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Analysis of Corporate Control: How Good is the Straffin Index?

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Abstract In this paper, Straffin index as a viable voting power index, particularly in corporate analysis is described. Voting power distribution anomaly, practical limitation and lack of clear meanings are listed as possible factors hindering the application of the index. More importantly, this paper introduces ways to determine voter affiliations, two possible lines of enquiry into the meaning behind this index and speculates as to the possible consequences.

Keywords Corporate Analysis; Voting Power; Banzhaf; Shapley-Shubik; Straffin Index.

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1 Introduction

Most studies in corporate analysis fail to recognise the importance of voting power index and if they do, the choices invariably settle to a few established indexes, mostly Banzhaf or Shapley-Shubik index. Das [1] wrote in page 1, "Despite the importance of the field, it is a subject that is not studied widely enough...". This paper helps to address this matter and in particular seeks to explore the potential of Straffin index as a viable analysis technique from a corporate view point. In this respect, this paper contributes to the literature on corporate control in two ways. First, to the authors' knowledge, since the index was first introduced, very few if any have attempted to explore the index from the corporate perspective. Second, and most importantly, this paper a) suggests a framework for determining voter's affiliation b) proposes lines of investigation into the meanings behind the Straffin index and speculates as to the possible consequences.

This article opens with discussion and illustration on the voting power concept and the related indices. Next, we illustrate the Straffin index. We then lay down the framework to determine voter's degree of affiliation before proceeding to highlight the challenges that one faces in trying to apply the index. Finally, this paper proposes possible solutions to one of the challenges i.e. lack of meaning, and contemplates the possible ramifications before closing with the discussion and conclusion.

2 Voting Power Concept

The voting power concept is a field of cooperative game theory. It is widely applied in political studies but infrequently in corporate analysis. The basic idea originates from the probability concept. The greater is the chance of being able to influence policy, the higher is the voting power. This probability is derived from shareholdings distribution and the rules governing the passage of resolutions. Power is conceived as a priori concept.

In 1946, Penrose [2] pioneered the concept and the index. The index defines power as being the number of times a shareholder is deemed important i.e. pivotal over the total number of coalitions involving that shareholder. In this regard, a pivot is defined as when withdrawal turns a winning into a losing coalition. The following section calculates the index.

Assume a company with the following shareholders: 'A' - 43 percent, 'B' - 48 percent and 'C' - 9 percent and a simple majority of more than 50 percent for the passage of resolutions is the rule. Table 1 lists the winning coalitions and important shareholders. In summary, a shareholder is pivotal twice and since a shareholder is involved in four coalitions i.e. 2^{n-1}, the Penrose index is 'A'= 2/4 or 0.50, 'B'= 2/4 or 0.50 and 'C'= 2/4 or 0.50.

Table 1 Winning Coalitions and important shareholders (underlined)

			Total
A	B		92%
A	C		52%
B	C		57%
C	B	A	100%

Note: 'A' –43.0%; 'B' – 48.0%; 'C' – 9.0%. Majority of 50 percent plus 1 share is required to win an election.

Table 2 Voting Power Index under Different Distributions

	Distribution 1		Distribution 2	
Shareholder	Ownership %	Voting power [1]	Ownership %	Voting Power [1]
A	44.0	0.50	21.0	0.25
B	47.0	0.50	47.0	0.75
C	9.0	0.50	21.0	0.25
D	-	-	11.0	0.25

Assume now that a new 'A' has sold some of the shares to 'C' and a new investor 'D' resulting in them having 21 percent, 21 percent and 11 percent share respectively. These generate a totally different voting power index, as summarised in Table 2.

Table 2 elucidates the non-monotonic relationships between shareholding sizes and voting power. In distribution 1, 'C' and 'B' share a similar level of voting power, despite the latter being four times bigger. Increases in size do not warrant an increase in voting power. Going from distributions 1 to 2, 'C', despite more than doubling in size, suffers reduction in voting power from 0.5 to 0.25. On the other hand, 'B', despite maintaining similar shareholding size, increases its voting power from 0.5 to 0.75. As for 'D', the 11 percent shareholding provides a similar level of voting power as 'A' and 'C', despite being almost half the size.

The previous exercise illustrates the volatile link between shareholding size and voting power. In the real corporate world, the number of shareholders can reach a few thousand, multiplying the volatility. As a result, a small change in the scenarios may alter the entire landscape of the voting power. To determine one's own voting power can be problematical, never mind determining that of the other voters. Consequently, voting power uncertainty complicates a shareholders voting strategy in company meetings. All voting power indices essentially share these characteristics.

