MATEMATIKA, 2011, Volume 27, Number 1, 79–96

Economic Design of Cumulative Sum Control Chart for Non-Normally Correlated Data

M. H. Lee
School of Engineering, Computing and Science, Swinburne University of Technology, Sarawak Campus, Jalan Simpang Tiga, 93350 Kuching, Sarawak, Malaysia.
e-mail: mhlee@swinburne.edu.my

Abstract Traditionally, one usually assumes that the measurements within a sample are independently and normally distributed when designing the control charts. However, this assumption may not be tenable in practice. In this study, the economic design of cumulative sum (CUSUM) control chart for non-normally correlated data is developed. The optimal design parameters are determined such that the cost function is minimized, in which the Burr distribution is used to represent various types of non-normal distribution. The numerical results show that skewness coefficient, kurtosis coefficient and correlation coefficient affect the economic design of the CUSUM chart.

Keywords Economic design; cumulative sum chart; non-normality; correlation; genetic algorithm

2010 Mathematics Subject Classification 62K05, 62P30.

1 Introduction

Control charts are one of the most important tools in statistical process control since Shewhart proposed the first control chart in the 1920s. One of the important technical decisions for the control charts is its design. Design of the control charts is referred to the selection of design parameters such as sample size, sampling interval and control limit. It is important because the design parameters affect the underlying process to be controlled. Recently the economic design of control charts is an important issue in statistical process control. The economic design procedure is to derive the optimal design parameters such that the expected cost per hour for the cost model is minimized. Duncan [1] proposed the first cost model for economically determining the design parameters for $\bar{X}$ chart that minimizes the average cost when a single out-of-control state (assignable cause) exists. Since then, the economic designs of control charts have received much attention. The economic design model for cumulative sum (CUSUM) chart was first studied by Taylor [2]. After that, the economic design of CUSUM chart has been developed and received considerable attention. Goel and Wu [3] presented a procedure for the economic design of CUSUM chart to control the mean of a process with a normally distributed quality characteristic. Chiu [4] proposed a production model for quality surveillance, which is based on CUSUM chart using the decision interval criterion. Chung [5] presented a search algorithm for the economic design of CUSUM chart using the one-dimensional H-pattern search technique. Simpson and Keats [6] used two-level fractional factorial designs to identify highly significant parameters in the Lorenzen and Vance economic control chart model under CUSUM condition. Pan and Chen [7] proposed a new way of monitoring and evaluating the environmental performance using the economic design of CUSUM chart. Nenes and Tagaras [8] proposed a model for the design of CUSUM chart for monitoring the process mean in short production runs.

When designing the control charts, one usually assumes that the measurements within each sample are normally distributed. However, this assumption may not be acceptable in practice. The normality assumption indicates that if the measurements are really normally distributed, the sample mean $\bar{X}$ is also normally distributed. If the measurement are not normally distributed, the sample mean $\bar{X}$ will be approximately normally distributed only when the sample size is sufficient large according to the central limit theorem but the sample size is not always sufficiently large due to the sampling cost. Therefore, if the measurements are not normally distributed, the traditional design approach may not be appropriate for designing control charts [9]. Lashkari and Rahim [10] and Haridy and El-Shabrawy [11] independently considered the economic design of CUSUM chart to maintain the current control of non-normal process means. Rahim [12] considered the economic design of $\bar{X}$ chart under non-normality. For all these designs, the non-normal probability density function of the process is more complicated than the Burr distribution approach presented by Burr [13]. The Burr distribution is a simple distribution function that has the capability of approximating a wide range of distributions, including normal and non-normal distributions. The advantage of the Burr distribution is its closed-form cumulative distribution function that simplifies the computations of run length distribution or Type I and Type II error probabilities of the control charts. The applications of the Burr distribution to represent various non-normal distributions in the design of control charts can be found in the literature [9]. Chou et al. [14] employed the Burr distribution to conduct the economic-statistical design of $\bar{X}$ chart for non-normal data, and from the sensitivity analysis, they found out that small values of the skewness coefficient have no significant effect on the optimal design; however, a larger value of skewness coefficient leads to a slightly large sample size and sampling interval, as well as wider control limits. Meanwhile, an increase on the kurtosis coefficient results in an increase on the sample size and wider control limit. After that, Chen [15] proposed a cost-quality model based on the Burr distribution for constructing the economic-statistical design for variable sampling intervals $\bar{X}$ chart, and his overall finding indicates that the designed variable sampling interval chart always outperforms the traditional chart with respected to the expected cost per unit time. Then, Chou et al. [16] studied the effect of non-normality on the design of warning limit $\bar{X}$ chart by combining the Gordan and Weindling's cost model with the Burr distribution. In their study, it is observed that negative skewness leads to wider control and warning limits. In addition, larger kurtosis results in a longer significant run length and wider control and warning limits. After that, Chen [17] proposed a cost model where the Burr distribution is employed to deal with the economic design of variable sampling intervals $\bar{X}$ chart. In comparison with the traditional chart, variable sampling interval chart requires to sample more often with larger control limits and smaller sample size. After than, Chou et al. [18] used the Burr distribution to determine the appropriate control limits or sample size for acceptance control chart under non-normality. From their presented example, ignoring the effect of non-normality in data leads to a higher type I or type II error probability. Then, Chen and Yeh [19] developed the economic design of $\bar{X}$ chart under Weibull shock models where the Burr distribution is employed to calculate for non-normal data. Their research outcome for non-normally distributed data is different from the original consequence of the example present by Banerjee and Rahim [20] for normal assumption. In summary, their proposed method requires slightly larger sample size. However, the sampling interval and the control limit width become shorter. Furthermore, the skewness coefficient significantly affects both the sample size and the sampling interval, but does not affect the control limit. However, the kurtosis coefficient does not significantly affect the design parameters of the $\bar{X}$ chart. In general, all the above studies show that the non-normality affects the design of control charts.

