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Universal Portfolio using the Best Current-Run Parameter

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Abstract The mixture-current-run universal portfolio introduced by Tan and Lim can extract the best daily wealth when it is implemented on different types of universal portfolios like the Helmbold and chi-square divergence universal portfolios. In this paper, we demonstrate another application of the mixture-current-run universal portfolio in extracting the best daily wealth due to the best parameter from the same parametric family of universal portfolios. The range of the parameter is discretized and the mixture-current-run universal portfolio is implemented on the different universal portfolios corresponding to the discretized parameters. Empirically, we demonstrate the performance of the portfolio by running it on some selected stock-data sets from the local stock exchange. The problem of selecting the best parameter at the outset is avoided by using the best current-run parameter in daily trading of the market and at the same time the best daily wealth is added to the accumulated wealth.

Keywords Investment Wealth; Mixture-Current-Run Universal Portfolio; Best Parameter.

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1 Introduction

Universal portfolios adopt the approach that does not depend on the stochastic model underlying the true distribution of the stock prices. Mixing two or more universal portfolios can extract the advantages of each portfolio to be exploited in a single mixture portfolio. The mixture-current-run (MCR) universal portfolio introduced by Tan and Lim [1] follows the current run of the portfolio that achieves the best single-day wealth return. In this paper, we study the application of the MCR universal portfolio in mixing two or more universal portfolios of the same type to estimate the best-performing parameter corresponding to the run of the best daily wealth. In [2,3], we know that the parameters η in the Helmbold universal portfolio and ξ in the chi-square divergence (CSD) universal portfolio are important factors influencing the performance of the universal portfolios. An improper choice of η or ξ may lead to a lower investment wealth. It is a difficult task to choose a good parameter at the beginning of the investment period. The MCR universal portfolio can be applied to deal with the above difficulty by mixing two or more universal portfolios of the same type with different values of the scalar parameter. A pre-defined range of the parameter is discretized and the MCR universal portfolio is applied by mixing the two or more universal portfolios of the same type with the discretized parameters. We can identify the best parameter among the discretized parameters by keeping track of their performance daily. The current parameter generating the run of the best single-day wealth return is estimated to be the best current-run parameter.

2 Application of the Mixture-Current-Run Universal Portfolio

Considering investment in a market of *m* stocks, a portfolio vector $\mathbf{b}_n = (b_{n1}, b_{n2}, ..., b_{nm})$ on the *n*th trading day is a vector satisfying $b_{ni} \geq 0$ for *i* = 1, 2 . . . , *m* and $\sum_{i=1}^{m} b_{ni} = 1$. The component b_{ni} is the proportion of the current wealth invested in stock i for i = 1, 2 . . . , m. Let $\mathbf{x}_n = (x_{n1}, x_{n2}, ..., x_{nm})$ denotes the price-relative vector on the nth trading day where x_{ni} is the ratio of the closing price of the ith stock to its opening price on the nth trading day for i = 1, 2 . . . , m. The wealth achieved at the end of the nth trading day is

$$S_n = \prod_{j=1}^{n} \mathbf{b}_j^t \mathbf{x}_j$$

where the initial wealth S_0 is assumed to be one unit. The single-day wealth on day j is

$$\mathbf{b}_j^t \mathbf{x}_j = \sum_{i=1}^{m} b_{ji} x_{ji} \ .$$

for j = 1, 2, . . . , n. The constant rebalanced portfolio investment strategy uses the same portfolio vector **b** for each trading day. The best constant rebalanced portfolio (BCRP) maximizes the wealth S1975(**b**) over all constant rebalanced portfolio **b**. The optimal wealth achieved by the BCRP is defined as

$$S_n^* = \max_b S_n(\mathbf{b}) = \max_b \prod_{j=1}^{n} \mathbf{b}_j^i \mathbf{x}_j \ .$$

