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Strong Local Colorings of Coronas

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Abstract In this paper, we study strong local colorings of some important families of coronas. A local coloring of a graph G of order at least 2 is a function $c : V(G) \to N$ such that for every set $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, there exists two distinct vertices $u, v \in S$ such that $|c(u) - c(v)| \geq m_s$, where m_s is the size of the induced subgraph $< S >$. The value of a local coloring c is the maximum color it assigns to a vertex of G. The local chromatic number of G is the minimum value of any local coloring of G and we denote it by $\chi_\ell(G)$. A local coloring of G with value $\chi_\ell(G)$ is called a minimum local coloring of G. If a minimum local coloring of G uses all the $\chi_\ell(G)$ colors then it is called a strong local coloring of G. If every minimum local coloring of G uses all the $\chi_\ell(G)$ colors then G is called strong local colorable and in this case, its local chromatic number is called strong local chromatic number and is denoted by $\chi_{s\ell}(G)$. In this paper, we have considered some important families of coronas and determined the strong local chromatic number, if it exists; otherwise, we have proved that they are not strong local colorable but local colorable and determined their local chromatic number.

Keywords Local coloring; strong local coloring; local chromatic number; strong local chromatic number; strong local colorable.

1 Introduction

Let G_1 and G_2 be two graphs. Let $V(G_1) = \{v_1, ..., v_k\}$ and take k copies of G_2.The corona $G_1 \circ G_2$ is the graph obtained by joining each v_i to every vertex of the i^{th} copy of $G_2, 1 \leq i \leq k$.

A coloring of a graph G is an assignment of colors to the vertices of G such that no two distinct adjacent vertices have the same color. A k-coloring of a graph is a coloring which uses k-colors. Here, we impose no condition on colors that are assigned to non-adjacent vertices. The chromatic number of a graph is the minimum number of colors required to color the vertices of the graph. Many variations and generalizations have been studied by researchers. We may replace the local requirement that adjacent vertices in a coloring must be assigned distinct colors by a more global requirement. A coloring can be considered as a function c: $V(G) \to N$ such that $|c(u) - c(v)| \geq 1$ for any two adjacent vertices u and v and $|c(u) - c(v)| \geq 0$ for any two non-adjacent vertices u and v. Precisely, a coloring can be considered as a function $c : V(G) \to N$ such that for every 2-element set $S = \{u, v\}$ of vertices of G,

$$|c(u) - c(v)| \geq m_S,$$

where m_S is the size of the induced subgraph $< S >$.

Gary Chartrand et al. has introduced [1] the study of local colorings of graphs. A k-local coloring of a graph G with order ≥ 2 is a function $c : V(G) \to N$ such that for each set S

$\subseteq$ *V(G)* with $2 \leq |S| \leq k$, there exists two distinct vertices $u, v \in$S such that

$$|c(u) - c(v)| \geq m_S,$$

where $2 \leq k \leq n$. The value of a k-local coloring c is the maximum color it assigns to a vertex of G and is denoted by $\ell_{c_k}(c)$. The k-local chromatic number of G is the minimum value of any k-local coloring of the graph G. That is,

$$\ell_{c_k}(G) = \min\{ \ell_{c_k}(c)$$

where the minimum is taken over all k-local colorings of G. Clearly, for every integer k with $2 \leq k \leq n, \chi(G) \leq \ell_{c_k}(G)$ and $\ell_{c_2}(G) = \chi\ (G)$. So the study will be more interesting for $k \geq 3$. Gary Chartrand et al. [1] refer a 3-local coloring of a graph as simply local coloring and write $\ell_{c_3}(G) = \chi_\ell(G)$. We also follow this assumption.

We note that if H is a subgraph of Gthen $\chi_\ell(H) \leq \chi_\ell(G)$.

Coronas are obtained by a graph operation and here we show that how their structures admit strong local coloring.

2 Strong Local Colorings of Graphs

Murugan [4] has introduced strong local coloring, strong local colorable graphs and strong local chromatic number.