Another basic assumption in designing the control charts is that the measurements within each sample are independently distributed. However, it may not be tenable in practice (Grant and Leavenworth [21]). Therefore, Neuhardt [22] considered the effect of correlated data within subgroups that can be defined for the purposes for statistical process control, and he showed that the effect of correlated measurements within the subgroup is shown to increase the type I error rate for $\bar{X}$ chart. After that, Yang and Hancock [23] extended Neuhardt's work to determine the effect of correlated data on $\bar{X}$, S, R and S^2 charts, and their results show that if a positive correlation exists but is not recognized in $\bar{X}$ chart, then the actual Type I error will be larger than assumed; whereas failure to recognize the correlation will also slightly increase the Type I error in S, R and S^2 charts. However, theoretical analysis and Monte Carlo simulation studies show that these effects are not substantial. So, the traditional control limits for S, R and S^2 charts do not need to be revised even if correlation exists. Then, Liu et al. [24] developed a minimum-loss design of $\bar{X}$ chart for correlated data within a sample by incorporating Taguchi's quality loss function. From their results of sensitivity analysis, they found out that as the measurements in the sample are positively correlated, hightly correlated data result in a smaller sample size and a frequent sampling interval; however, as the measurements in the sample are negatively correlated, highly correlated data yield a smaller sample size and a narrower control limits. After that, Liu et al. [25] studied the effect of correlated data on the design of warning limit $\bar{X}$ chart by combining the cost model given in Gordan and Weindling [26] with Yang and Hancock's correlation model. Based on their study, it is observed that among the parameters in the economic design, only the significant run length is affected by the correlated data. Highly correlated data or independent data result in a longer run in the warning band. After that, Chen and Chiou [27] proposed a cost model combining the multivariate normal distribution model given by Yang and Hancock [23] with cost model of Bai and Lee [28] to develop the design of variable sampling intervals $\bar{X}$ chart for correlated data. Their results indicate that variable sampling interval charts outperform the traditional charts for large mean shift when correlation is present. In addition, there is a different between the design parameters of variable sampling interval charts when correlation is present or absent. Recently, Chen et al. [29] studied the effect of correlation on the design parameters for the economic design of $\bar{X}$ chart with variable sample sizes and variable sampling intervals. The magnitude of mean shift has a significant effect on the optimal values of the shortest and longest sampling intervals, and the cost saving. Correlation coefficient within the sample is highly associated with the optimum chart. From the results from all the above studies, it can be seen that correlation generally affects the design of the control charts.

Recently, genetic algorithms (GA) have been widely applied for the optimal design of quality control charts. GA is considered as an appropriate technique for solving the optimization problems. Aparisi and García-Díaz [30] used GA to carry out the design of EWMA chart for in-control, indifference and out-of-control regions. Chen [31] applied GA to minimize the cost model of variable sampling interval T^2 control chart. Chou et al. [32] applied GA to search the optimal design parameters for the economic design of EWMA chart using variable sampling intervals with sampling at fixed times. Lin et al. [33]

employed GA to obtain the solution for the economic design of variable sampling intervals $\bar{X}$ chart with A&L switching rule. Chen [34] applied GA to the cost model for the economic design of T^2 chart with variable sample sizes and sampling intervals. Kaya [35] used GA to determine the sample size for attribute control chart. Torng et al. [36] applied GA to solve the economic design model of double sampling $\bar{X}$ chart for the determination of optimal design parameters.

In this study, an approach for the economic design of CUSUM chart for non-normally correlated data is presented. The major concern of this study is to give a general idea on how non-normality parameters (i.e., skewness and kurtosis coefficients) and correlation parameter (i.e., correlation coefficient) affect the economic design of the CUSUM chart. The goal of the economic design of the CUSUM chart is to find the optimal design parameters for minimizing the expected cost per hour, given the process and cost parameters of the cost model. The optimal values of the design parameters based on the cost model are determined by using GA. In the next section, a review of the CUSUM chart will be given. Lorenzen and Vance's cost model will be briefly discussed in Section 3. The Burr distribution and the correlation model given in Yang and Hancock will be briefly reviewed in Section 4 and Section 5, respectively. The transition probabilities of Markov chain approach for the CUSUM chart will be derived in Section 6. An example will be provided in Section 7 to illustrate the solution procedure for the economic design of the CUSUM chart for non-normally correlated data. A sensitivity analysis will also be performed in Section 7 to investigate the effects of non-normality and correlation on the optimal economic design of the CUSUM chart. Finally, some concluding remarks will be provided in the last section.

2 The CUSUM Chart

The CUSUM chart was introduced by Page [37], and has been widely used for monitoring the process shift of production process. Consider a process in which the distribution of the observations from the process is normally distributed with mean μ and standard deviation σ. Then the $t-$th CUSUM statistic is

$$S_t = \max(0, \bar{X}_t + S_{t-1} - k) \quad \text{for} \quad t \quad = 1, 2, 3, ... \tag{1}$$

where $\bar{X}_t$ is the t-th subgroup mean, μ_0 is the in-control mean, and n is the sample size, the initial value of S_t is usually set as 0, and k is the reference value of the CUSUM chart.

When a CUSUM chart is used to monitor a process, a sample of fixed size n is taken every h hours, and the CUSUM statistic S_t is calculated at each sampling point. If the plotted CUSUM statistic S_t goes beyond the control limit L, a signal will be given. Then a search for an assignable cause is initiated. Otherwise, the process is considered as being in-control, and the next sample is continually taken at the next sampling point.

3 The Cost Model

In this study, the unified cost model developed by Lorenzen and Vance [38] is adopted to the economic design of the CUSUM chart for non-normally correlated data. In this cost model, a process is assumed to start in an in-control state with mean μ_0 and standard deviation σ. The occurrence of the assignable cause results in a shift in the process mean from μ_0

to $\mu_0 + \delta\sigma$, where δ is the magnitude of shift in the process mean. The time between occurrences of the assignable cause is assumed following an exponential distribution with a mean of $1/\theta$ hours. The expected cost per hour of the cost model is expressed as

$$C = \frac{C_0/\theta + C_1\left(-\tau + ne + \mathrm{ARL}_1 + \gamma_1 T_1 + \gamma_2 T_2\right) + sy/\mathrm{ARL}_0 + W}{1/\theta + (1-\gamma_1)\, sT_0/\mathrm{ARL}_0 - \tau + ne + \mathrm{ARL}_1 + T_1 + T_2}$$

$$+ \left(\frac{a+bn}{h}\right)\frac{1/\theta - \tau + ne + \mathrm{ARL}_1 + \gamma_1 T_1 + \gamma_2 T_2}{1/\theta + (1-\gamma_1)\, sT_0/\mathrm{ARL}_0 - \tau + ne + \mathrm{ARL}_2 + T_1 + T_2} \tag{2}$$

where C_0 = quality cost per hour for an in-control process; C_1 = quality cost per hour for an out-of-control process ($C_1 > C_0$); $1/\theta$ = mean time the process is in-control; a = fixed cost per sample; b = cost per unit sampled; y = cost per false alarm; W = cost to locate and correct the assignable cause; e = time to sample and chart one item; T_0 = expected search time after a false alarm; T_1 = expected time to discover the assignable cause; T_2 = expected time to correct the process; γ_1 is a binary variable = 1 if production continues during searches, and 0 if production ceases during searches; γ_2 is a binary variable = 1 if production continues during correction, and 0 if production ceases during correction; ARL_1 = out-of-control average run length; ARL_0 = in-control average run length; s = expected number of samples taken while in-control = $e^{-\theta h}/(1-e^{-\theta h})$; and τ = expected time of occurrence of the assignable cause between two samples while in-control.

