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Diagram Groups over Union of Two Semigroup Presentations of Natural Numbers by Adding Relations

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Abstract For any given semigroup presentation P , we may obtain diagram groups with base point u, where u is a word in generating set. The purpose of this paper is to determine the graphs $\Gamma_{p_n}(n \in N)$, which are obtained from diagram group for union of two semigroup presentations with two and three different initial generators. The general polynomial of vertices, edge in these graphs with three and two different initial generators by adding a relation will be computed.

Keywords Diagram groups, semigroup presentation; generators; relation; initial generators.

1 Introduction

We obtained the general polynomial of component in graphs for semigroup presentation $P = \langle x, y, z \mid x = y, y = z, x = z \rangle$ and also we obtain the spanning tree in graphs of diagram group over this semigroup presentation using lifting method (refer to [1], and [2]. In this research we want to determine the semigroup presentation of union of two semigroup presentation of natural numbers with three and two different initial generators by adding a relation.

Let $P_1 = \langle x, y, z \mid x = y, y = z, x = z \rangle$, $P_2 = \langle a, b \mid a = b \rangle$ be semigroup presentations. Now we consider the new semigroup presentation

$$P = \langle x, y, z, a, b \mid x = y, y = z, x = z, a = b, x = a \rangle$$

obtained from union of initial generators and relations of P_1 and P_2 by adding a relation $x = a$(for more detail see [3–5].

In section 2, we will determine the graphs $\Gamma_{p_n}(n \in N)$ obtained from semigroup presentation$P = \langle x, y, z, a, b \mid x = y, y = z, x = z, a = b, x = a \rangle$.

In section 3, the general polynomial of vertices, edges will be computed.

2 Determining the Graphs $\Gamma_{p_n}(n \in N)$

Let $P = \langle x, y, z, a, b \mid x = y, y = z, x = z, a = b, x = a \rangle$ be a semigroup presentation which is obtained from union of initial generators and relations of P_1 and P_2 by adding a relation $x = a$. Note that the graph $\Gamma(P_1)$ obtained from Semigroup presentation P_1 is just a collection of subgraphs $\Gamma_{p_{1_n}}$ where $\Gamma_{p_{1_n}}$ contains all vertices of length n and respective edges. Similarly we obtain $\Gamma_{p_{2_n}}$ for semigroup presentation P_2. Now we may obtain the graph Γ_{p_n} for semigroup presentation P, the graph $Gamma_{p_n} = \Gamma_{p_{1_n}} \cup \Gamma_{p_{2_n}} \cup \{(u, x_1 \to a_1, v)\}$ such length $uv = n - 1$. If W_n is a vertex in Γ_{p_n} then $w_n g, g \in \{x, y, z, a, b\}$is a vertex in $\Gamma_{p_{n+1}}$.

Similarly if $(u, R_\varepsilon \to R_{-\varepsilon}, v)$ is an edge in Γ_{p_n}, then $(u, R_\varepsilon \to R_{-\varepsilon}, vg)$ is the respective edges in $\Gamma_{p_{n+1}}$. Thus $\Gamma_{p_{n+1}}$ is just five copy of Γ_{p_n} together with five vertices and edges $(u, x_1 \to a_1, vg), g \in \{x, y, z, a, b\}$. For example the graph of Γ_{p_1} is shown in Figure 1.

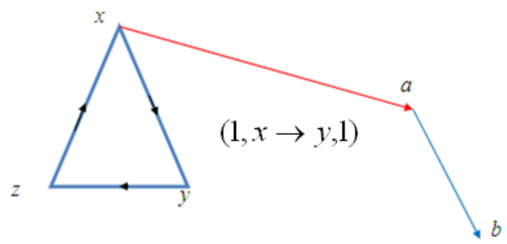

Figure 1: Graph Γ_{p_1}

The graph of Γ_{p_2} is just five copies of Γ_{p_1} as shown in Figure 2.

Similarly we may obtain the graph for $\Gamma_{p_n}(n \in N)$.

Note that Γ_{p_2} is five copies of Γ_{p_1} and each vertex in each copy are joined together, respectively by considering the relation $x_1 = a_1$. Similarly, with five copies of Γ_{p_2}, we may obtainΓ_{p_3}. Repeat similar procedures for Γ_{p_4} and so on.

3 General Polynomials of Component in Graph $\Gamma_{p_n}(n \in N)$, and Theorems

Lemma 3.1: Let$P = \langle x, y, z, a, b \mid x = y, y = z, x = z, a = b, x = a\rangle$ be the presentation. If u and v are two positive words on$\{x, y, z, a, b\}$, then $\pi_1(K(S), u)$= $\pi_1(K(S), v)$ if and only if length(u)= length(v).

Proof: See the proof in [6].

Lemma 3.2: Let the semigroup presentation $P_1 = \langle x, y, z \mid x = y, y = z, x = z\rangle$. The general polynomial of the number of vertices in $\Gamma_{p_n}(n \in N)$is $v_n = 3^n$, wherev_i is the number of vertices in$\Gamma_{p_i}(i = 1, 2, 3, \ldots)$.