The concept can also be presented in mathematical notation. A shareholder is identified as i and a company with n shareholders; represented by a set $N = \{1, 2, ..., n\}$ and $v = a$ simple weighted voting game. T represents a coalition among the shareholders, w for each shareholders percentage of shares and q denotes the winning requirement. The rules for winning or losing coalition is represented $w(T)$ as is $\geq q$ and $w(T) < q$ respectively. A shareholder (i) is said to be pivotal when without him i.e. withdrawal, the weight is less than the quota i.e. $w(T) < q$ but with him i.e. $T + \{i\}$ the weight is more than the quota i.e. $w(T_i) \geq q$. Applying these notations;

i	A, B and C				
n	3				
w_A	42.93%,	w_B	47.21%,	w_C	9.86%
q	50.1 percent				
T	Shapley-Shubik:	$n!$	and	Penrose/Banzhaf:	2^{n-1}

2.1 Shapley-Shubik Index

The index introduced in 1954, by Shapley and Shubik [3] has been one of the earliest and most widely used voting power indexes. Conceptually, this index measures power by the probability of a player being pivotal over all possible coalitions that include *all* the voters. Specifically, this pivot is when the *arrival* of a shareholder results in the formation of a winning coalition. Another way of looking at it is when *withdrawal* turns a winning into a losing coalition. The *order of arrival* of a shareholder into various coalitions is important in the determination of the Shapley-Shubik index [4]. In the context of corporate election, this mimics one shareholder vote at a time. The shareholder whose vote results in a winning coalition being formed is called the pivotal shareholder. The shareholders who vote after this pivotal shareholder are ignored as *dummy* since the winning coalition has been formed. Accordingly, this allows only one pivot per winning coalition. Therefore the total number of pivots always equals the number of coalitions i.e. the index always sums to unity. In the voting power jargon, the index is said to be *normalised*, which becomes a useful feature since it permits inter-group comparison amongst voting bodies. Similarly, comparing results in percentages are easier to visualise than results in absolute form.

Table 3 describes the pivots. The last column indicates that each shareholder is pivotal in two coalitions. In addition, six possible coalitions exist i.e. . Hence, the index for each shareholder is calculated as 2/6 or 0.33.

Table 3 Shapley-Shubik Index - Pivotal Shareholders

Arrivals 1st	2nd	3rd	Pivotal shareholder
A	B	C	B
A	C	B	C
B	A	C	A
B	C	A	C
C	A	B	A
C	B	A	B

Note: 1. A–42.9%; B-47.2%; C–9.9%. Majority required to win an election is more than 50.0%.

Typically, Greek letter Φ (phi) and the following definition represent the index;

$$\phi_1 = \frac{(T-1)!(n-T)!}{n!}\left[v(T)-v(T_i)\right]$$

i= swings for T

The notation $(T-1)!$ denotes the number of possible orderings for voters prior to i and $(n-T)!$ denotes possible orderings subsequent to i. In principle, these two notations sum to $n!$. Partitioning signifies the sequent i is computed. $v(T)-v(T_i)$ indicates the manner the coalition is analysed i.e. whether a game with T players i.e. $v(T)$ without a member i i.e. $v(T_i)$ produces a swing for player i. Finally, if this results in a swing, the contribution of player i is said to be full i.e. one. And since only a single swing for each ordering appears and the contribution due to this swing is one, the total of swings will always be equal to $n!$. For that reason, the sum of Shapley-Shubik index will always equal one.

2.2 Penrose-Banzhaf Index

Another widely applied voting power index introduced in 1946, by Penrose [2]. A shareholder being pivot over the total number of coalitions involving that shareholder defines this index. In contrast to Shapley-Shubik index, the order in which a shareholder arrives or withdraws from a coalition does not define the pivot. The definition results in a single pivotal shareholder per coalition whose withdrawal would render the coalition impotent. Consequently, in contrast to the Shapley-Shubik, the Penrose-Banzhaf index can have more than one pivotal shareholder in a winning coalition. Since the total number of pivots does not necessarily equal the number of winning coalitions, the index does not always sum to unity. In other words, the index stands in absolute form. The parallel in corporate election i.e. AGM, suggests all shareholders vote at once.

