with α-cuts, $A_\alpha = [a^-_\alpha, a^+_\alpha]$ and $B_\alpha = [b^-_\alpha, b^+_\alpha]$ respectively, the minimum of A and B is a fuzzy number denoted by $MIN(A,B)$ defined as, $MIN(A,B) = A_\alpha(\wedge)B_\alpha \equiv [a^-_\alpha, a^+_\alpha] \wedge [b^-_\alpha, b^+_\alpha] = [a^-_\alpha \wedge b^-_\alpha, a^+_\alpha \wedge b^+_\alpha]$.

On the other hand, the maximum of A and B denoted by $MAX(A,B)$ is defined as, $MAX(A,B) = A_\alpha(\vee)B_\alpha \equiv [a^-_\alpha, a^+_\alpha] \vee [b^-_\alpha, b^+_\alpha] = [a^-_\alpha \vee b^-_\alpha, a^+_\alpha \vee b^+_\alpha]$, [11].

3 Fuzzy Jaccard Ranking Method

Based on the psychological ratio model of similarity from Tversky [12], which is defined as

$$S_{\alpha,\beta}(X,Y) = \frac{f(X \cap Y)}{f(X \cap Y) + \alpha f(X \cap \overline{Y}) + \beta f(Y \cap \overline{X})},$$

various index of similarity measures have been proposed which depend on the values of α and β. For $\alpha = \beta = 1$, the psychological ratio model of similarity becomes the Jaccard index similarity measure which is defined as $S_{1,1}(X,Y) = \frac{f(X \cap Y)}{f(X \cup Y)}$. Typically, the function f is taken to be the cardinality function. The objects X and Y described by the features are replaced with fuzzy sets A and B which are described by the membership functions. The fuzzy Jaccard index similarity measure is defined as $S_J(A,B) = \frac{|A \cap B|}{|A \cup B|}$ where $|A|$ denotes the cardinality of fuzzy set A, $\cap$ and $\cup$ can be replaced by t-norm and s-norm respectively. The fuzzy Jaccard ranking procedure by Setnes and Cross [9] is presented as follows:

Step 1: For each pair of triangular fuzzy numbers A_i and A_j where $i,j = 1,2,\ldots,n$, find the fuzzy minimum and fuzzy maximum between A_i and A_j.

Step 2: Calculate the evidences of $E(A_i \geq A_j)$, $E(A_j \leq A_i)$, $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$ which are defined based on fuzzy Jaccard index as $E(A_i \geq A_j) = S_J(MAX(A_i,A_j),A_i)$, $E(A_j \leq A_i) = S_J(MIN(A_i,A_j),A_j)$, $E(A_j \geq A_i) = S_J(MAX(A_i,A_j),A_j)$ and $E(A_i \leq A_j) = S_J(MIN(A_i,A_j),A_i)$. To simplify, C_{ij} and c_{ji} are used to represent $E(A_i \geq A_j)$ and $E(A_j \leq A_i)$, respectively. Likewise, C_{ji} and c_{ij} are used to denote $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$ respectively.

Step 3: Calculate the total evidences $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$ which are defined based on the mean aggregation concept as

$$E_{total}(A_i \geq A_j) = \frac{C_{ij} + c_{ji}}{2} \quad \text{and} \quad E_{total}(A_j \geq A_i) = \frac{C_{ji} + c_{ij}}{2}.$$

To simplify, $E_{\geq}(i,j)$ and $E_{\geq}(j,i)$ are used to replace $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$ respectively.

Step 4: For two triangular fuzzy numbers, compare the total evidences in *Step 3* which will result in the ranking of the two triangular fuzzy numbers A_i and A_j as follows:

i. $A_i \succ A_j$ if and only if $E_{\geq}(i,j) > E_{\geq}(j,i)$.
ii. $A_i \prec A_j$ if and only if $E_{\geq}(i,j) < E_{\geq}(j,i)$.
iii. $A_i \approx A_j$ if and only if $E_{\geq}(i,j) = E_{\geq}(j,i)$.