A minimum local coloring of a graph with $\chi_\ell(G\ )$= k colors need not use all the colors to produce a local coloring of the graph. However colors 1 and k must be used.

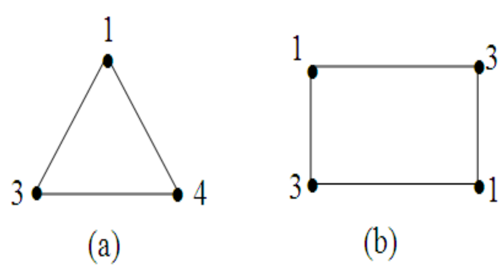

Figure 1: Minimum Local Coloring

We note that χ_ℓ(C$_3$) =4 and in Figure 1(a), we have a minimum local coloring of C$_3$ with 4 colors but the color 2 is not used. Also, χ_ℓ(C$_4$) = 3 and in Figure 1(b), we have a minimum local coloring of C$_4$ with three colors, but color 2 is not used.

When we consider $C_4 + \{e\}$, its local chromatic number is 4. In Figure 2(a), we have a minimum coloring with all four colors but in Figure 2(b), we have a minimum coloring without color 3.

We observe that the local chromatic number of C_5 is 3 and any minimum coloring of C$_5$ must use all the 3 colors. This suggests the problem of studying minimum local colorings which uses all the $\chi_\ell(G)$ colors and the graphs all of whose minimum local colorings uses $\chi_\ell(G)$ colors.

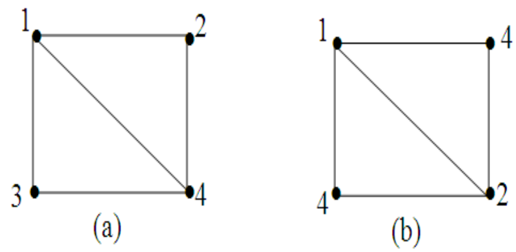

Figure 2: Different Minimum Local Coloring

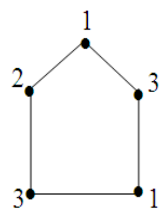

Figure 3: Strong Local Coloring

If a minimum local coloring of a graph G uses all the $\chi_\ell(G)$ colors then it is called a strong local coloring of G. If every minimum local coloring of G uses all the $\chi_\ell(G)$ colors then G is called strong local colorable and in this case its local chromatic number is called strong local chromatic number and we denote it by $\chi_{s\ell}(G)$.

3 Coronas

Definition 1: Let G_1 and G_2 be two graphs. Let $V(G_1) = \{v_1, ..., v_k\}$ and take k copies of $G_{2.}$ The corona $G_1 \,^\circ G_2$ is the graph obtained by joining each v_i to every vertex of the i^{th} copy of G_2, $1 \leq i \leq k$.

Theorem 1 *For k, $n \geq 2$, the corona $P_k \,^\circ K_{1,n}$ is strong local colorable and its strong local chromatic number is 4.*

Proof: Let the vertices of P_k be $v_1, v_2, ..., v_k$. Color v_i, i odd and $1 \leq i \leq k$ with color 2 and v$_i$, i even and $2 \leq i \leq k$ with color 4. For each odd i and $1 \leq i \leq k$, consider the vertices of $K_{1,n}$ which are joined with v_i. Color the central vertex of these $K_{1,n}$ say w_i , with 1 and all other vertices with 4. For each i , $i$even and $2 \leq i \leq k$, consider the vertices of $K_{1,n}$ which are joined with v_i. Color the central vertex of these $K_{1,n}$, say w_i, by 3 and all other vertices with 1. Since $m_s \leq 3$ for any $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, this coloring is a local coloring and value of this coloring is 4. Therefore $\chi_\ell \leq 4$. Since K_3 is a subgraph of this graph, $\chi_\ell \geq 4$. Hence, $\chi_\ell = 4$.