Closely related to the Penrose index is the index developed in 1965, by Banzhaf [5]. While total pivots are similar, the former index is defined over the number of coalitions with the latter over the total number of pivots. Changing the denominator of the Banzhaf index achieved similar results to that of the Penrose index hence the name Penrose-Banzhaf index. The Banzhaf index stands normalised since the aggregate number of pivots from individual shareholders is always equal to the total number of pivots. An important point to note is that normalising fails to capture the true strength of voting power in a similar way that percentage may hide the real number [6].

Table 4 illustrates pivotal shareholders for both indices. Notice that each shareholder is pivotal on two occasions. Additionally, a shareholder is involved in four coalitions i.e. 2^{n-1}. Consequently, the Penrose-Banzhaf index for each shareholder is 'A'= 2/4 or 0.50, 'B'= 2/4 or 0.50 and 'C'= 2/4 or 0.50. As the Banzhaf index involves six pivots, the index for each shareholder is 'A'= 2/6, 'B' = 2/6 and 'C' = 2/6.

Table 4 Penrose-Banzhaf Index - Pivotal Shareholders

Winning Coalitions, Pivotal Shareholder (Underlined)		
$\underline{A}$	$\underline{B}$	
$\underline{A}$	$\underline{C}$	
$\underline{B}$	$\underline{C}$	
C	B	A

Note: 1.'A' −42.9%; 'B' - 47.2%; 'C' − 9.9%. Majority required to win an election is more than 50.0%.

Normally, the Greek symbol β (beta) represents Penrose/Banzhaf index. Probabilistically, the index follows this probabilistic model:

$$\beta' = n/2^{n-1}.$$

The denominator, 2^{n-1}, denotes the number of coalitions and the nominator, η, represents the number of pivots for each shareholder. This η may not be equal to the denominator hence the index remains in absolute form. The total number of shareholders being pivotal over the total combined number of pivots for all the shareholders is known as a normalised Banzhaf index (i.e. presenting each shareholder index relative to others). In some studies, the term Banzhaf index remains normalised:

$$\beta = \eta/\sum\eta.$$

These indices can be calculated directly i.e. manually or by multilinear extension technique (Appe ndix A).

2.3 Straffin Index

While the Shapley-Shubik, Penrose and Banzhaf indexes have been widely discussed, discussion of the Straffin index remains limited. This study discusses the index in view of its possible application in corporate analysis.

In 1977, Straffin [4] proposed what has become to be known as the Straffin index. In terms of definition, this index comes under partial homogeneity assumption. Technically, it is a combination of the Shapley-Shubik index and the Penrose-Banzhaf index [7]. The idea is that these two different indices, as discussed in the previous section, reflect different ways of analysing the same different voting patterns but a common thread can be found in these two analyses in the theory of probabilistic voting. In other words, even though the two indices are dissimilar, they stand connected in some ways due to the manner of voting distribution.

This section explains the connection. In the Banzhaf index the voting distribution is assumed to be binomial. This can be translated as the probability of a voter voting 'yes' or 'no' is the same. It is 1/2. In contrast, the assumption of Shapley-Shubik starts with uniform distribution i.e. $Pi = P$. Consequently, while P is assumed to be 1/2 in Banzhaf, it is not essential in deriving the Shapley-Shubik index. The only assumption that is needed for this index is that the probability distribution of voting 'yes' or 'no' is the same amongst voters. Thus if P is 1/2 for a voter in the Shapley-Shubik index, then the P for the other voter is also 1/2. These underlying voting distributions explain why the two indices are probabilistically connected.

Straffin further elaborated the meanings behind this difference. In the binomial poll distribution $P = 1/2$ implies voters vote independently of each other while in the uniform distribution $Pi = P$ implies voters vote homogenously. The terms independence and homogenous are now normally associated with Banzhaf and Shapley-Shubik index respectively. Nevertheless, although the latter index assumes a homogenous voting pattern, this does not mean one vote influences the other but simply that the voters share what Straffin [4] explained in page 12 as "[*i*]uniform standard or set of values". Straffin defines this state of affairs, where there appears to be a commonality of values between voters, as a state of homogeneity. Straffin proposes a more general index based on partial homogeneity. The Shapley-Shubik is a special case of this general index in that Shapley-Shubik assumes complete homogeneity in the "standard of values". Penrose-Banzhaf is another special case in that the voters need not share a common set of values, and they vote independently of each other.