Step 5: For n triangular fuzzy numbers, develop $n \times n$ binary ranking relation $R_{>}(i,j)$, defined as

$$R_{>}(i,j) = \begin{cases} 1, & E_{\geq}(i,j) > E_{\geq}(j,i) \\ 0, & \text{otherwise.} \end{cases}$$

Step 6: Develop a column vector $[O_i]$ where O_i is the total element of each row of $R_{>}(i,j)$ defined as $O_i = \sum_{j=1}^{n} R_{>}(i,j)$ for $j = 1,2,\ldots,n$.

Step 7: The total ordering of the triangular fuzzy numbers A_i corresponds to the order of the elements O_i in the column vector $[O_i]$.

4 Extension of Jaccard Ranking Method

We propose an extension of Jaccard procedure as follows:

Step 1: For each pair of fuzzy numbers (triangular, trapezoidal or general form) A_i and A_j where $i, j = 1, 2, \ldots, n$, find the fuzzy minimum and fuzzy maximum between A_i and A_j.

Step 2: Calculate the evidences of $E(A_i \geq A_j)$, $E(A_j \leq A_i)$, $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$ which are defined based on fuzzy Jaccard index as $E(A_i \geq A_j) = S_J(MAX(A_i, A_j), A_i)$, $E(A_j \leq A_i) = S_J(MIN(A_i, A_j), A_j)$, $E(A_j \geq A_i) = S_J(MAX(A_i, A_j), A_j)$ and $E(A_i \leq A_j) = S_J(MIN(A_i, A_j), A_i)$. To simplify, C_{ij} and c_{ji} are used to represent $E(A_i \geq A_j)$ and $E(A_j \leq A_i)$ respectively. Likewise, C_{ji} and c_{ij} are used to denote $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$ respectively.
Step 3: Calculate the total evidences $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$ which are defined by $E_{total}(A_i \geq A_j) = \beta C_{ij} + (1-\beta)c_{ji}$ and $E_{total}(A_j \geq A_i) = \beta C_{ji} + (1-\beta)c_{ij}$, with β represents the degree of optimism. $E_{\geq}(i, j)$ and $E_{\geq}(j, i)$ are used to replace $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$ respectively.

Step 4: For each pair of fuzzy numbers, compare the total evidences in *Step 3*.
i. $A_i \succ A_j$ if and only if $E_{\geq}(i, j) > E_{\geq}(j, i)$.
ii. $A_i \prec A_j$ if and only if $E_{\geq}(i, j) < E_{\geq}(j, i)$.
iii. $A_i \approx A_j$ if and only if $E_{\geq}(i, j) = E_{\geq}(j, i)$.

Step 5: For n fuzzy numbers, an ordinal scaling method which is a paired comparison approach is applied [13]. $\frac{1}{2}n(n-1)$ pairs of ranking results from *Step 4* are used for the ranking purposes. Arrange all the pair wise ranking from *Step 4* in a way that the sign '$\succ$' is in the same direction. The fuzzy number which ranks first, second, third,..., n-th should appear $n-1, n-2, n-3, \ldots, 0$ times on the left-hand side of the sign '$\succ$'.

The above procedure is illustrated in Figure 1.

5 Implementation

In this section, eight sets of numerical examples are presented to illustrate the validity and advantages of the Jaccard extension ranking method. Tables 1 and 2 show the ranking results for Sets 1-5 and Sets 6-8 respectively.