We denote the BCRP by $\mathbf{b}^*$ where

$$S_n(\mathbf{b}^*) = \max_b S_n(\mathbf{b}) \ .$$

In this paper, we focus our study on two special universal portfolios, namely, the multiplicative Helmbold universal portfolio in [2], and the additive chi-square divergence (CSD) universal portfolio in [3]. The Helmbold universal portfolio is defined by:

$$b_{n+1,i(\eta)} = \frac{b_{ni} \exp\left(\eta \dfrac{x_{ni}}{\mathbf{b}_n^t \mathbf{x}_n} \right)}{\sum_{j=1}^{m} b_{nj} \exp\left(\eta \dfrac{x_{nj}}{\mathbf{b}_n^t \mathbf{x}_n} \right)}$$

for i = 1, 2, . . . , m, where the parameter η is a real number and the initial starting portfolio b_1 is given. The CSD universal portfolio is defined by:

$$b_{n+1,i}(\xi) = b_{ni} \left[\frac{\xi(x_{ni} - \mathbf{b}_n^t \mathbf{x}_n}{\mathbf{b}_n^t \mathbf{x}_n} + 1 \right]$$

for i = 1, 2, . . . , m, where the parameter ξ is any chosen real number such that $b_{ni} \geq 0$ for i = 1, 2, . . . , m and the initial starting portfolio $\mathbf{b}_1$ is given.

The portfolio

$$b_n = \sum_{i=1}^{k} p_{ni} \mathbf{b}_n^i$$

is a mixture of the k universal portfolios, $b_n^1, b_n^2, ..., b_n^k$ if the weights { p_{ni} } where $p_{ni} \geq 0$ for i = 1, 2, . . . , k and $\sum_{i=1}^{m} p_{ni} = 1$ for i = 1, 2, 3, . . . , are chosen according to some decision rule. From [1], the mixture-current-run (MCR) universal portfolio on different types of universal portfolios is a mixture universal portfolio where the weights { p_{ni} } are chosen according to the rule that given in $\mathbf{x}_n$,

$$p_{n+1,i} = 1 \text{ if } \max_l \{(\mathbf{b}_n^l)^t \mathbf{x}_n\} = (\mathbf{b}_n^i)^t \mathbf{x}_n$$

where l = 1, 2, . . . , k and

$$p_{n+1,i} = 0 \quad \text{for all } l \neq i .$$

The estimation of the best-performing parameter within a pre-defined range corresponding to the run of the best daily wealth can be done by using the application of the MCR universal portfolio in mixing two or more universal portfolios of the same type. Consider a universal portfolio with the parameter θ defined within a certain range of values, say $\theta_1 \leq \theta \leq \theta_k$. We can form k universal portfolios of the same type using k different values of the same scalar parameter, that is $\mathbf{b}_n^l = \mathbf{b}_n(\theta_l)$ where l = 1, 2, . . . , k. The k different values of the same scalar parameter can be obtained by discretizing the range [θ_1, θ_k] by

$$\theta_i = \theta_{i-1} + \frac{\theta_k - \theta_1}{k-1} \quad (1)$$

for i = 2, 3, . . . , k. The portfolio with parameter θ_i creates a run on days n, n+1, . . . , n+s in the mixture if

$$\max_l \{(\mathbf{b}_{n+r}(\theta_l))^t \mathbf{x}_{n+r}\} = (\mathbf{b}_{n+r}(\theta_i))^t \mathbf{x}_{n+r} \quad \text{for } r = 0, 1, 2, \ldots, s$$

for some positive integers r, s. The run of the portfolio with parameter θ_i is terminated when there exists a smallest integer $u > s$ such that

$$\max_l \{(\mathbf{b}_{n+u}(\theta_l))^t \mathbf{x}_{n+u}\} = (\mathbf{b}_{n+u}(\theta_j))^t \mathbf{x}_{n+u}$$

where $j \neq i$. The current parameter generating the run of the best single-day wealth is estimated to be the best current-run parameter. The universal portfolio using the best current-run parameter follows the run of the portfolio with parameter θ_i, that is

$$\mathbf{b}_{n+v} = \mathbf{b}_{n+v}(\theta_i) \quad \text{for } v = 1, 2, \ldots, u$$

for days n + 1, n + 2, . . . , n + u and changes to the run of the portfolio with parameter θ_j on day n + u + 1, that is

$$\mathbf{b}_{n+u+1} = \mathbf{b}_{n+u+1}(\theta_j) \quad \text{where } j \neq i .$$

In other words, the universal portfolio using the best current-run parameter on day $n + 1$ is $\mathbf{b}_{n+1} = \mathbf{b}_{n+1}(\theta_i)$ if the portfolio with parameter θ_i, namely $\mathbf{b}_n(\theta_i)$ achieves the maximum single-day wealth on day n.