4 The Burr Distribution

In this study, the Burr distribution is used to represent various types of non-normal distributions. The cumulative distribution function of the Burr distribution [13] is

$$F(y) = \begin{cases} 1 - \dfrac{1}{(1+y^c)^q} & \text{for } y \geq 0 \\ 0 & \text{for } y < 0 \end{cases} \tag{3}$$

where c and q are greater than one. Different combinations of c and q cover a wide range of skewness and kurtosis coefficient of various practical data distributions such as normal, Gamma and Beta. For example, the Burr distribution will be approximately as a normal distribution with $c = 4.85437$ and $q = 6.22665$ [13].

Burr [13] tabulated the expected value, the standard deviation, the skewness coefficient and the kurtosis coefficient of the Burr distribution for various combination of c and q. After the sample skewness and kurtosis coefficient are estimated for a data set, the tables in Burr [13] may be used to obtain the mean M and standard deviation S of the corresponding Burr distribution. When the process is in-control ($\mu = \mu_0$), the Burr distribution Y may be transformed to the sample mean $\bar{X}$ by the standardized transformation as follows:

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{Y - M}{S}$$

$$\bar{X} = \mu_0 + (Y - M)\frac{\sigma/\sqrt{n}}{S} \tag{4}$$

When the process mean has shifted ($\mu = \mu_0 + \delta\sigma$), the standardized transformation is as follows:

$$\frac{\bar{X} - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}} = \frac{Y - M}{S}$$

$$\bar{X} = \mu_0 + \delta\sigma + (Y - M)\frac{\sigma/\sqrt{n}}{S} \tag{5}$$

5 Yang and Hancock's Correlation Model

According to Yang and Hancock's model, it is assumed that each sample or subgroup is a realization of the random vector $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}$, which has a multivariate normal distribution $\mathbf{N}(\mu, \mathbf{V})$, where $\mu = \{\mu_i\}$, $i = 1, \ldots, n$, is the vector of mean values, and

$$\mathbf{V} = \{v_{ij}\},\ i,j = 1, \ldots, n,$$

is the covariance matrix. In addition, $\mathbf{V} = \sigma^2\mathbf{R}$, where $\mathbf{R} = \{r_{ij}\}$, $i,j = 1, \ldots, n$, is the correlation matrix. Based on these assumptions, the sample mean $\bar{X}$ can be shown to be normally distributed with a mean and variance as follows:

$$E(\bar{X}) = \mu \tag{6}$$

$$Var(\bar{X}) = \frac{\sigma^2}{n}\left[1 + (n-1)\rho\right] \tag{7}$$

where $\rho = \sum_{i \neq j} r_{ij}/n(n-1)$ is the correlation coefficient. The mean and variance of $\bar{X}$ are still valid even when the measurements are not normally distributed [9].

6 Transition Probabilities

In this study, the Markov chain approach of Prabhu et al. [39] is modified to evaluate the performance of the CUSUM chart for non-normally correlated data. The CUSUM chart can be modeled as a Markov chain with states denoted as 0, 1, 2, ..., m, where state m is the absorbing state. The interval between 0 and L is discretized into m states, such that the width of the interval for each state is $d = 2L/(2m-1)$, except the width of state 0 is $d/2$. Based on Equations (1), (3), (5), (6) and (7), the transition probability p_{ij} for $i = 0, 1, 2, \ldots, (m-1)$ and $j = 1, 2, \ldots, (m-1)$ in the transition probability matrix can be expressed as

$$\begin{aligned}
p_{ij} &= \Pr\left[(j-0.5)d < S_t < (j+0.5)d \,|S_{t-1} = id\right] \\
&= \Pr\left[(j-i-0.5)d + k < \bar{X} < (j-i+0.5)d + k\right] \\
&= \Pr\left\{M + \frac{\left[(j-i-0.5)d + k\right]S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}} < Y < M + \frac{\left[(j-i+0.5)d + k\right]S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\} \\
&= \left[1 - \frac{1}{\left(1 + \left\{M + \frac{[(j-i+0.5)d+k]S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\}^c\right)^q}\right] - \left[1 - \frac{1}{\left(1 + \left\{M + \frac{[(j-i-0.5)d+k]S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\}^c\right)^q}\right]
\end{aligned}$$

$$p_{ij} = \frac{1}{\left(1 + \left\{M + \frac{[(j-i-0.5)d+k]S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\}^c\right)^q} - \frac{1}{\left(1 + \left\{M + \frac{[(j-i+0.5)d+k]S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\}^c\right)^q} \tag{8}$$

Similarly, the transition probability p_{ij} for $i = 0, 1, 2, \ldots, (m-1)$ and $j = m$ can be expressed as

$$\begin{aligned} p_{im} &= \Pr\left\{Y > M + \frac{[(m-i-0.5)d+k]\,S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\} \\ &= 1 - \Pr\left\{Y < M + \frac{[(m-i-0.5)d+k]\,S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\} \\ &= \frac{1}{\left(1+\left\{M + \frac{[(m-i-0.5)d+k]\,S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\}^{c}\right)^{q}} \end{aligned} \tag{9}$$

and the transition probability p_{ij} for $i \quad = 0, 1, 2, (m-1)$ and $j \quad = 0$ can be expressed as

$$\begin{aligned} p_{i0} &= \Pr\left\{Y < M + \frac{[(-i+0.5)d+k]\,S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\} \\ p_{i0} &= 1 - \frac{1}{\left(1+\left\{M + \frac{[(-i+0.5)d+k]\,S - \delta S\sqrt{n}}{\sqrt{1+(n-1)\rho}}\right\}^{c}\right)^{q}} \end{aligned} \tag{10}$$

where Y is the probability density function of the Burr random variable.

