MATEMATIKA, 2012, Volume 28, Number 2, 133–149

Effect of Double Stratification on Mixed Convection in a Micropolar Fluid

[1]D. Srinivasacharya and [2]Ch. Ram Reddy
[1,2]Department of Mathematics, National Institute of Technology, Warangal-506004, Andhra Pradesh, India
e-mail: [1]dsrinivasacharya@yahoo.com; [2]chittetiram@gmail.com

Abstract The effects of thermal and solutal stratification on mixed convection heat and mass transfer along a vertical plate embedded in a micropolar fluid has been studied. The nonlinear governing equations and their associated boundary conditions are initially cast into dimensionless forms by pseudo - similarity variables. The resulting system of equations is then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The effect of coupling number, mixed convection parameter and double stratification parameters on local skin-friction and wall couple stress coefficients are discussed. An analysis of the results obtained shows that the flow field is influenced appreciably by coupling number, mixed convection parameter, thermal and solutal stratification parameters.

Keywords Mixed convection; Micropolar fluid; Thermal and Solutal Stratification.

2010 Mathematics Subject Classification 76R05, 76R10, 76A05, 76N20.

1 Introduction

The analysis of mixed convection boundary layer flow along a vertical plate has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Extensive studies of mixed convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation for Newtonian fluids have been undertaken by several authors. Somers [1] considered theoretical results for combined thermal and mass transfer from a flat plate. Theoretical solution of heat transfer by mixed convection about a vertical flat plate has been obtained by Kliegel [2]. Szewczyk [3] studied the combined forced and free convection laminar flow. Lloyd and Sparrow [4] used a local similarity method to solve the mixed convection flow on a vertical surface and showed that the numerical solutions ranged from pure forced convection to mixed convection. Kafoussias [5] presented analysis of the effects of buoyancy forces in a laminar uniform forced-convective flow with mass transfer along a semi-infinite vertical plate. A detailed review of mixed convective heat and mass transfer can be found in the book by Bejan [6].

The study of non-Newtonian fluid flows has gained much attention from researchers because of its applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid also have great importance in engineering applications such as thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available convection heat and mass transfer problems to include the non-Newtonian effects. Many of the non-Newtonian fluid models describe the nonlinear relationship between stress and the rate of strain. But the fluid model introduced by Eringen [7] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The main advantage of using micropolar fluid model compared to other non-Newtonian fluids is that it takes care of the rotation of fluid particles by means of an independent kinematic vector called the microrotation vector. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and porous media is presented by Lukaszewicz [8]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. The boundary layer flow of a micropolar fluid over a semi-infinite plate has been obtained by Ahmadi [9]. Takhar and Soundalgekar [10] considered the heat transfer on a semi-infinite plate of micropolar fluid. Laminar mixed convection boundary layer flow of a micropolar fluid from an isothermal vertical flat plate has been analyzed by Jena and Mathur [11]. Asymptotic boundary layer solutions are presented for the combined convection from a vertical semi-infinite plate to a micropolar fluid by Gorla *et al.* [12]. Tian-Yih Wang [13] proposed a mixed-convection parameter ξ to analyze the steady laminar mixed convection heat transfer between non-Newtonian fluids and a vertical plate with constant wall heat flux.

In many problems of practical interest, natural convection flows arise in a thermally-stratified environment. The input of thermal energy in enclosed fluid regions, clue to the discharge of hot fluid or heat removal from heated bodies, often leads to the generation of a stable thermal stratification. Stratification of fluid arises due to temperature variations, concentration differences or the presence of different fluids. Several investigations have explored the importance of convective heat and mass transfer in doubly stratified porous media using Darcian and non-Darcian models. Previous studies (for a comprehensive review see Gebhart *et al.* [14]) have shown that stratification increases the local heat transfer coefficient and decreases the velocity and buoyancy levels. Another considerable effect of the stratification on the mean field is the formation of a region with a temperature deficit (i.e. a negative dimensionless temperature) and flow reversal in the outer part of the boundary layer. This phenomenon was first shown theoretically by Prandtl [15] for an infinite wall and later on by Jaluria and Himasekhar [16] for semi-infinite walls. Combined heat and mass transfer process by natural convection along a vertical wavy surface in a thermal and mass stratified fluid saturated porous enclosure has been numerically studied by Rathish Kumar and Shalini [18]. Recently, Murthy *et al.* [17], Lakshmi Narayana and Murthy ([19], [20]) reported that the temperature and concentration became negative in the boundary layer depending on the relative intensity of the thermal and solutal stratification.

Only few experimental studies were carried out on vertical free convection in a stratified environment. Jaluria and Gebhart [21] studied the stability of the flow adjacent to a vertical plate dissipating a uniform heat flux into a stratified medium both theoretically and experimentally. For this case a theoretical similarity solution exists, in which the ambient stratification varies like $x^{1/5}$, where x is the downstream coordinate. Unlike the case of linear stratification, the flow reversal and temperature deficit in this case (Jaluria and Gebhart [21]), where the variation of the ambient temperature is relatively weak, are extremely small. More recently, natural convection heat and mass transfer along a vertical plate embedded in a doubly stratified micropolar fluid saturated non-Darcy porous medium is studied numerically using the Keller box method by Srinivasacharya and RamReddy [22].