Now we show that all the four colors 1,2,3 and 4 should be used in any minimum local coloring of this graph.

Since v_1, w_1 and each adjacent vertex of w_1 ($\neq v_1$) forms a K_3, the colors 1,4 should be used to the vertices of this K_3. Suppose one color, say 3, is not used in a minimum local coloring of this graph (A similar argument can be given if 2 is not used). We note that since the coloring is minimum local coloring, the same color has to be given to the vertices of $K_{1,n}$ which are adjacent to w_1 ($\neq v_1$) and no two consecutive colors can be given to w_1 and to any vertex of $K_{1,n}$ which are adjacent to w_1 ($\neq v_1$). Similarly no two consecutive colors can be given to v_1 and to any vertex of $K_{1,n}$ which are adjacent to w_1 ($\neq v_1$). So v_1 is colored with 1 or 2. Suppose v_1 is colored with 2, then w_1 will receive color 1 and vertices of $K_{1,n}$ which are adjacent to w_1 ($\neq v_1$), will receive color 4. So v_2 should receive color 1 or 4. The above argument holds for the K_3 with vertices v_2 , w_2 and any vertex of $K_{1,n}$ which are adjacent to w_2 *($\neq v_2$)*. Therefore v_2 should receive color 1 and w_2 color 2. This is a contradiction, since there is a path of length 2 namely $v_1v_2w_2$ whose vertex colors violates the definition of local coloring.

A similar argument can be given if v_1 receives color 1.Hence all the four colors 1,2,3, and 4 must be used in any minimum local coloring of this graph. Hence $P_k\ ^\circ K_{1,n}$ are strong local colorable and $\chi_{s\ell} = 4.\square$

Theorem 2 *The corona $P_k\ ^\circ P_m$, $m \geq 3$, $k \geq 2$ is strong local colorable and its strong local chromatic number is 4.*

Proof: Consider $P_k\ ^\circ P_m$, $m \geq 3$ and $k \geq 2$. Let the vertices of P_k be $v_1, v_2, ..., v_k$. Color v_i , i odd and $1 \leq i \leq k$, with color 4 and v_i , i even and $2 \leq i \leq k$, with color 1. For each i, i odd and $1 \leq i \leq k$, consider the vertices of P_m which are joined with v_i. Color these vertices alternatively with color 1 and 3. For each i,i even and $2 \leq i \leq k$, consider the vertices of P_m which are joined with v_i. Color these vertices alternatively with color 4 and 2. Since $m_s \leq 3$ for any $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, this coloring is a local coloring and value of this coloring is 4. Therefore$\chi_\ell \leq 4$. Since K_3 is a subgraph of this graph, $\chi_\ell \geq 4$. Hence $\chi_\ell = 4$.

Now we show that all the four colors 1,2,3 and 4 should be used in any minimum local coloring of this graph.

Suppose, one color, say 2, is not used in a minimum local coloring of this graph (a similar argument can be given if 3 is not used). We cannot color v_1 with 1; if we do so then we cannot color the vertices of P_m which are adjacent with v_1 with 3 and 4. So we have to color v_1 with 3 or 4.

Suppose, we color v_1 with 4 (a similar argument can be given if we color v_1 with 3) then the vertices of P_m which are adjacent with v_1 have to be colored with 1 and 3 alternatively. In this case, v_2 should be colored only with 1. Now the vertices of P_m which are adjacent with v_2 cannot be colored the remaining colors 3 and 4.

Hence all the four colors 1,2,3 and 4 should be used in any minimum local coloring of this graph. Hence $P_k\ ^\circ P_m$, $m \geq 3$, $k \geq 2$ are strong local colorable and $\chi_{s\ell} = 4.\square$

Corollary 1: In the Theorem 2, when $m = 2$ the graph is not strong local colorable and its local chromatic number is 4.