The in-control transition probability matrix $\mathbf{Q}_0$ is obtained by using p_{ij} as its elements based on the in-control condition ($\delta = 0$). It is an ($m\ m$) matrix excluding the last row and the last column since they correspond to the absorbing state. Then the in-control average run length is given by

$$\mathrm{ARL}_0 = \mathbf{r}'\left(\mathbf{I} - \mathbf{Q}_0\right)^{-1}\mathbf{1} \tag{11}$$

where $\mathbf{r}$ is the (m initial probability vector containing 1 for the first element and zeros otherwise; $\mathbf{I}$ is the (mm) identity matrix; and $\mathbf{1}$ is the (m vector of all 1's. The in-control steady-state probability vector $\mathbf{B} = (b_0, b_1, b_2, b_{m_{-1}})\prime$ is obtained by solving the following equation (Prabhu et al. [39]):

$$\mathbf{B} = \tilde{\mathbf{Q}}'_0\mathbf{B} \quad \text{subject to} \quad \mathbf{1}'\mathbf{B} = 1 \tag{12}$$

where $\tilde{\mathbf{Q}}_0$ is obtained by dividing each row of $\mathbf{Q}_0$ by the sum of the elements of that row. For calculating the out-of-control average run length, it is assumed that the chart statistic has reached its stationary distribution at the time when the process shift occurs. Then the out-of-control average run length is calculated as

$$\mathrm{ARL}_1 = \mathbf{B}'\left(\mathbf{I} - \mathbf{Q}\right)^{-1}\mathbf{1} \tag{13}$$

where $\mathbf{Q}$ is the (mm) out-of-control transition probability matrix which can be obtained similarly as $\mathbf{Q}_0$, except the elements p_{ij} of $\mathbf{Q}$ are evaluated based on the out-of-control condition ($\delta > 0$).

7 An Example and the Sensitivity Analysis

In this section, an application example from a set of hypothetical process and cost parameters is provided to illustrate the solution obtained from the economic design of the CUSUM chart for non-normally correlated data.

Consider a factory produces yogurt drink which is contained in bottles. The target quantity of yogurt drink for each bottle is 0.02 liters. The yogurt drink is inserted into fifteen bottles at a time in the production process, and the fifteen bottles of yogurt drink will be packed in a box later. Before the fifteen bottles of yogurt drink are packed, the first four bottles are sampled to check if the quantity of yogurt drink is 0.02 liters. The process standard deviation is estimated to be 0.0012 liters. The recent 50 successive boxes are viewed as a random sample from a multivariate distribution with the first four bottle quantity averages are $\mu' = (0.012, 0.025, 0.018, 0.022)$, and the corresponding covariance matrix is

$$\mathbf{V} = \begin{bmatrix} 2.5\times10^{-7} & 8.8\times10^{-8} & 1.6\times10^{-7} & 9.9\times10^{-8} \\ 8.8\times10^{-8} & 2.5\times10^{-7} & 1.3\times10^{-7} & 7.1\times10^{-8} \\ 1.6\times10^{-7} & 1.3\times10^{-7} & 2.4\times10^{-7} & 9.3\times10^{-8} \\ 9.9\times10^{-8} & 7.1\times10^{-8} & 9.3\times10^{-8} & 2.6\times10^{-7} \end{bmatrix}$$

Based on the covariance matrix, the correlation coefficient is estimated to be $\rho = 0.4$.

Suppose that the hourly cost for operating in the in-control state and the out-of-control state are $C_0 = \$10$ and $C_1 = \$100$, respectively. Process shifts occur according to an exponential distribution with the frequency of about one every hundred hours of operation. Thus $\theta = 0.01$. The fixed cost of sampling is $a = \$0.5$, and the variable cost of sampling is $b = \$0.1$. The cost of investigating a false alarm is $y = \$50$, and the cost to investigate a true action signal is $W = \$25$. It takes approximately three minutes (0.05 hours) to take in and analyze a sample of observations. It takes about $T_1 = 2$ hours to investigate an action signal, and it takes about $T_2 = 2$ hours to correct the process. The process continues in operation during the search and repair of an assignable cause. Past historical data indicate that the skewness and kurtosis coefficients of the quantity of a bottle of yogurt drink are approximately 0.4836 and 3.3801, respectively, which can be described by the Burr distribution with $c = 3$ and $q = 6$.

The economic design of the CUSUM chart optimizes the design parameters: sample size n, sampling interval h, control limit L, and reference value k. Optimization is carried out using the GA by minimizing the cost function in Equation (2) for each sample size n subject to the following constraints:

$$0.01 \le h \le 2, 0.0001 \le L \le 5, 0.01 \le k \le 2$$

where n is ranged from 2 to 20. Table 1 shows the optimal design parameters of the CUSUM chart with the corresponding minimum cost. As shown in Table 1, the expected cost per hour C given in Equation (2) reaches a minimum of \$17.54 for non-normally correlated data ($n = 2, h = 0.44, L = 4.89, k = 0.35$), and a minimum of \$16.78 for the condition under independent and normality assumption ($n = 2, h = 0.85, L = 1.69, k = 1.12$). Accordingly, the cost error of the independent and normality assumption is (17.54 – 16.78)/17.54 100% = 4.33%.

Table 2 shows the optimal design parameters of the CUSUM chart with different magnitude of shift $\delta = 0.5$, 1.0, 1.5 and 2.0. From this table, it is noted that the design parameters

Table 1: The optimal design parameters (sample size n, sampling interval h, control limit L, and reference value k) of CUSUM chart and the corresponding minimum expected cost per hour C for non-normally correlated data ($a = 0.5$, $b = 0.1$, $C_0 = 10$, $C_1 = 100$, $y = 50$, $W = 25$, $e = 0.05$, $T_0 = 0$, $T_1 = 2$, $T_2 = 2$, $\theta = 0.01$, $\gamma_1 = \gamma_2 = 1$, $\delta = 1$, $c = 3$, $q = 6$, $\rho = 0.4$)

	n	h	L	k	C
CUSUM chart for non-normally correlated data	2	0.44	4.89	0.35	17.54
CUSUM chart under normality and independent assumption	2	0.85	1.69	1.12	16.78

(i.e., the sampling interval, the control limit, the reference value and the expected cost per hour) are significantly affected by the size of shift, except the sample size where the sample size is constant ($n = 2$) for all the given shift sizes. When the magnitude of shift increases from 0.5 to 2.0, the sampling interval becomes longer from 0.37 to 0.77, the control limit becomes narrower from 5.00 to 2.03, the reference value becomes larger from 0.09 to 1.87, and the corresponding expected cost per hour become lower from 18.88 to 15.77. This shows that larger magnitude of shift leads to a longer sampling interval and a larger reference value but leads to a smaller control limit and a lower expected cost per hour. It is also noted that the sample size does not affected by the magnitude of shift.