The previous section describes scenarios appropriate for the two indices. However, when voter splits appear i.e. some vote homogenously while others vote independently, suitability ends. Accordingly, if users can identify this split and apply the index accordingly, a more appropriate measure of the distribution of voting power will be obtained. Thus, when 'C' and 'B' vote together i.e. choose the same decision, Straffin provides a better index. In the 2004 study by Kauppi and Widgren [7], the application of this index (known as a modified Shapley-Shubik index in their study) produces an index that better reflects the pattern of the European Union budget allocation than does the normal Shapley-Shubik Index.

In corporate scenarios, partial homogeneity is the split that exists among the shareholders as a result of certain common values. For example, it may be reasonable to assume that family members will vote together instead of against each other. It is also reasonable to assume that various government investment agencies vote en-bloc as do shareholders who exhibit similar values on environmental or political issues. Some of the shareholders therefore may vote homogenously while others vote independently.

Previous sections point out that the Banzhaf and Shapley-Shubik indexes reflect voting patterns of $P = 1/2$ and P is homogenous respectively. Multilinear extension technique has made this reflection possible. In particular, setting $P = 1/2$ in a polynomial will get to the Banzhaf index whilst integrating a polynomial will get to the Shapley-Shubik index. Appendix A calculates the Straffin index using multilinear extension technique as per illustrations in Straffin [8], Owen [9] and Roth [10]. In this calculation, 'B' and 'C' are assumed to vote homogenously while 'A' votes independently. Table 5 summarises the results.

Table 5 Voting Power Indices

Shareholders and percentage	Shapley-Shubik	Penrose-Banzhaf	Straffin index (B and C vote homogenously)
A 42.9%	0.33	0.50	0.67
B 47.2%	0.33	0.50	0.75
C 9.9%	0.33	0.50	0.75

Formally, the Straffin index in mathematical notation is a partial homogeneity structure of *N*; a partition of *N* into disjoint subsets; $P = \{S_1, ..., S_m\}$. Technically, voting power under *P*-partial homogeneity assumption defines this index;

$K(G, P)$ = voting power index under *P*-partial homogeneity assumption.
where: K = voting power index
G = Game.
P = Probability. *P* is not known. But for the homogeneity assumption, According to Straffin [4] "*simply assign the same P to the members of each subset*…" And for independent assumption "*select that P from [0,1] independently*…" (Page 114).

This paper tries to keep the technical matters to the minimum, focusing instead on the essential aspects necessary for a greater understanding for corporate analysis.

3 Determining Voting Pattern

While the technical aspects have been detailed in many previous studies, the practical aspect of determining voter's affiliation i.e. estimating or assigning the degree of homogeneity in the calculation of *P* has been missing. Fulfilling this absence may boost the appeal of the index. In this respect, this paper suggests two ways of determining *P* in corporate settings. First, the expert estimate. This method has its advantages particularly in the absence of prior records e.g. lack of data on new companies. Additionally, it can also be faster and cheaper to estimate *P* instead of detailing massive voting patterns of thousands of shareholders i.e. the 'ocean'. It can also make it easier to estimate *P* for companies especially when the controlling shareholders keep changing hands. In short, the method quickens the process of determining voter behaviour. Nevertheless, as can be expected, it can be very subjective, leading to bias and manipulation.

A second method of ascertaining *P* is to examine from historical voting patterns. Table 6 illustrates the method of examining voting patterns. Assume there are three shareholders with equal weight i.e. equal percentage of shareholdings and four policies. Suppose that Shareholder 'A' 'disagrees' while Shareholders 'B' and 'C' agree on Policy 1. The figures in italics express the overall patterns. Shareholder 'A' votes 'alone' in three of the four polices i.e. P_A = 3/4 while shareholder 'B' and 'C' vote together in three out of four policies i.e. $P_{BC} = \frac{3}{4}$. Statistically, it can be concluded that shareholder 'A' voted almost independently while shareholders 'B' and 'C' voted roughly homogenously. Partial homogeneity obtains, giving rise to the case for the Straffin power index to be considered.