Now color v_i, i odd and $1 \leq i \leq k$, with color 1 and v_i , i even and $2 \leq i \leq k$ with color 4. Assign colors 3 and 4 to the vertices of P_m which are adjacent with v_i, i odd and $1 \leq i \leq k$ and color 1 and 3 to the vertices of P_m which are adjacent with v_i, i even and $2 \leq i \leq k$. Clearly this is a minimum local coloring, which is not strong. Hence $\chi_\ell = 4$.

Theorem 3 *If nis odd and $n > 3$ and $k > 1$, the corona $P_k \,^\circ C_n$isstrong local colorable and its strong local chromatic number is 5.*

Proof: Consider $P_k \,^\circ C_n$, n odd and $n > 3$ and $k > 1$. Let the vertices of P_k be $v_1, v_2, ..., v_k$. Color v_i , i odd and $1 \leq i \leq k$, with color 1 and v_i , i even and $2 \leq i \leq k$, with color 5.

Let the vertices of the cycle which are adjacent with v_i be $u_{i1}, u_{i2}, ..., u_{in}$, $1 \leq i \leq k$. For each i,i odd and $1 \leq i \leq k$, color u_{ij} , j odd and $1 \leq j \leq n\text{-}2$, with color 5 and color u_{ij} , j even and $2 \leq j \leq n\text{-}1$, with color 3 and u_{in} with color 4. For each i, i even and $2 \leq i \leq k$, color u_{ij}, j odd and $1 \leq j \leq n\text{-}2$, with color 1 and color u_{ij} , j even and $2 \leq j \leq n\text{-}1$, with color 3 and u_{in} with color 2. Since $m_s \leq 3$ for any $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, this coloring is a local coloring and value of this coloring is 5. Therefore $\chi_\ell \leq 5$. Since K_3 is a subgraph of this graph, $\chi_\ell \geq 4$.

Suppose there is a local coloring of this graph with four colors, then the color 4 may be given to v_i or to the vertices of u_{ij}, $1 \leq j \leq n$. Suppose the color 4 is given to v_i , for some i , the color 1 should be given to alternate vertices of u_{ij} so that no two adjacent vertices receive the same color. Then the remaining vertices of u_{ij} , $1 \leq j \leq n$, cannot be colored with the remaining colors 2 and 3 so that the coloring is local.

Suppose color 4 is assigned to alternate vertices of u_{ij} so that no two adjacent vertices receive the same color then color 1 may be assigned to v_i or some vertices u_{ij} , $1 \leq j \leq n$. In both the cases, the remaining vertices cannot be colored with the remaining colors 2 and 3 so that the coloring is local. Thus, there is no local coloring with four colors. Hence χ_ℓ= 5.

Now we show that all five colors 1,2, 3, 4 and 5 should be used in any minimum local coloring of this graph.

Since 5 and 1 must be used in any local coloring of this graph $P_k \,^\circ C_n$, n odd and $n > 3$, it is sufficient to show that colors 2, 3 and 4 must be used in any minimum local coloring of this graph. Several cases are to be considered.

Case 1: Suppose 4 is not used in a minimum local coloring of $P_k \,^\circ C_n$, n odd and $n > 3$. Then the colors used are 1,2,3 and 5.

Case 1(a): Color 5 is used to some v_i. Then we should alternate the colors 3 and 1 to the vertices of $u_{i1}, ..., u_{in-1}$ and the color 2 to u_{in} . Now consider the adjacent vertex of v_i on the path P_k, say v_j. v_j cannot be colored with 5 and can be colored with any one of the colors 1,2 or 3. In this case, the vertices u_{j1}, ... , u_{jn} cannot be colored so that the coloring is a local coloring.

Case 1(b): Color 5 is used to some of the vertices $u_{ij}, j = 1, 2, ..., n$, which are adjacent with v_i. Then one of the colors 1,2 or 3 should be assigned to v_i. In this case, the remaining vertices of $u_{ij}, j = 1, 2, ..., n$ cannot be colored with the remaining colors so that the coloring is a local coloring.