Table 2: Effect of magnitude of shift δ on the optimal design parameters (sample size n, sampling interval h, control limit L, and reference value k) of CUSUM chart and the corresponding minimum expected cost per hour C for non-normally correlated data

δ	n	h	L	k	C
0.5	2	0.37	5.00	0.09	18.88
1.0	2	0.44	4.89	0.35	17.54
1.5	2	0.61	3.15	1.03	16.33
2.0	2	0.77	2.03	1.87	15.77

A sensitivity analysis is conducted here to study the effects of non-normality and correlation on the optimal economic design of the CUSUM chart. The combinations of the skewness coefficient α_3, kurtosis coefficient α_4, and correlation coefficient ρ follow the values given by Chou et al. [9]. The values of skewness coefficient α_3 and kurtosis coefficient α_4 are given in column 2 and 3, respectively in Tables 3-6 with the corresponding values of (c, q) given in column 1. The correlation coefficient ρ with the values of -0.8, -0.4, 0, 0.4, and 0.8 are given in column 4, 5, 6, 7, and 8, respectively in Tables 3-6. Tables 3-6 give the optimal sample size n, optimal sampling interval h, optimal control limit L, and optimal reference value k for the CUSUM chart in this study, respectively.

Table 3 shows the optimal sample size n for various combinations of skewness coefficient α_3, kurtosis coefficient and correlation coefficient ρ. It can be seen that higher negatively correlated data give a smaller sample size for the given range of correlation coefficient. For example, for $(c,\ q)$ = (6, 11), the sample size decreases from 5 to 3 when the negative correlation coefficient increase from -0.0 to -0.4; then the sample size decreases from 3 to 2 when the negative correlation coefficient further increases from -0.4 to -0.8 (Group I). If the data are positively correlated, the positive correlation coefficient does not significantly affect the sample size. It can be noticed that all the groups (Group I-VI) in Table 3 give constant sample size. This means that the non-normality does not give significant effect on the sample size. For example, the sample sizes of Group IV (skewness coefficient increases from 0.434 to 1.014 and kurtosis coefficient is near to a constant and more than 4.0) are constant with the values of 2, 3, 5, 2, and 2 for correlation coefficient ρ = -0.8, -0.4, 0.0, 0.4, and 0.8, respectively. Note that for correlation coefficient $\rho = 0.0$, although the sample size is 4 for $(\alpha_3,\ \alpha_4)$ = (0.434, 4.106) but the sample sizes are all 5's for other combinations of $(\alpha_3,\ \alpha_4)$. Thus, from the results in this study we can consider that there is no significant changes for sample size in Group VI for correlation coefficient $\rho = 0.0$. The results of Chou et al. [9] for $\bar{X}$ chart show that the higher positively and negatively correlated data reduce the sample size. Meanwhile the results in this paper show that only negatively correlated data reduce the sample size of the CUSUM chart, thus the CUSUM chart in this study is more robust than the $\bar{X}$ chart of Chou et al. [9] when the data are positively correlated. Table 3 also shows that the sample size of the CUSUM chart in this study is not significantly affected by the skewness and kurtosis coefficients.

Table 4 shows the optimal sampling interval h for various combinations of skewness coefficient α_3, kurtosis coefficient α_4, and correlation coefficient ρ. From Table 4, it can be observed that higher positively correlated data give a slightly shorter sampling interval. For example, for Group III with $(\alpha_3,\ \alpha_4)$ = (-0.465, 3.430), the sampling interval decreases from 0.83 to 0.47 when the positive correlation coefficient increases from 0.0 to 0.4, and the sampling interval decreases from 0.47 to 0.44 when the positive correlation coefficient increases further from 0.4 to 0.8. If the data are negatively correlated, the sampling interval is not significantly affected by correlation. If the data are negatively correlated, a change from positive value to negative value in the skewness coefficient results in a shorter sampling interval. For example, for Group III (α_3 changes from -0.465 to 0.484 and α_4 near a constant), the sampling interval decreases from 0.81 to 0.72 for correlation coefficient ρ = -0.8 whereas the sampling interval decreases from 0.93 to 0.87 for correlation coefficient ρ = -0.4. Therefore, this study shows that if the data are positively correlated, there is no specific effect of the skewness and kurtosis coefficients on the sampling interval; whereas Chou et al. [9] show that an increase on the kurtosis coefficient usually leads to a longer sampling interval when the data are positively correlated for $\bar{X}$chart. Thus for positively correlated data, the design of CUSUM chart in this study is more robust than the design of $\bar{X}$chart of Chou et al. [9] in terms of sampling interval.

Table 5 gives the optimal control limit L for various combinations of skewness coefficient α_3, kurtosis coefficient α_4, and correlation coefficient ρ. When the data are negatively correlated, an increase in skewness coefficient α_3 from negative to positive (Group I and III) or an increase in skewness coefficient α_3 (Group IV) gives a larger control limit. For example,

Table 3: Optimal values for sample size (n) for various combinations of skewness coefficient α_3, kurtosis coefficient α_4 with corresponding combinations of Burr parameters (c, q) and correlation coefficient ρ