Set 1: $A = (0.2, 0.5, 0.9)$, $B = (0.1, 0.6, 0.8)$.
Set 2: $A = (0.15, 0.7, 0.8)$, $B = (0.35, 0.5, 1)$.
Set 3: $A = (-0.5, -0.3, -0.1)$, $B = (0.1, 0.3, 0.5)$.
Set 4: $A = (1, 2, 5)$, $B = [1, 2, 2, 4]$ with membership function,

$$f_B(x) = \begin{cases} \sqrt{1-(x-2)^2}, & 1 \leq x \leq 2 \\ \sqrt{1-\frac{1}{4}(x-2)^2}, & 2 \leq x \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

Set 5: $A = (0.1, 0.2, 0.4, 0.5)$, $B = (0.2, 0.3, 0.4)$.
Set 6: $A = (1, 2, 5)$, $B = (0, 3, 4)$ and $C = (1.5, 2, 4.5)$.
Set 7: $A = (3, 5, 12)$, $B = (1, 7, 10)$, $C = (0, 1, 3, 7)$.
Set 8: $A = (2, 6.5, 9, 12.5)$, $B = (5, 6, 13)$ and $C = (1, 7, 10, 12)$.

Find fuzzy maximum and fuzzy minimum
$MAX(A_i, A_j)$ & $MIN(A_i, A_j)$

Calculate evidences
$E(A_i \geq A_j), E(A_j \leq A_i), E(A_j \geq A_i)$ & $E(A_i \leq A_j)$

Calculate total evidences
$E_{\geq}(i, j) = \beta.C_{ij} + (1 - \beta)c_{ji}$ &
$E_{\geq}(j, i) = \beta.C_{ji} + (1 - \beta)c_{ij}$

Pair wise ranking results
$A_i \approx A_j \leftrightarrow E_{\geq}(i, j) = E_{\geq}(j, i)$
$A_i \succ A_j \leftrightarrow E_{\geq}(i, j) > E_{\geq}(j, i)$ &
$A_i \prec A_j \leftrightarrow E_{\geq}(i, j) < E_{\geq}(j, i)$

Ordering n fuzzy numbers
by ordinal scaling

Figure 1: Jaccard Extension Ranking Procedure

Table 1: Comparative Result for Sets 1-5

	Fuzzy Number	Set 1	Set 2	Set 3	Set 4	Set 5
Cheng	A	0.726	0.765	0.583	2.707	0.583
[1]	B	0.724	0.778	0.583	2.473	0.583
Results		$A \succ B$	$A \prec B$	$A \approx B$	$A \succ B$	$A \approx B$
Yao & Wu	A	0.525	0.588	-0.3	2.5	0.3
[2]	B	0.525	0.588	0.3	*	0.3
Results		$A \approx B$	$A \approx B$	$A \prec B$	-	$A \approx B$
Chu & Tsao	A	0.262	0.293	-0.15	1.244	0.15
[4]	B	0.262	0.293	0.15	1.182	0.15
Results		$A \approx B$	$A \approx B$	$A \prec B$	$A \succ B$	$A \approx B$
Chen & Chen	A	1.423	1.437	0.636	3.162	1.206
[5]	B	1.386	1.508	1.236	*	1.267
Results		$A \succ B$	$A \prec B$	$A \prec B$	-	$A \prec B$
Abbasbandy	A	1.05	1.175	0.6	5	0.6
& Asady [3]	B	1.05	1.175	0.6	*	0.6
Results		$A \approx B$	$A \approx B$	$A \approx B$	-	$A \approx B$
Asady &	A	0.525	0.588	-0.3	2.5	0.3
Zendehnam [7]	B	0.525	0.588	0.3	*	0.3
Results		$A \approx B$	$A \approx B$	$A \prec B$	-	$A \approx B$
Chen & Chen	A	0.406	0.402	0.446	*	0.424
[6]	B	0.400	0.411	0.747	*	0.473
Results		$A \succ B$	$A \prec B$	$A \prec B$	-	$A \prec B$
Wang & Lee	A	0.533	0.55	-0.3	2.667	0.3, 0.5
[8]	B	0.5	0.617	0.3	2.424	0.3, 0.5
Results		$A \succ B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \approx B$
Original Jaccard	$E_{\geq(A,B)}$	0.895	0.733	0	0.890	0.583
[9]	$E_{\geq(B,A)}$	0.867	0.826	1	0.806	0.583
Results		$A \succ B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \approx B$
Extension of Jaccard	$E_{\geq(A,B)}$	0.895	0.733	0	0.893	0.5
$\beta = 0$	$E_{\geq(B,A)}$	0.867	0.826	1	0.795	0.667
Results		$A \succ B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \prec B$
Extension of Jaccard	$E_{\geq(A,B)}$	0.895	0.733	0	0.890	0.583
$\beta = 0.5$	$E_{\geq(B,A)}$	0.867	0.826	1	0.806	0.583
Results		$A \succ B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \approx B$
Extension of Jaccard	$E_{\geq(A,B)}$	0.895	0.733	0	0.888	0.667
$\beta = 1$	$E_{\geq(B,A)}$	0.867	0.826	1	0.816	0.5
Results		$A \succ B$	$A \prec B$	$A \prec B$	$A \succ B$	$A \succ B$