Case 2: Suppose 3 is not used in some minimum local coloring of $P_k \,^\circ C_n$, n odd and $n > 3$. Then the colors used are 1,2,4 and 5.

Case 2(a): Color 5 is used to some v_i. Then the vertices $u_{ij}, j = 1, 2, ..., n$ cannot be colored with the remaining colors 1,2 and 4 so that the coloring is a local.

Case 2(b): Color 5 is used to some of the vertices $u_{ij}, j = 1, 2, ..., n$ which are adjacent with v_i. Then we cannot use color 4 for the vertices in $u_{ij}, j \neq n$ or v_i. Now the remaining vertices cannot be colored with 1 and 2 as the coloring is local.

Case 3: Suppose 2 is not used in some minimum local coloring of

$P_k \circ C_n, n$ odd and $n > 3$. Then the colors used are 1,3,4 and 5. The proof of this case is very similar to case 2.

Hence the coronas $P_k \circ C_n$, n odd and $n > 3$ and $k > 1$ are strong local colorable and its strong local chromatic number is 5.□

Corollary 2 : The corona $P_k \circ C_3$ is not strong local colorable and its local chromatic number is 5.

Proof: Let the vertices of P_k be $v_1, v_2, ..., v_k$ and the vertices adjacent to v_i be u_{i1}, u_{i2} and u_{i3} . For each i, i odd and $1 \leq i \leq k$, color v_i with color 5 and u_{i1}, u_{i2} and u_{i3} with color 1,2 and 4 respectively. For each i, i even and $2 \leq i \leq k$, color v_i with color 1 and u_{i1}, u_{i2} and u_{i3} with colors 2,4 and 5 respectively. Clearly this is a local coloring of this graph and so $\chi_\ell \leq 5$. Since K_3 is a subgraph of this graph, $\chi_\ell \geq 4$. But this graph cannot have a local coloring with 4 colors that is 1,2,3 and 4 since K_4 is a subgraph of this graph and local chromatic number of K_4 is 5. Hence $\chi_\ell = 5$.

Since color 3 is not present in this minimum coloring, $P_k \circ C_3$ is not strong local colorable. □

Theorem 4 *If n is even, the corona $P_k \circ C_n$, $k \geq 2$ is not strong local colorable and its local chromatic number is 5.*

Proof: Consider $P_k \circ C_n$, n even. Let the vertices of P_k be $v_1, v_2, ..., v_k$. Color v_i, i odd and $1 \leq i \leq k$ with color 1 and v_i , i even and $2 \leq i \leq k$, with color 5. For each i , i odd and $1 \leq i \leq k$, the adjacent vertices of v_i which are on the cycle be colored alternatively with 5 and 3. For each i, i even and $2 \leq i \leq k$, the adjacent vertices of v_i which are on the cycle be colored alternatively with 1 and 3. Since $m_s \leq 3$ for any $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, this coloring is a local coloring and value of this coloring is 5. Therefore $\chi_\ell \leq 5$. Since wheel is a subgraph of $P_k \circ C_n$, n even, and its local chromatic number is 5, $\chi_\ell(P_k \circ C_n) \geq 5$, n even. Hence $\chi_\ell(P_k \circ C_n) = 5$ for even n.

Since the color 2 and 4 are not present in this minimum local coloring, this graph is not strong local colorable. □

Theorem 5 *If n is even and $n \geq 6$, the corona $P_k \circ W_n$ is strong local colorable and its strong local chromatic number is 6.*