(c, q)	α_3	α_4	ρ					Note
			-0.8	-0.4	0	0.4	0.8	
(6, 11)	-0.254	3.027	2	3	5	2	2	Group I
(5, 5)	0.040	3.070	2	3	6	2	2	α_3: from − to +
(3, 11)	0.329	3.006	2	3	6	2	2	α_4: close to normal
(4, 11)	0.050	2.866	2	3	6	2	2	Group II
(6, 4)	-0.019	3.169	2	3	6	2	2	α_3: close to normal
(10, 2)	0.044	3.646	2	3	5	2	2	α_4: increasing
(10, 7)	-0.465	3.430	2	3	5	2	2	Group III
(10, 3)	-0.208	3.418	2	3	5	2	2	α_3: from − to +
(5, 3)	0.277	3.485	2	3	5	2	2	α_4: near a constant
(3, 6)	0.484	3.380	2	3	6	2	2	
(6, 2)	0.434	4.106	2	3	4	2	2	Group IV
(5, 2)	0.635	4.630	2	3	5	2	2	α_3: increasing
(2, 10)	0.884	4.122	2	3	5	2	2	α_4: near a constant,
(2, 7)	1.014	4.707	2	3	5	2	2	and >4.0
(2, 8)	0.958	4.443	2	3	4	2	2	Group V
(2, 6)	1.094	5.118	2	3	5	2	2	α_3: close to one
(9, 1)	1.060	7.215	2	3	5	2	2	α_4: increasng
(2, 4)	1.432	7.356	2	3	5	2	2	Group VI
(2, 3)	1.909	12.460	2	3	4	2	2	α_3: increasing, >>0
(1, 9)	2.940	19.760	2	3	4	1	2	α_4: increasing, >>0
(1, 6)	3.810	38.670	2	3	4	2	2	

the control limit L increases from 0.88 to 1.24 when the skewness coefficient increases from 0.434 to 1.014 for correlation coefficient ρ = -0.8 where the kurtosis coefficient is near a constant and more than 4.0 (Group IV). If the data are positively correlated, an increase in skewness coefficient from negative to positive (Group I and III) gives a larger control limit. For example, the control limit L increases from 2.32 to 4.89 for correlation coefficient ρ = 0.4 when the skewness coefficient changes from -0.465 to 0.484 and kurtosis coefficient is near a constant (Group III). Therefore, the results from this study show that if the data are correlated, the control limit is not significantly affected by the correlation coefficient; whereas the results of Chou et al. [9] show that the correlation coefficient significantly affects the control limit of the $\bar{X}$ chart when the data are positively and negatively correlated. Therefore, the control limit for CUSUM chart in this study is more robust than the control limit of $\bar{X}$ chart of Chou et al. [9] in terms of correlation coefficient. Table 6 gives the optimal reference value k for various combinations of skewness coefficient α_3, kurtosis coefficient α_4, and correlation coefficient ρ. From this table, it can be noticed that if the data are correlated, the reference value is not significantly affected by the correlation coefficient. The reference value is also not significantly affected by the skewness and kurtosis coefficients if the data are correlated.

8 Conclusion

The economic design of the CUSUM chart for non-normally correlated data is presented based on the Lorenzen and Vance's cost model. The cost model is employed with the Burr distribution and Yang and Hancock's correlation model. Appling GA to the cost model, the design parameters are optimally determined such that the expected cost per hour is minimized. The magnitude of shift has a significant effect on the optimal design parameters of the CUSUM chart. According to the sensitivity analysis in this study, the following conclusions can be drawn:

(i) If the data are negatively correlated, higher negative correlation coefficient gives a smaller sample size. If the data are positively correlated, the positive correlation coefficient does not significantly affect the sample size. The non-normality does not give significant effect on the sample size.

(ii) Higher positively correlated data give a slightly shorter sampling interval. If the data are negatively correlated, the sampling interval is not significantly affected by correlation. If the data are negatively correlated, a change from positive value to negative value in the skewness coefficient results in a shorter sampling interval. If the data are positively correlated, there is no specific effect of the skewness and kurtosis coefficients on the sampling interval.

(iii) If the data are correlated, the control limit is not significantly affected by the correlation coefficient. When the data are negatively correlated, an increase in skewness coefficient from negative to positive or an increase in skewness coefficient gives a larger control limit. If the data are positively correlated, an increase in skewness coefficient from negative to positive gives a larger control limit.

(iv) The reference value is not significantly affected by non-normality and correlation.

Table 4: Optimal values for sampling interval (h) for various combinations of skewness coefficient α_3, kurtosis coefficient α_4 with corresponding combinations of Burr parameters (c, q) and correlation coefficient ρ

(c, q)	α_3	α_4	ρ					Note
			-0.8	-0.4	0	0.4	0.8	
(6, 11)	-0.254	3.027	0.80	0.92	0.80	0.46	0.43	Group I
(5, 5)	0.040	3.070	0.76	0.90	0.85	0.45	0.42	α_3: from − to +
(3, 11)	0.329	3.006	0.74	0.88	0.82	0.44	0.42	α_4: close to normal
(4, 11)	0.050	2.866	0.76	0.90	0.85	0.45	0.42	Group II
(6, 4)	-0.019	3.169	0.77	0.90	0.85	0.45	0.43	α_3: close to normal
(10, 2)	0.044	3.646	0.76	0.89	0.76	0.45	0.43	α_4: increasing
(10, 7)	-0.465	3.430	0.81	0.93	0.83	0.47	0.44	Group III
(10, 3)	-0.208	3.418	0.78	0.91	0.79	0.45	0.43	α_3: from − to +
(5, 3)	0.277	3.485	0.74	0.88	0.74	0.44	0.42	α_4: near a constant
(3, 6)	0.484	3.380	0.72	0.87	0.80	0.44	0.42	
(6, 2)	0.434	4.106	0.73	0.87	0.65	0.44	0.43	Group IV
(5, 2)	0.635	4.630	0.71	0.86	0.72	0.44	0.43	α_3: increasing
(2, 10)	0.884	4.122	0.68	0.84	0.70	0.44	0.42	α_4: near a constant,
(2, 7)	1.014	4.707	0.68	0.83	0.69	0.44	0.42	and >4.0
(2, 8)	0.958	4.443	0.68	0.83	0.62	0.44	0.42	Group V
(2, 6)	1.094	5.118	0.67	0.83	0.69	0.44	0.42	α_3: close to one
(9, 1)	1.060	7.215	0.68	0.86	0.71	0.45	0.43	α_4: increasing
(2, 4)	1.432	7.356	0.66	0.82	0.69	0.44	0.43	Group VI
(2, 3)	1.909	12.460	0.65	0.81	0.62	0.45	0.39	α_3: increasing, >>0
(1, 9)	2.940	19.760	0.60	0.73	0.61	0.40	0.45	α_4: increasing, >>0
(1, 6)	3.810	38.670	0.67	1.00	0.62	0.47	0.46	