'$*$': the ranking method cannot calculate the ranking value
'$-$': no conclusion for the ranking result

In Table 1, we have the following results: In Sets 1 and 2, for Yao and Wu's [2], Chu and Tsao's [4], Abbasbandy and Asady's [3] and Asady and Zendehnam's [7] methods, the ranking order is $A \approx B$. This is the shortcoming of [2], [4], [3] and [7] that cannot discriminate the ranking between two different fuzzy numbers. However, the Jaccard extension method has the same result as other five techniques. For Set 3, Cheng's [1] and Abbasbandy and Asady's [3] also cannot discriminate the ranking between A and B. The Jaccard extension method has $A \prec B$, which is similar to the other methods. In Set 4, Yao and Wu's [2], Chen and Chen's [5], Abbasbandy and Asady's [3], Asady and Zendehnam's [7] and Chen and Chen's [6] methods cannot rank the general fuzzy number B. The score value for A in Chen and Chen's [6] method also cannot be obtained since it depends on the minimum value of the horizontal centroid point of A and B. The results for other methods are $A \succ B$ and similar with Jaccard extension method. For Set 5, all the methods have $A \approx B$, except [5], [6] and the Jaccard extension method with $\beta = 0$ and $\beta = 1$.

Based on Table 2, we have the following results: In Set 6, Yao and Wu's [2], Chu and Tsao's [4], Abbasbandy and Asady's [3] and Asady and Zendehnam's [7] methods rank the three fuzzy numbers equally. Cheng's [1], Wang and Lee's [8] and Jaccard original [9] have $A \approx C \succ B$. However, the Jaccard extension with $\beta = 0$ and $\beta = 1$ can discriminate the ranking between A, B and C. Jaccard extension with $\beta = 0$ has $C \succ A \succ B$ and consistent with [5]. While, Jaccard extension with $\beta = 1$ has $A \succ C \succ B$ and similar with [6]. For Set 7, the ranking for all the methods is $A \succ B \succ C$, except [2], [3]and [7] have $A \approx B \succ C$. In Set 8, for [2], [3] and [7], the ranking order is $A \approx B \approx C$ while [9] has $B \succ A \approx C$. [1], [5], [8] have $B \succ A \succ C$ and similar to Jaccard extension with $\beta = 0$. However, [4] and [6] have $C \succ A \succ B$ and similar to Jaccard extension with $\beta = 1$.