Table 6 Shareholders Voting Affiliation

Shareholder Policy	A		B		C	
Policy 1	Disagree		Agree		Agree	
2	Agree		Disagree		Disagree	
3	Agree		Disagree		Disagree	
4	Agree		Disagree		Agree	
Affiliation	*Alone*	$P_A = 3/4$	*Alone*	$P_B = 1/4$	*Alone*	$P_C = 0$
	With B	$P_{AB} = 0$	*With A*	$P_{AB} = 0$	*With A*	$P_{CA} = 1/4$
	With C	$P_{AC} = 1/4$	*With C*	$P_{BC} = 3/4$	*With B*	$P_{CB} = 3/4$
	With B & C	$P_{ABC} = 0$	*With A & C*	$P_{ABC} = 0$	*With A & B*	$P_{ABC} = 0$

Obviously, analysing historical voting patterns is a more objective way of estimating P than seeking expert opinion about voting intentions. This also allows for a more comprehensive analysis. For example, voting patterns can be further analysed according to various policies e.g. dividend, financing and investment. A drawback of this method is that it requires the availability of data that may not always be there. Typically, details of the voting pattern i.e. who has voted on a particular agenda are not subject to the legal requirement of formal disclosure by the management. Additionally, problems of aggregation introduce noise into the interpretation of voting patterns. Potentially, minor policies can skew the voting patterns on the more important polices such those on dividend, capital restructuring and major investment. Most importantly, ownership structures in a publicly owned company are rarely static. As existing shareholders exit and new investors emerge, historical data on voting patterns become less reliable as a guide to future voting patterns.

4 Straffin Index and the Issues

Straffin [4] suggested the index be applied according to shareholders voting behaviour. He wrote on page 117, "[i]..⋯we need not simply apply either β or ϕ". The application of this index he explained "$[i]$...may give a better idea of the distribution of power than either pure classical index[1]". In other words, in situations where coalitions exist among voters the application of Straffin index produces better results than either the Banzhaf index or the Shapley-Shubik index since it better captures voter behaviour than the other two indices. His suggestion has received some support in the literature, in study on individual power and the effect of hierarchical structure in an organisation by Braham and Steffen [11] and study on individual power in a committee by Grondahl [12]. However, unlike the Shapley-Shubik and Banzhaf indices, very few studies have applied the Straffin index. The reasons have been identified as follows: i) voting power distribution anomaly ii) practical limitations in collecting data and iii) problems in the interpretation of meanings.

4.1 Voting Power Distribution Anomaly

Currently, in the voting power fraternity, the index is accepted in *a-priori* form i.e. only voter's size and winning quota are incorporated into the calculation of voting power indices. A logical local monotonicity requirement is that voters with equal weight should have equal power; alternatively no

[1] i.e. the Shapley-Shubik and Banzhaf index.

voters with larger weight should have less power than voters with smaller weight. However, the Straffin index violates this property.

The Straffin index assumes that some voters will vote in a cohesive manner. Consequently, smaller shareholders can now be accorded with larger voting power than the bigger shareholders. In the example used in this paper, the Straffin index results in smaller shareholders (B and C) having bigger power than the larger shareholder (A) at 0.75 versus 0.67 (Table IV). This paradox hinders the use of Straffin index [13, 14], but it is not unique to the Straffin index.

4.2 Practical Limitation

Naturally, when one studies power it invites comparisons and questions such as who has greater power and whether or not power has increased. Examples are widespread; matching voting power of United Nations members [2], the power of the President, Senate and House in the U.S congress [3], the power of each country in the European Union [15] and the power of a shareholder group [16, 17]. Commonly, these studies analyse voting power under differing circumstances.

One of these circumstances is the analysis of power pre and post coalition. Consistency requires that the pre and post coalition voting power index should be the same. Nevertheless, it stands that Straffin index best measures voting power only when coalitions occur. The crux of the problem is the reliability of information about the degree of homogeneity that is likely to obtain in voting prior to the formation of coalitions. Without evidence of the formation of coalitions i.e. pre coalition, voting power analysis has to rely on other indices e.g. Shapley-Shubik index and Banzhaf index. Note that the Straffin, Shapley-Shubik and Banzhaf indexes make different underlying assumptions on issues such as independent versus homogenous voting patterns (discussed in an earlier section). One faces a theoretical dilemma if a study continues to employ a different index for pre and post analysis. Consequently, the inability to be used in both pre and post analysis limits the application of the Straffin index.

4.3 The Meaning/s?

Another possible reason for the lack of application of the Straffin index is the absence of an explanation of its underlying meaning. Two concepts of voting power, I-Power and P-Power have been employed to glean an intuitive understanding of Penrose-Banzhaf and Shapley-Shubik indices [18]. In detail, I-Power stands for the ability to influence the direction of the voting body. On the other hand, P-Power resembles payoffs or rewards in helping a coalition win an election.