Table 5: Optimal values for control limit (L) for various combinations of skewness coefficient α_3, kurtosis coefficient α_4 with corresponding combinations of Burr parameters (c, q) and correlation coefficient ρ

(c, q)	α_3	α_4	ρ					Note
			-0.8	-0.4	0	0.4	0.8	
(6, 11)	-0.254	3.027	0.50	0.44	1.37	2.78	3.60	Group I
(5, 5)	0.040	3.070	0.67	0.55	1.75	3.83	4.84	α_3: from − to +
(3, 11)	0.329	3.006	0.79	0.61	2.12	4.51	5.00	α_4: close to normal
(4, 11)	0.050	2.866	0.63	0.52	1.69	3.78	4.74	Group II
(6, 4)	-0.019	3.169	0.65	0.55	1.69	3.71	4.66	α_3: close to normal
(10, 2)	0.044	3.646	0.72	0.62	2.12	4.05	4.95	α_4: increasing
(10, 7)	-0.465	3.430	0.45	0.40	1.20	2.32	2.92	Group III
(10, 3)	-0.208	3.418	0.60	0.52	1.67	3.38	4.15	α_3: from − to +
(5, 3)	0.277	3.485	0.80	0.64	2.39	4.49	5.00	α_4: near a constant
(3, 6)	0.484	3.380	0.89	0.67	2.41	4.89	5.00	
(6, 2)	0.434	4.106	0.88	0.68	2.93	4.79	5.00	Group IV
(5, 2)	0.635	4.630	0.98	0.72	2.92	5.00	5.00	α_3: increasing
(2, 10)	0.884	4.122	1.15	0.76	3.25	5.00	5.00	α_4: near a constant,
(2, 7)	1.014	4.707	1.24	0.80	3.41	5.00	5.00	and >4.0
(2, 8)	0.958	4.443	1.20	0.79	3.61	5.00	5.00	Group V
(2, 6)	1.094	5.118	1.29	0.81	3.48	5.00	5.00	α_3: close to one
(9, 1)	1.060	7.215	1.27	0.74	3.30	5.00	5.00	α_4: increasng
(2, 4)	1.432	7.356	1.45	0.86	3.67	5.00	5.00	Group VI
(2, 3)	1.909	12.460	1.57	0.86	4.11	5.00	5.00	α_3: increasing, >>0
(1, 9)	2.940	19.760	1.72	1.94	4.77	5.00	5.00	α_4: increasing, >>0
(1, 6)	3.810	38.670	1.71	0.35	4.62	5.00	5.00	

Table 6: Optimal values for reference value (k) for various combinations of skewness coefficient α_3, kurtosis coefficient α_4 with corresponding combinations of Burr parameters (c, q) and correlation coefficient ρ

(c, q)	α_3	α_4	ρ					Note
			-0.8	-0.4	0	0.4	0.8	
(6, 11)	-0.254	3.027	0.70	0.82	1.17	0.72	0.66	Group I
(5, 5)	0.040	3.070	0.65	0.81	1.09	0.51	0.45	α_3: from − to +
(3, 11)	0.329	3.006	0.62	0.82	1.00	0.41	0.43	α_4: close to normal
(4, 11)	0.050	2.866	0.67	0.82	1.13	0.52	0.47	Group II
(6, 4)	-0.019	3.169	0.66	0.81	1.11	0.53	0.47	α_3: close to normal
(10, 2)	0.044	3.646	0.64	0.80	0.91	0.46	0.42	α_4: increasing
(10, 7)	-0.465	3.430	0.71	0.81	1.21	0.81	0.79	Group III
(10, 3)	-0.208	3.418	0.67	0.81	1.04	0.58	0.55	α_3: from − to +
(5, 3)	0.277	3.485	0.61	0.80	0.84	0.40	0.41	α_4: near a constant
(3, 6)	0.484	3.380	0.59	0.80	0.91	0.35	0.42	
(6, 2)	0.434	4.106	0.58	0.80	0.65	0.35	0.40	Group IV
(5, 2)	0.635	4.630	0.55	0.79	0.70	0.32	0.39	α_3: increasing
(2, 10)	0.884	4.122	0.52	0.80	0.66	0.33	0.40	α_4: near a constant,
(2, 7)	1.014	4.707	0.49	0.79	0.63	0.32	0.38	and >4.0
(2, 8)	0.958	4.443	0.50	0.79	0.54	0.32	0.39	Group V
(2, 6)	1.094	5.118	0.48	0.79	0.61	0.31	0.37	α_3: close to one
(9, 1)	1.060	7.215	0.46	0.80	0.60	0.29	0.35	α_4: increasing
(2, 4)	1.432	7.356	0.43	0.78	0.55	0.28	0.34	Group VI
(2, 3)	1.909	12.460	0.38	0.79	0.41	0.25	0.31	α_3: increasing, >>0
(1, 9)	2.940	19.760	0.31	0.40	0.29	0.04	0.20	α_4: increasing, >>0
(1, 6)	3.810	38.670	0.29	1.02	0.27	0.15	0.17	

The conclusions above show that correlation gives significant effect on the sample size and sampling interval but correlation does not give significant effect on the control limit and reference value; whereas non-normality gives significant effect on the sampling interval and control limit but non-normality does not give significant effect on the sample size and reference value. In general, both non-normality and correlation affect the economic design of the CUSUM chart. Therefore, estimation of non-normality and correlation coefficient is an important topic on the economic design of the CUSUM chart.

References

[1] Duncan, A. J. The economic design of $\bar{X}$ charts used to maintain current control of a process. *Journal of the American Statistical Association.* 1956. 51: 228−242.

[2] Taylor, H. M. The economic design of cumulative sum control charts. *Technometrics.* 1968. 10: 479−488.

[3] Goel, A. L. and Wu, S. M. Economically optimum design of CUSUM charts. *Management Science.* 1973. 19: 1271−1282.

[4] Chiu, W. K. The economic design of CUSUM charts for controlling normal means. *Applied Statisitics.* 1974. 23: 420−433.

[5] Chung, K. J. Economically optimal determination of the parameters of CUSUM charts (cumulative sum control chart). *International Journal of Quality and Reliability Management.* 1992. 9: 8−17.