6 Conclusion

This paper extends and improves the Jaccard index similarity measure proposed by Setnes and Cross [9] in ranking all shapes of fuzzy numbers such as triangular, trapezoidal and general form of fuzzy numbers. The Jaccard extension ranking method can rank fuzzy numbers effectively which have failed to be ranked by some previous ranking methods such as Cheng's [1], Yao and Wu's [2], Chu and Tsao's [4], Chen and Chen's [5], Abbasbandy and Asady's [3], Asady and Zendehnam's [7], Chen and Chen's [6] and Wang and Lee's [8]. The fuzzy total evidence which applies the degree of optimism instead of the mean aggregation has solved the issue in case when two non-identical fuzzy numbers have the same core and one is symmetrical included in the other and is regarded as equal in the original Jaccard ranking method. The extension Jaccard ranking method can rank the non-identical fuzzy numbers based on the values of index of optimism in which the original Jaccard ranking method cannot discriminate the ranking result. Ordinal scaling used in ordering n fuzzy numbers has simple calculation and has eliminated the procedure of developing binary ranking relation $R_{>}(i, j)$ and column vector $[O_i]$. Thus, it can be concluded that the extension of Jaccard method can rank triangular, trapezoidal and general fuzzy numbers which improves not only to the original Jaccard ranking method but also to some other previous ranking methods.

Table 2: Comparative Result for Sets 6-8

	Fuzzy Number	Set 6	Set 7	Set 8
Cheng [1]	A	2.707	6.683	7.466
	B	2.394	6.022	8.014
	C	2.707	2.924	7.328
Results		$A \approx C \succ B$	$A \succ B \succ C$	$B \succ A \succ C$
Yao & Wu [2]	A	2.5	6.25	7.5
	B	2.5	6.25	7.5
	C	2.5	2.75	7.5
Results		$A \approx B \approx C$	$A \approx B \succ C$	$A \approx B \approx C$
Chu & Tsao [4]	A	1.244	3.111	3.766
	B	1.244	3.120	3.733
	C	1.244	1.313	3.817
Results		$A \approx B \approx C$	$A \succ B \succ C$	$C \succ A \succ B$
Chen & Chen [5]	A	3.162	6.868	7.564
	B	2.829	6.216	8.223
	C	3.244	3.155	7.360
Results		$C \succ A \succ B$	$A \succ B \succ C$	$B \succ A \succ C$
Abbasbandy & Asady [3]	A	5	12.5	15
	B	5	12.5	15
	C	5	5.5	15
Results		$A \approx B \approx C$	$A \approx B \succ C$	$A \approx B \approx C$
Asady & Zendehnam [7]	A	2.5	6.25	7.5
	B	2.5	6.25	7.5
	C	2.5	2.75	7.5
Results		$A \approx B \approx C$	$A \approx B \succ C$	$A \approx B \approx C$
Chen & Chen [6]	A	0.342	3.826	1.161
	B	0.077	3.164	1.042
	C	0.337	0.679	1.317
Results		$A \succ C \succ B$	$A \succ B \succ C$	$C \succ A \succ B$
Wang & Lee [8]	A	2.667, 0.467	6.667, 0.467	7.449, 0.506
	B	2.333, 0.533	6, 0.52	8, 0.467
	C	2.667, 0.467	2.888, 0.454	7.310, 0.522
Results		$A \approx C \succ B$	$A \succ B \succ C$	$B \succ A \succ C$
Original Jaccard [9]	A	1	2	0
	B	0	1	2
	C	1	0	0
Results		$A \approx C \succ B$	$A \succ B \succ C$	$B \succ A \approx C$
Extension of Jaccard $\beta = 0$	A	1	2	1
	B	0	1	2
	C	2	0	0
Results		$C \succ A \succ B$	$A \succ B \succ C$	$B \succ A \succ C$
Extension of Jaccard $\beta = 0.5$	A	1	2	0
	B	0	1	2
	C	1	0	0
Results		$A \approx C \succ B$	$A \succ B \succ C$	$B \succ A \approx C$
Extension of Jaccard $\beta = 1$	A	2	2	1
	B	0	1	0
	C	1	0	2
Results		$A \succ C \succ B$	$A \succ B \succ C$	$C \succ A \succ B$

'$*$': the ranking method cannot calculate the ranking value
'$-$': no conclusion for the ranking result

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