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A Lot-for-Lot Model with Multiple Instalments for a Production System under Time-Varying Demand Process

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Abstract In manufacturing systems, the quantity of raw material needed in production is dependent on the production size. In this paper we consider a manufacturing system which procures raw materials from suppliers and processes them to make a finished product. The problems are to determine an ordering policy for raw materials (a lot with multiple instalments) and a production policy for the finished product to satisfy a deterministic time-varying demand process. We find an optimal solution for the model by using the Solver of Microsoft Excel. We present some numerical examples for a discussion.

Keywords Batch size, Manufacturing system, Time-varying demand, Optimization, Integrated inventory.

1 Introduction

In reality, a quantity of raw materials (QR) needed in any production is dependent on a production size (QM). It is desirable to consider QR and QM simultaneously. Sarker et al. [3] have shown that this joint ordering policy provides lower cost under certain condition compared with the separate ordering policies.

Sarker and Khan [4] have developed three joint ordering policy models. The first model is lot-for-lot with there is no demand during a production time and the whole lot of product will be delivered immediately after the production time for each batch finished. In the second model, the ordering quantity of raw material is equal to the raw material required for multiple lot of product. The last model is similar to the second, however in this model the delivery of the finished product is considered in multiple instalments.

Recently, Omar and Smith [1] developed the joint ordering policy model for the case where demand for finished product is deterministic linearly increasing. This is a general model for lot-for-lot policy where the supply of raw material and demand for finished product are continuous.

In this paper we extend the Omar and Smith model [1] by considering a case where one lot of raw material will be delivered with multiple instalments during the production time due to either limitation in transportation or warehouse facilities, for a lot (or batch) of production quantity. In this model we assumed that the number of instalment, m, is the same for each batch production. For each batch we determine an economic ordering quantity of raw material with m instalments together with an economic quantity of manufacturing quantity which minimize the total cost. We used the spreadsheets Solver of Microsoft Excel to find an optimal policy for the model.

2 Mathematical Formulation

In this section, a general cost model is developed. The following costs are considered;

- raw material ordering cost
- manufacturing set-up cost
- raw materials inventory carrying cost
- finished product inventory carrying cost

To develop the model, the following terminology is used:

- The demand rate of finished product at time t in $(0, H)$ is $f(t)$. H is the time horizon after which no demand will be met.
- The finite production rate is P units per unit time and $P > f(t)$ (to ensure no shortage).
- There is a fixed manufacturing set-up cost of c_p for each production run.
- There is an ordering cost of c_j for raw material j.
- There is a carrying inventory cost of h_p per unit per unit time for finished goods.
- There is a carrying inventory cost of h_j per unit per unit time for raw material j.
- QM_i is the production quantity of the $(i+1)$st batch.
- QR_{ijk} is the raw material quantity for raw material j of the $(i+1)$st batch and for k instalment.
- n is the total number of batch replenishment (and therefore we define $t_n = H$).
- m_i is the number of instalment for each batch where $m_i = m_{i+1} = m$.
- r_j is the amount/quantity of raw material j required in producing one unit of a finished product.

From Omar and Smith [1], for lot-for-lot model, the total cost for n-batch if $f(t) = a+bt$ and $j = 1$ is

$$\begin{aligned} TRC(n) &= n(c_p + c_1) \\ &+ h_p \sum_{i=0}^{n-1} \frac{(t_{i+1} - t_i)^2}{2} \left\{ [a + \frac{b}{3}(2t_{i+1} + t_i)] - \frac{1}{P}[a + \frac{b}{2}(t_{i+1} + t_i)]^2 \right\} \\ &+ \frac{h_1 r_1}{2P} \sum_{i=0}^{n-1} (t_{i+1} - t_i)^2 [a + \frac{b}{2}(t_{i+1} + t_i)]^2 \end{aligned} \quad (1)$$

In our model, for each batch i, the raw material j will be delivered in m_i instalments (small lots) for a lot of production size where m_i is positive integer. Generally m_i might be different for all i, however as mentioned before we assumed that $m_i = m_{i+1} = m$ for some i. Figure 1 gives a graphical representation of the model for the $(i + 1)$st batch when $m = 2$.

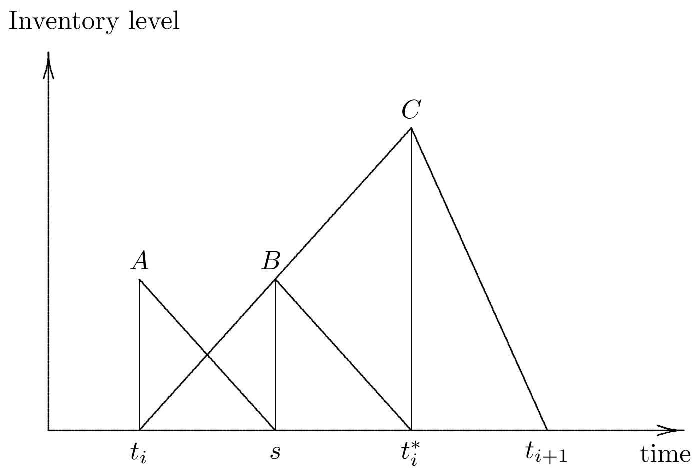

Figure 1: Plot of stok of raw material and finished product against time

Figure 1 shows the graf of raw material and finished product against time for the $(i+1)$st batch. The raw material will be fully consumed at the end of the production time. We assumed that during production finished product becomes immediately available to meet the demand process. For a finite P, the time-weighted stockholding for finished product is given by the area of curve t_iCt_{i+1} while for the raw material is the area of triangles t_iAs and sBt_i^*.