Proof: By induction on n, and refer to [2].

Lemma 3.3: Let the presentation $P_2 = \langle a, b \mid a = b\rangle$. The general polynomial of the number of vertices in $\Gamma_{p_n}(n \in N)$ is $v_n = 2^n$, wherev_i is the number of vertices in $\Gamma_{p_i}(i = 1, 2, 3, \ldots)$.

Proof: By induction on n.

Theorem 3.4: Let the presentation $P = \langle x, y, z, a, b \mid x = y, y = z, x = z, a = b, x = a\rangle$. The general polynomial of the number of vertices in $\Gamma_{p_n}(n \in N)$is $v_n = 5^n$, wherev_i is the number of vertices in$\Gamma_{p_i}(i = 1, 2, 3, \ldots)$.

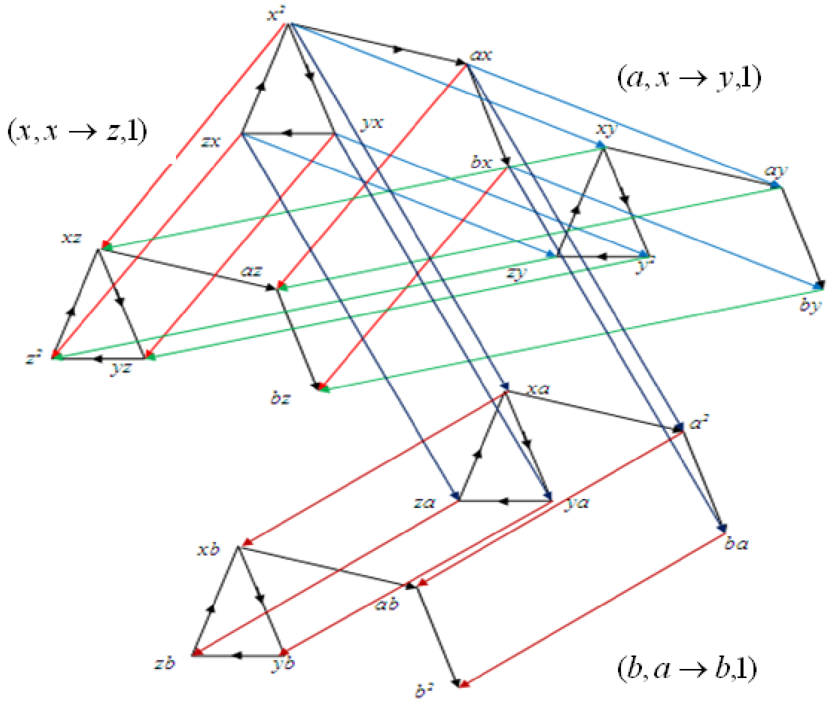

Figure 2: Graph Γ_{p_2}

Proof: By induction, for $k = 1$ the number of all vertices in Γ_{p_1} is 5, thus for $k = 1$ is true (Figure 1). Now assume $v_k = 5^k$ be the number of all vertices in Γ_{p_k}. We will prove that the number of all vertices in $\Gamma_{p_{k+1}}$is$v_{k+1} = 5^{k+1}$. By definition $\Gamma_{p_{k+1}}$ is five copies of $\Gamma_{p_k}$and assumption, then the number of vertices of $\Gamma_{p_{k+1}}$is $v_{k+1} = 5.5^k = 5^{k+1}$.

Theorem 3.5: The general polynomial of the number of all edges in $\Gamma_{p_n}(n \in N)$ is $e_n = 5e_{n-1} + 5^n$, where e_i is the number of all edges in$\Gamma_{p_i}(i = 1, 2, 3, \ldots)$.

Proof: By definition Γ_{p_n} is five copies of $\Gamma_{p_{n-1}}$. Thus if there is e_{n-1} edges in $\Gamma_{p_{n-1}}$, then the number of edges in Γ_{p_n} is $5e_{n-1}$ plus all edges between the vertices in Γ_{p_n}, which is 5^n. Thus the number of all edges in Γ_{p_n} is $e_n = 5e_{n-1} + 5^n$.

Corollary 3.6: The general polynomial of the number of all edges in $\Gamma_{p_n}(n \in N)$ is $e_n = n5^n$, where e_i is the number of all edges in Γ_{p_i} $(i = 1, 2, 3, \ldots)$.

Proof: By induction, the number of all edges in Γ_{p_1} is 3 then$e_1 = 5$. Now let $e_k = k5^k$ be the number of all edges in Γ_{p_k}. We will prove that the number of all edges in $\Gamma_{p_{k+1}}$, is $e_{k+1} = (k + 1)5^{k+1}$. By using the theorem 3.5,

$$e_{k+1} = 5e_k + 5^{k+1} = 5(k5^k) + 5^{k+1} = (k + 1)5^{k+1}.$$

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