It is not clear how to interpret the Straffin index in terms of the above ideas of voting power. This inquiry is vital for the Straffin index to gain wide acceptance. For example, in applying the Straffin index, for voter ‘A’ who votes independently, influence underlies the meaning of power. In contrast, for voter ‘B’ and ‘C’ who vote homogenously, reward underlies the meaning of power. This line of reasoning suggests that, for each voter, the meaning of power may change between influence and reward depending on the voting behaviour in each voting exercise. Similarly, this suggests accepting that in a single voting body, two different voting patterns, namely homogenous and independent, and two different techniques of determining the pivot, namely order of arrival and withdrawal, can exist simultaneously. Alternatively, there may well be a new underlying meaning totally unrelated to the meaning that underlies these two indices. Naturally, the implication of this yet to be proposed alternative meaning remains unclear.

Figure 1 illustrates the anomaly facing a researcher employing the Straffin index in analysing the dynamic distribution of power within a single company. Take the case of analysing voting power in situation 1 against situation 2. In situation 1, the power index of voter ‘A’ is based on the assumption of homogenous voting distribution, the meaning of power comes as payoff and the pivot is based on the *order of arrival* while in situation 2, the power index is based on altogether different assumptions. Resort to the Straffin index introduces a dilemma in making comparisons. There appears to be an anomaly.

On the other hand, the anomaly facing the Straffin index is absent in the Shapley-Shubik and Banzhaf index. The comparisons in the later indices are based on a set of assumptions that remain constant throughout the analysis. For this reason, if the Straffin index is to be accepted, it requires similar unifying and consistent underlying assumptions. The index can only be used if one either accepts this anomaly or manages to find an alternative explanation of the index.

Company X:
Assumptions on Voting Pattern, Meaning and Pivotal Basis

Situation 1	Situation 2		Situation 1	Situation 2
Voter A	Voter A	Voting Pattern	Homogenous	Independent
		Meaning	Reward (Voter A)	Influence (Voter A)
		Pivotal Basis	Order of Arrival	Withdrawal
		Voting Pattern	Homogenous	Homogenous
Voter B	Voter B	Meaning	Reward (Voter B)	Reward (Voter B)
		Pivotal Basis	Order of Arrival	Order of Arrival
		Voting Pattern	Independent	Homogenous
Voter C	Voter C	Meaning	Influence (Voter C)	Reward (Voter C)
		Pivotal Basis	Withdrawal	Order of Arrival

Note:
** Coalition*

Intra-company Comparisons

Figure 1 Intra-company comparison of assumptions

5 Conclusions

This paper begins by addressing voting power indices that are employed in political analysis. In corporate analysis, power indices are absent in the literature. The paper attempts to rectify this imbalance by exploring potential applications of voting power indices in corporate analysis. Next, the paper discusses two popular indexes, namely Penrose-Banzhaf index and Shapley-Shubik index, and explains that gaining 'influence' best reflects the former while gaining 'reward' best echoes the latter index. Finally, the paper elaborates on the Straffin index for which comprehensive discussion is sparse even in the literature in political science and the applications are even fewer. The paper evaluates the merit of this index in comparison with Penrose-Banzhaf and Shapley Shubik indexes. The strength and drawback of the Straffin index are discussed. More work is needed to resolve these drawbacks and this paper has taken a tentative step by proposing possible lines of enquiries into the interpretation of the meaning of this index. To this end, this paper hopes to pave the way for greater understanding on the Straffin index and greater appreciation of the voting power concept among corporate researchers.

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Appendix A: Straffin Index as per Multilinear Extension

Straffin used multilinear extension technique to illustrate this voting power index. The used of this technique permits the calculation of index even without knowing the precise *P*. Let *g* be a game whose subset of $N=\{1,2,...,n\}$ and *f* is a function for *g* and *S* is any random coalition. As these *n* players will have 2^n number of subsets, substituting the component to either 0 or 1 result in $\{0,1\}^n$ dimensional space i.e. the cube. The corners of this cube will be the value of *f*. The value of *f* at each corner can then be extended to other corners in the cube. Multilinear extension of *g* is defined by;

$$f(x_1,x_2,x_3)=\sum_{S\subset N}\left\{\prod_{i\in S}x_i\prod_{i\notin S}(1-x_i)\right\}g(S)\quad \text{for}\quad 0\le x_i\le 1,\qquad i=1,2,...,n$$