Using a simple calculus, it can be shown that the area of triangles t_iAs and sBt_i^* is minimum when the interval between the first and the second instalment are the same. It is similar for $m = 3$. From these results, we assumed that the interval between the consecutive instalment are the same. Hence from Figure 1, we have

$$P(s - t_i) = P(t_i^* - s) = \frac{1}{2} \int_{t_i}^{t_{i+1}} f(t)dt. \quad (2)$$

It follows that a time-weighted stockholding of raw material $j = 1$ is

$$2r_1\left(\frac{1}{2}\frac{t_i^*-t_i}{2}\frac{1}{2}\int_{t_i}^{t_{i+1}} f(t)dt\right) = \frac{r_1}{4P}\left(\int_{t_i}^{t_{i+1}} f(t)dt\right)^2, \quad (3)$$

since $P(t_i^* - t_i) = \int_{t_i}^{t_{i+1}} f(t)dt$.

Similarly, if $m = 3$, the time-weighted inventory is

$$\frac{r_1}{6P}\left(\int_{t_i}^{t_{i+1}} f(t)dt\right)^2 = \frac{r_1}{2mP}\left(\int_{t_i}^{t_{i+1}} f(t)dt\right)^2.$$

For n-batch with m instalments, the total time-weight inventory is

$$\frac{r_1}{2mP}\sum_{i=0}^{n-1}\left(\int_{t_i}^{t_{i+1}} f(t)dt\right)^2.$$

It follows that the total ordering and holding cost of raw material for n-batch with m instalments is

$$\begin{aligned} nmc_1 \quad &+ \quad \frac{h_1r_i}{2mP}\sum_{i=0}^{n-1}\left(\int_{t_i}^{t_{i+1}} f(t)dt\right)^2 \\ &= \quad nmc_1 + \frac{h_1r_i}{2mP}\sum_{i=0}^{n-1}\left[a(t_{i+1}-t_i)+\frac{b}{2}(t_{i+1}^2-t_i^2)\right]^2 \\ &= \quad nmc_1 + \frac{h_1r_i}{2mP}\sum_{i=0}^{n-1}\left[a(t_{i+1}-t_i)+\frac{b}{2}(t_{i+1}-t_i)(t_{i+1}+t_i)\right]^2 \\ &= \quad nmc_1 + \frac{h_1r_i}{2mP}\sum_{i=0}^{n-1}(t_{i+1}-t_i)^2\left[a+\frac{b}{2}(t_{i+1}+t_i)\right]^2 \end{aligned} \quad (4)$$

Hence the total cost of the model if $f(t) = a + bt$ is

$$\begin{aligned} TC_s \quad = \quad & nc_p + h_p\sum_{i=0}^{n-1}\frac{(t_{i+1}-t_i)^2}{2}\left\{[a+\frac{b}{3}(2t_{i+1}+t_i)] - \frac{1}{P}[a+\frac{b}{2}(t_{i+1}+t_i)]^2\right\} \\ & nmc_1 + \frac{h_1r_1}{2mP}\sum_{i=0}^{n-1}(t_{i+1}-t_i)^2\left[a+\frac{b}{2}(t_{i+1}+t_i)\right]^2. \end{aligned} \quad (5)$$

2.1 Numerical Examples

To demonstrate the effectiveness of this model, we present some numerical examples. We do a comparison with the lot-for-lot model to show the applicability of the model. Consider the case where demand is linearly increasing, $f(t) = a + bt$, with $a = 20$ and $b = 2$ for

Table 1: Comparison between lot-for-lot models with a single and multiple instalments

c_r	h_r	n	Lot for a lot	n	m	Multiple lot for a lot
3.0	1	6	*156.8358	6	1	156.8358
2.7	1	6	*155.0358	6	1	155.0358
2.5	1	6	*153.8358	6	1	153.8358
1.5	1	6	147.8358	6	2	*143.7220
1.0	1	7	144.4283	6	2	*137.7220
0.7	1	7	142.3283	6	3	*133.9423

$H = 5$. The other parameters are $c_p = 10, h_p = 2, r_1 = 1$ and $P = 50$. While parameters c_r and h_r are varies as shown in the Tables 1 and 2.

Table 1 gives the optimal policy for the models with different values of c_r. As expected, the model with multiple instalments become more superior than the model with a single instalment when c_r decrease. For example when $c_r = 0.7$, an optimal number of batches from the model with a single instalment is 7 with the production time starting at 0, 0.7585, 1.4988, 2.2234, 2.9343, 3.6331, 4.3213 and 5. The total cost for this optimal policy is 142.3283. On the other hand, the model with multiple instalments gives an optimal number of batches is 6 with 3 instalments. The production time starting at 0, 0.8571, 1.7006, 2.5339, 3.3599, 4.1811 and 5, with the total minimum cost for this policy is 133.9423.

Table 2 gives an optimal policy for these models with different values of h_r. The model with multiple instalments is superior when the values of h_r are 1.5, 1.8 and 2.0.

Table 2: Comparison between lot-for-lot models with a single and multiple instalments

c_r	h_r	n	Lot for a lot	n	m	Multiple lot for a lot
2	0.1	1	*126.5284	5	1	126.5284
2	0.2	6	*129.3344	6	1	129.3344
2	0.3	6	*132.4657	6	1	132.4657
2	0.5	6	*137.7220	6	1	137.7220
2	1.5	7	162.6467	6	2	*156.2830
2	1.8	7	169.3707	6	2	*160.2155
2	2.0	7	173.8511	6	2	*162.8358

3 Conclusion

A model lot-for-lot with multiple instalments has been proposed in this paper. To avoid complexity and to make this model more applicable, we assumed that the number of instalment is same for each batch. Our numerical results shown that the model with multiple instalments is superior for some cases.

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