3 Empirical Results

Four stock-price data sets D, E, F and G are selected from the Kuala Lumpur Stock Exchange for this study. There is a total of 1975 trading days which covers the period of 2 January 2003 until 30 December 2010. Each data set consists of ten companies and listed in Table 1. We run the Helmbold universal portfolio, Helmbold universal portfolio using the best current-run parameter, chi-square divergence (CSD) universal portfolio and CSD universal portfolio using the best current-run parameter on data sets D, E, F and G with the uniform initial starting portfolio b_1 = (0.1000, 0.1000, . . . , 0.1000).

Table 1 List of Companies in the Data Sets *D*, *E*, *F* and *G*

Set D	Set E	Set F	Set G
YTL Corporation	YTL Corporation	YTL Power International	Malaysian Airline System
UMW Holdings	UMW Holdings	PPB Group	Hong Leong Financial Group
MMC Corporation	MMC Corporation	Petronas Dagangan	IOI Corporation
YTL Power International	YTL Power International	Digi.com	YTL Power International
PPB Group	PPB Group	Hong Leong Financial Group	Kuala Lumpur Kepong
Petronas Dagangan	Petronas Dagangan	Malaysian Airline System	Petronas Dagangan
Digi.com	Digi.com	Kuala Lumpur Kepong	MMC Corporation
Malayan Banking	Hong Leong Financial Group	PLUS Expressway	PPB Group
Malaysian Airline System	Malaysian Airline System	IOI Corporation	Digi.com
Kuala Lumpur Kepong	IOI Corporation	Sime Darby	Sime Darby

From Table 2, the maximum value of S1975 achieved by the Helmbold universal portfolios for data sets D, E, F and G are S_{1975} = 18.2486 at η = 0.4138, S_{1975} = 22.9859 at η = −2.3639, S_{1975} = 15.7558 at η = −9.4444, and S_{1975} = 19.9357 at η = −83.1143 respectively. In this paper, we focus our study on the range of the parameters [−10, 10]. The wealth S_{1975} achieved by the Helmbold universal portfolio using the best current-run parameter, where η_1 = −10, η_k = 10 and k = 21, for data sets d, E, F and G are 26.6274, 27.6025, 22.4733, and 29.6287 respectively. The Helmbold universal portfolios using the best current-run parameter outperform the Helmbold universal portfolios with best parameter for all the four data sets.

Four data sets D, E, F and G, the valid intervals for the parameter such that $b_{ni} \geq 0$ for all i = 1, 2, . . . ,m are [−5.1308, 4.6077], [−5.1203, 4.6078], [−4.9553, 5.3162], and [−5.1030, 4.7028] respectively in Table 2. The CSD universal portfolios achieve the maximum wealth of S_{1975} = 18.2431, 29.1040, 22.3262, and 25.5834 at ξ = 0.3769, −2.8760, −4.9553, and −3.7942 respectively for the corresponding data sets *D*, *E*, *F* and *G*. The corresponding wealth S_{1975} achieved by the CSD universal portfolio using the best current-run parameter, where $\xi_1 = -10$, $\xi_k = 10$ and k = 21, are 29.6337, 31.9852, 25.0480, and 43.4262 respectively which are much higher that the maximum wealth achieved by the CSD universal portfolios.