Proof: Let the vertices of P_k be $v_1, v_2, ..., v_k$, the central vertex of the wheel which is adjacent with $v_i$be w_i and the other vertices of the wheel which are adjacent with v_i are $u_{i1}, u_{i2}, ..., u_{in-1}$. Color v_i, i odd and $1 \leq i \leq k$ with color 6 and v_i, i even and $2 \leq i \leq k$ with color 2. For each odd i and $1 \leq i \leq k$, color w_i with color 5, color u_{ij} , j odd and $1 \leq j \leq n-3$, with color 1, color u_{ij},j even and $2 \leq j \leq n-2$, with color 3 and color u_{in-1} by color 2. For each even i and $2 \leq i \leq k$, color w_i with color 1, color u_{ij}, j odd and $1 \leq j \leq n-3$ with color 4, color u_{ij}, j even and $2 \leq j \leq n-2$, with color 6 and color u_{in-1}

by color 5. Since $m_s \leq 3$ for any $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, this coloring is a local coloring and value of this coloring is 6. Therefore$\chi_\ell \leq 6$. Since wheel is a subgraph of $P_k \circ W_n$, n even and $n \geq 6$ and its local chromatic number is 5, $\chi_\ell \geq 5$. Suppose there is a coloring of this graph with 5 colors1,2,3,4,5 then all the 5 colors should be used, since n is even and this cannot be a local coloring since v_is are adjacent to all vertices of i^{th} copy of the wheel. Hence $\chi_\ell = 6$.

Now we show that all the six colors 1,2,3,4,5 and 6 should be used in any minimum local coloring of this graph. Since color 6 and 1 must be used in any local coloring of this graph $P_k \circ W_n$, n even and $n \geq 6$, it is sufficient if we show that the colors 2,3,4,and 5 must be used in any minimum local coloring of this graph. We note that since v_i is adjacent to all the vertices of the i^{th} copy of the wheel, we need at least five colors to give a coloring of this graph. Some cases arise as follows:

Case 1: Suppose color 5 is not used in a minimum local coloring of $P_k \circ W_n, n$ even and $n \geq$ *6.* Then the colors used are 1,2,3,4 and 6. For each i, since $v_i$is adjacent to all the vertices of the i^{th} copy of the wheel, v_i can receive only the color 6. In this case, we cannot color other v_is with the remaining colors. Hence color 5 should be used in any minimum local coloring of this graph.

Case 2: Suppose color 4 is not used in a minimum local coloring of $P_k \circ W_n, n$ even and $n \geq 6$. Then the colors used are 1,2,3,5 and 6.

Case 2a : Suppose color 6 is used to v_i for some i. Then w_i should receive color 5. The vertices $u_{ij}, 1 \leq j \leq n-1$, will receive the colors 1,2 or 3. Now consider the adjacent vertex v_j of v_i . v_j cannot be colored with 6 or 5. So v_j may be colored with 1,2 or 3. In any case, w_j and $u_{j\alpha}, 1 \leq \alpha \leq n-1$ cannot be colored with the remaining colors so that the coloring is local.

Case 2b: Suppose color 6 is not used to v_i then this color should be used to color w_i only while color 5 should be used to color this v_i; otherwise the colors 1,2 and 3 will apply to the vertices of K_3. This contradicts the definition of local coloring. Now consider the adjacent vertex v_j of v_i . $v_j$which cannot be colored with 6 or 5. So v_j may be colored with 1,2 or 3. In any case, $w_j$and $u_{j\alpha}, 1 \leq \alpha \leq n-1$ cannot be colored with the remaining colors so that the coloring is local.

Case 3 : Suppose color 3 is not used in a minimum local coloring of $P_k \circ W_n, n$ even and $n \geq 6$. Then the colors used are 1,2,4,5 and 6. The colors 4,5 or 6 cannot be used to color any v_i since colors 4,5 and 6 will color the vertices of K_3, which contradicts the definition of local coloring. So colors 1 or 2 must be used to color v_is. Suppose for some i, v_i is colored with 1 then w_i must be colored with 2. Now consider the adjacent vertex v_j of $v_i.v_j$ cannot be colored with 1 or 2. Thus, color 4,5 or 6 may be used to color v_j . Then colors 4,5 and 6 will color the vertices of a K_3, which contradicts the definition of local coloring.

A similar argument can be given if v_i is colored with 2.