[6] Simpson, J. R. and Keats, J. B. Sensitivity study of the CUSUM control chart with an economic model. *International Journal of Production Economics.* 1995. 40: 1−19.

[7] Pan, J. N. and Chen, S. T. The economic design of CUSUM chart for monitoring environmental performance. *Asia Pacific Management Review.* 2005. 10: 155−161.

[8] Nenes, G. and Tagaras, G. The economically designed CUSUM chart for monitoring short production runs. *International Journal of Production Research.* 2006. 44: 1569−1587.

[9] Chou, C. Y., Chen C. H. and Liu, H. R. Economic design of $\bar{X}$ charts for non-normally correlated data. *International Journal of Production Research.* 2001. 39: 1931−1941.

[10] Lashkari, R. S. and Rahim, M. A. An Economic design of cumulative sum charts to control non-normal process means. *Computers and Industrial Engineering.* 1982. 6: 1−18.

[11] Haridy, A. M. A and El-Shabrawy, A. Z. The economic design of cumulative sum charts used to maintain current control of non-normal process means. *Computers and Industrial Engineering.* 1996. 31: 783−790.

[12] Rahim, M. A. Economic model of $\bar{X}$-chart under non-normality and measurement errors. *Computers and Operations Research,* 1985. 12: 291−299.

[13] Burr, I. W. Cumulative frequency distribution. *Annals of Mathematical Statistics.* 1942. 13: 215−232.

[14] Chou, C. Y., Chen, C. H. and Liu, H. R. Economic-statistical design of $\bar{X}$ charts for non-normal data by considering quality loss. *Journal of Applied Statistics.* 2000. 27: 939−951.

[15] Chen, Y. K. An evolutionary economic-statistical design of VSI $\bar{X}$ control charts under non-normality. *International Journal of Advanced Manufacturing Technology.* 2003. 22: 602−610.

[16] Chou, C. Y., Chen, C. H. and Liu, H. R. Effects of non-normality on the economic design of warning limit $\bar{X}$ charts. *Quality Engineering.* 2004. 16: 567−575.

[17] Chen, Y. K. Economic design of $\bar{X}$ control charts for non-normal data using variable sampling policy. *International Journal of Production Economics.* 2004. 92: 61−74.

[18] Chou, C. Y., Chen, C. H. and Liu, H. R. Acceptance control charts for non-normal data. *Journal of Applied Statistics.* 2005. 32: 25−36.

[19] Chen, F. L. and Yeh, C. H. Economic design of control charts with Burr distribution for non-normal data under Weibull failure mechanism. *Journal of the Chinese Institute of Industrial Engineers.* 2006. 23: 200−206.

[20] Banerjee, P. K. and Rahim, M. A. Economic design of $\bar{X}$ control charts under Weibull shock models. *Technometrics.* 1988. 30: 407−414.

[21] Grant, E.L. and Leavenworth, R. S. *Statistical Quality Control.* London: McGraw-Hill. 1988.

[22] Neuhardt, J. B. Effects of correlated sub-samples in statistical process control. *IIE Transactions.* 1987. 19: 208−214.

[23] Yang, K. and Hancock, W. M. Statistical quality control for correlated samples. *International Journal of Production Research.* 1990. 28: 595−608.

[24] Liu, H. R., Chou, C. Y. and Chen, C. H. Minimum-loss design of $\bar{X}$ chart for correlated data. *Journal of Loss Prevention in the Process Industries.* 2002, 15: 405−411.

[25] Liu, H. R., Chou, C. Y. and Chen, C. H. The effect of correlation on the economic design of warning limit $\bar{X}$ charts. *International Journal of Advanced Manufacturing Technology.* 2003. 22: 306−312.

[26] Gordon, G. R.and Weindling, J. I. A cost model for economic design of warning limit control chart schemes. *AIIE Transactions,* 1975. 7: 166−171.

[27] Chen, Y. K. and Chiou, K. C. Optimal Ddesign of VSI $\bar{X}$ control charts for monitoring correlated samples. *Quality and Reliability Engineering International.* 2005. 21: 757−768.

[28] Bai, D. S. and Lee, H. T. An economic design of variable sampling interval $\bar{X}$ control charts. *International Journal of Production Economics,* 1998. 54: 57−64.

[29] Chen, Y. K., Hsieh, K. L. and Chang, C. C. Economic design of VSSI $\bar{X}$ control charts for correlated data. *International Journal of Production Economics.* 2007. 107: 528−539.

[30] Aparisi, F. and García-Díaz, J. C. Design and optimization of EWMA control charts for in-control, indifference, and out-of-control regions. *Computers and Operations Research.* 2007. 34: 2096−2108.

[31] Chen, Y. K. Economic design of variable sampling interval T^2 control charts – a hybrid Markov chain approach with genetic algorithms. *Expert Systems with Applications.* 2007. 33: 693−689.

[32] Chou, C. Y., Cheng, J. C. and Lai, W. T. Economic design of variable sampling intervals EWMA charts with sampling at fixed times using genetic algorithms. *Expert Systems with Applications.* 2008. 34: 419−426.

[33] Lin, H. H., Chou, C. Y. and Lai, W. T. Economic design of variable sampling interval $\bar{X}$ charts with A&L switching rule using genetic algorithms. *Expert Systems with Applications.* 2009. 36: 3048−3055.

[34] Chen, Y. K. Economic design of T^2 control charts with the VSSI sampling scheme. *Quality and Quantity.* 2009. 43: 109−122.

[35] Kaya, I. A genetic algorithm approach to determine the sample size for attribute control charts. *Information Sciences.* 2009. 179: 1552-1566.

[36] Torng, C. C., Lee, P. H., Liao, H. S. and Liao, N. Y. An economic design of double sampling $\bar{X}$ charts for correlated data using genetic algorithms. *Expert Systems with Applications.* 2009. 36: 12621-12626.

[37] Page, E. S. Continuous inspection schemes. *Biometrika.* 1954. 41: 100−115.

[38] Lorenzen, T. J. and Vance, L. C. The economic design of control charts: a unified approach. *Technometrics.* 1986. 28: 3−10.

[39] Prabhu, S. S., Runger, G. C. and Montgomery, D. C. Selection of the subgroup size and sampling interval for a CUSUM control chart. *IIE Transactions,* 1997. 29: 451−457