The definition is translated into the 'product of all winning coalition times 1-each member of a losing coalition. Applying the definition: A: 42.93% = x_1 B: 47.21% = x_2 and C: 9.86% = x_3 and the quota is 50.01% in [0, 1] normalisation, the multilinear extension of *g* is;

$$\begin{aligned} f(x_1,x_2,x_3) &= x_1x_2\left(1-x_3\right)+x_1x_3\left(1-x_2\right)+x_2x_3\left(1-x_1\right)+x_1x_2x_3 \\ &= x_1x_2+x_2x_3+x_1x_3-2x_1x_2x_3 \end{aligned}$$

x is the probability of voting 'yes' and $1-x$ is probability of voting 'no'. e.g. since x_1x_2 is a winning coalition, the linear equation will be $x_1x_2(1-x_3)$.

The power polynomials i.e. the i^{th} partial derivative for each shareholder are thus;

$$f_1\left(x_1,x_2,x_3\right)=x_2+x_3-2x_2x_3;\qquad f_2\left(x_1,x_2,x_3\right)=x_1+x_3-2x_1x_3;$$
$$f_3\left(x_1,x_2,x_3\right)=x_2+x_1-2x_1x_2$$

As an illustration, assumes 'A' vote independently whilst 'B' and 'C' vote homogenously. 'A' is critic al when; (x_2 Yes x_3 No) or (x_2 No x_3 Yes) or (x_2 Yes x_3 Yes). The polynomial is thus;

$$K'_A=\int_0^1\int_0^1 x_2(1-x_3)dx_2dx_3\quad\text{or}\quad\int_0^1\int_0^1(1-x_2)x_3dx_2dx_3\quad\text{or}$$
$$\int_0^1\int_0^1 x_2x_3dx_2dx_3$$

Since x_2 and x_3 vote homogenously integration of x_2 and x_3 together will get to K'.

$$K'_A=\int_0^1 x_2\left(1-x_3\right)dx+\int_0^1\left(1-x_2\right)x_3dx+\int_0^1 x_2x_3dx$$

So,= $2\left[\tfrac{1}{2}x^2-\tfrac{1}{3}x^3\right]_0^1+\tfrac{1}{3}x^3\Big|_0^1=\tfrac{2}{3}$ = 2/3 or 0.67

'B' is critical when; (x_1 Yes x_3 No) or (x_1 No x_3 Yes) or (x_2 Yes x_3 Yes). The polynomial is thus;

$$K'_B=\int_0^1\int_0^1 x_1(1-x_3)dx_1dx_3\quad\text{or}\quad\int_0^1\int_0^1(1-x_1)x_3dx_1dx_3\quad\text{or}\quad\int_0^1\int_0^1 x_1x_3dx_1dx_3$$

Since x_2 vote independently whilst x_2 vote homogenously setting x_2 at 1/2 directly into the polynomial and integrating x_3 will get to K'.

$$K_B' = \int_0^1 x_1 dx \ . \ \int_0^1 (1-x_3)dx + \int_0^1 (1-x_1)dx . \int_0^1 x_3 dx + \int_0^1 x_1 dx \ . \int_0^1 x_3 dx.$$
$$= 3 . \tfrac{1}{2} . \tfrac{1}{2} = \tfrac{3}{4} = 0.75$$

'C' is critical when; (x_1 Yes x_2 No) or (x_1 No x_2 Yes) or (x_2 Yes x_1 Yes). The polynomial is thus;

$$K_C' = \int_0^1\int_0^1 x_1(1-x_2)dx_1dx_2 \quad \text{or} \quad \int_0^1\int_0^1 (1-x_1)x_2dx_1dx_2 \quad \text{or} \quad \int_0^1\int_0^1 x_1x_2dx_1dx_2$$

Since x_1 vote independently whilst x_2 vote homogenously setting x_1 at 1/2 directly into the polynomial and integrating x_2 will get to K'.

$$K_C' = \int_0^1 x_1 dx \ . \ \int_0^1 (1-x_3)dx + \int_0^1 (1-x_1)dx . \int_0^1 x_3 dx + \int_0^1 x_1 dx \ . \int_0^1 x_3 dx.$$
$$= 3 . \tfrac{1}{2} . \tfrac{1}{2} = \tfrac{3}{4} = 0.75$$

In summary; K'_A=2/3 or 0.67 K'_B=3/4 or 0.75 K'_C=3/4 or 0.75

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