The wealth achieved by the Helmbold universal portfolio using the best current-run parameter and the CSD universal portfolio using the best current-run parameter in Table 2 do not exceed the wealth achieved by the BCRP for data sets *D* and *E*. On the other two data sets *F* and *G*, the Helmbold universal portfolio using the best current-run parameter and the CSD universal portfolio using the best current-run parameter in Table 2 outperform the BCRP. For data set *F*, the Helmbold universal portfolio using the best current-run parameter and the CSD universal portfolio using the best current-run parameter outperform the BCRP by 1.7564 units of wealth achieved and 4.3311 units of wealth achieved respectively. For data set *G*, the Helmbold universal portfolio using the best current-run parameter and the CSD universal portfolio using the best current-run parameter outperform the BCRP by 4.9906 units of wealth achieved and 18.7881 units of wealth achieved respectively.

Table 2 The maximum wealth achieved by the Helmbold and CSD universal portfolios with respective best parameter and the valid interval for its parameter, the wealth achieved by the Helmbold and CSD universal portfolios using the best current-run parameter with the pre-defined range and the wealth achieved by the BCRP after 1975 trading days for data sets *D*, *E*, *F* and *G*

Set D			
Type	**Valid Interval**	**Parameter**	S_{1975}
Helmbold	$(-\infty, \infty)$	0.4138	18.2486
Helmbold (best current-run parameter)		$\eta_1 = -10$ $\eta_k = 10$ $k = 21$	26.6274
CSD	[−5.1308, 4.6077]	0.3769	18.2431
CSD (best current-run parameter)		$\xi_1 = -10$ $\xi_k = 10$ $k = 21$	29.6337
BCRP	−	−	37.5867
Set E			
Type	**Valid Interval**	**Parameter**	S_{1975}
Helmbold	$(-\infty, \infty)$	−2.3639	22.9859
Helmbold (best current-run parameter)		$\eta_1 = -10$ $\eta_k = 10$ $k = 21$	27.6025
CSD	[−5.1203, 4.6078]	−2.8760	29.1040
CSD (best current-run parameter)		$\xi_1 = -10$ $\xi_k = 10$ $k = 21$	31.9852
BCRP	−	−	37.5867
Set F			
Type	**Valid Interval**	**Parameter**	S_{1975}
Helmbold	$(-\infty, \infty)$	−9.4444	15.7558
Helmbold (best current-run parameter)		$\eta_1 = -10$ $\eta_k = 10$ $k = 21$	22.4733
CSD	[−4.9553, 5.3162]	−4.9553	22.3262

CSD (best current-run parameter)		$\xi_1 = -10$ $\xi_k = 10$ $k = 21$	25.0480
BCRP	–	–	20.7169
Set G			
Type	**Valid Interval**	**Parameter**	S_{1975}
Helmbold	$(-\infty, \infty)$	−83.1143	19.9357
Helmbold (best current-run parameter)		$\eta_1 = -10$ $\eta_k = 10$ $k = 21$	29.6287
CSD	[−5.1030, 4.7028]	−3.7942	25.5834
CSD (best current-run parameter)		$\xi_1 = -10$ $\xi_k = 10$ $k = 21$	43.4262
BCRP	–	–	24.6381

4 Conclusion

In conclusion, the empirical study shows that the universal portfolio using the best current-run parameter seem to outperform the original universal portfolio that using the best parameter from hindsight. There are also exists some universal portfolios using the best current-run parameter outperform the BCRP. Although the valid intervals for the parameter of the CSD universal portfolio are restricted, for example, [−5.1308, 4.6077] for data set *D*, we still able to run the application of MCR on the bigger interval of parameter, that is [−10, 10] for this study. The parameter $\xi^{\circ} \notin$ [−5.1308, 4.6077] will only generate the portfolio with component $b_{ni} < 0$ after some stage n° where *n* is the trading day. Then the portfolios generated by ξ° after the trading day n° are disposed and the parameter ξ° is excluded from the consideration of the current-run parameter. As long as there is a valid parameteris one of the discretized parameter in (1), then the CSD universal portfolio using the best current-run parameter always generate the genuine portfolio which is $b_{ni} \geq 0$ for all $i - 1, 2, \ldots,$ m and $\sum_{i=1}^{m} b_{ni} = 1$ for all $n = 1, 2, 3, \ldots$. Selecting the best parameter at the outset is avoided by using the best current-run parameter.

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