Case 4: Suppose color 2 is not used in a minimum local coloring of $P_k \circ W_n, n$ even and $n \geq 6$. Then the colors used are 1,3,4,5 and 6.The colors 3,4,5 and 6 cannot be used to color any v_i since then 3 of the colors will color the vertices of a K_3, which contradicts the definition of local coloring. So color 1 must be used to color v_i. Then the remaining vertices cannot be colored with the remaining colors so that the coloring is local.

Thus all six colors, 1,2,3,4,5 and 6 should be used in any minimum local coloring of this graph. Hence, $P_k\ ^\circ W_n, n$ even and $n \geq 6$ are strong local colorable and $\chi_{s\ell} = 6.\Box$

Corollary 3: The corona $P_k\ ^\circ W_4$ are not strong local colorable and its local chromatic number is 7.

Proof: Consider the corona $P_k\ ^\circ W_4$. Let the vertices of P_k be $v_1, v_2, ..., v_k$. Each v_i is adjacent with every vertex of the i^{th} copy of W_4. Let $w_i$be the central vertex of the i^{th} copy of the wheel and other vertices are u_{i1}, u_{i2} and u_{i3} . Color v_i, i odd and $1 \leq i \leq k$ with color 7 and v_i, i even and $2 \leq i \leq k$ with color 1. For each odd i and $1 \leq i \leq k$, color w_i with color 5, u_{i1}, u_{i2} and $u_{i3}$with colors 1, 2 and 4 respectively. For each even i and $2 \leq i \leq k$, color w_i with color 7, u_{i1}, u_{i2} and u_{i3} with colors 2, 4 and 6 respectively. Clearly this is a local coloring of $P_k\ ^\circ W_4$and its value is 7. That is, $\chi_\ell(\ \mathrm{P}_k\ ^\circ \mathrm{W}_n) \leq 7$. Since K_5 is a subgraph of this graph and $\chi_\ell(K_5) = 7$ [1], we have ,$\chi_\ell(P_k\ ^\circ W_4) \geq 7$. Hence, $\chi_\ell(P_k\ ^\circ W_4) = 7.\Box$

Theorem 6 *If n is odd and $n \geq 5$, the corona $P_k\ ^\circ W_n$ is not strong local colorable and its local chromatic number is 6.*

Proof : Let the vertices of P_k be $v_1, v_2, ..., v_k$. Let the central vertex of the wheel which is adjacent with v_i be w_i and the other vertices of the wheel which are adjacent with v_i are $u_{i1}, u_{i2}, ..., u_{in-1}$. Color v_i, i odd and $1 \leq i \leq k$ with color 6 and v_i, i even and $2 \leq i \leq k$ with color 1. For each i, i odd and $1 \leq i \leq k$, color w_i with color 5 and alternate colors 1 and 3 to the vertices $u_{ij}, j = 1, 2, ..., n-1$. For each i, i even and $2 \leq i \leq k$, color w_i with color 6 and alternate colors 5 and 3 to the vertices $u_{ij}, j = 1, 2, ..., n-1$. Since $m_s \leq 3$ for any $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, this coloring is a local coloring and value of this coloring is 6. Therefore $\chi_\ell \leq 6$. Since wheel is a subgraph of $P_k\ ^\circ W_n$, n odd and $n \geq 5$ and its local chromatic number is 5,$\chi_\ell \geq 5$. Suppose there is a local coloring of this graph with five colors then no two of the colors of the triangle $w_i$with u_{ij} and $u_{ij+1}, 1 \leq j \leq n-2$ be consecutive, denoting the vertex of a triangle with its color. Then v_i cannot be colored with any one of the remaining two colors since v_i forms a triangle with any two vertices of w_i, u_{ij} and u_{ij+1} so that the coloring is local. Hence χ_ℓ= 6.

Since the colors 2 and 4 are not present in this minimum local coloring, this graph is not strong local colorable.$\Box$

In this work, we have studied strong local colorability of some coronas and determined their strong local chromatic number. We believe that this paper will spark interest in the study of strong local colorable graphs.

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