The aim of this investigation is to consider the effects of thermal and mass stratification on the mixed convection heat and mass transfer along a vertical plate with uniform heat and mass flux conditions embedded in a stable, micropolar fluid. The Keller-box method given in Cebeci and Bradshaw [23] is employed to solve the nonlinear system of this particular problem. The effects of micropolar, mixed convection, thermal and mass stratification parameters are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2 Mathematical Formulation

Consider the steady mixed convection heat and mass transfer along a vertical plate of length L embedded in a stable, a doubly stratified micropolar fluid with free stream velocity u_∞, temperature $T_{\infty,0}$ and concentration $C_{\infty,0}$. Choose the coordinate system such that x-axis is along the vertical plate and y-axis normal to the plate. The physical model and coordinate system are shown in Figure 1. The plate is maintained at uniform and constant heat and mass fluxes q_w and q_m respectively. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0} + Ax$ and $C_\infty(x) = C_{\infty,0} + Bx$ respectively, where A and B are constants and varied to alter the intensity of stratification in the medium.

By employing laminar boundary layer flow assumptions and the Boussinesq approximation, the governing equations for the micropolar fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = (\mu + \kappa)\frac{\partial^2 u}{\partial y^2} + \kappa\frac{\partial \omega}{\partial y} + \rho g^*(\beta_T(T - T_\infty) + \beta_C(C - C_\infty)) \tag{2}$$

$$\rho j\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right) = \gamma\frac{\partial^2 \omega}{\partial y^2} - \kappa\left(2\omega + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{5}$$

where u and v are the velocity components in x and y directions respectively, ω is the component of micro-rotation whose direction of rotation lies in the xy-plane, T is the temperature, C is the concentration, g^* is the acceleration due to gravity, ρ is the density, μ is the dynamic coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, α is the thermal diffusivity and D is the solutal diffusivity of the medium.

The boundary conditions are

$$u = 0, \quad v = 0, \quad \omega = 0, \quad q_w = -k\frac{\partial T}{\partial y}, \quad q_m = -D\frac{\partial C}{\partial y} \qquad at \qquad y = 0 \tag{6a}$$

$$u = u_\infty, \quad \omega = 0, \quad T = T_\infty(x), \quad C = C_\infty(x) \qquad as \qquad y \to \infty \tag{6b}$$

where the subscripts w, $(\infty, 0)$ and ∞ indicate the conditions at the wall, at some reference point in the medium, and at the outer edge of the boundary layer respectively. The boundary condition $\omega = 0$ in Eq. (6a), represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate.

In view of the Continuity Eq. (1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

Substituting Eq. (7) in Eqs. (2)-(5) and then using the following similarity transformations

$$\left.\begin{aligned}
&\xi = \frac{x}{L}, \quad \eta = \left(\frac{Re_L}{\xi}\right)^{1/2}\frac{y}{L}, \quad f(\xi,\eta) = \left(\frac{Re_L}{\xi}\right)^{1/2}\frac{\psi}{Lu_\infty},\\
&g(\xi,\eta) = \left(\frac{\xi}{Re_L}\right)^{1/2}\frac{L}{u_\infty}\omega,\\
&\theta(\xi,\eta) = \frac{T - T_\infty(x)}{\frac{q_w L}{k}\xi^{1/2}}Re_L^{1/2}, \quad \phi(\xi,\eta) = \frac{C - C_\infty(x)}{\frac{q_m L}{D}\xi^{1/2}}Re_L^{1/2},
\end{aligned}\right\} \tag{8}$$

we get the following nonlinear system of differential equations.

$$\left(\frac{1}{1-N}\right)f''' + \frac{1}{2}ff'' + \left(\frac{N}{1-N}\right)g' + Ri\xi^{3/2}(\theta + \mathcal{B}\phi) = \xi\left[f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right] \tag{9}$$

$$\lambda g'' + \frac{1}{2}fg' + \frac{1}{2}f'g - \left(\frac{N}{1-N}\right)\mathcal{J}\xi(2g + f'') = \xi\left[f'\frac{\partial g}{\partial \xi} - g'\frac{\partial f}{\partial \xi}\right] \tag{10}$$

$$\frac{1}{Pr}\theta'' + \frac{1}{2}f\theta' - \frac{1}{2}f'\theta - \varepsilon_1\xi^{1/2}f' = \xi\left[f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right] \tag{11}$$

$$\frac{1}{Sc}\phi'' + \frac{1}{2}f\phi' - \frac{1}{2}f'\phi - \varepsilon_2\xi^{1/2}f' = \xi\left[f'\frac{\partial \phi}{\partial \xi} - \phi'\frac{\partial f}{\partial \xi}\right] \tag{12}$$

where the primes indicate partial differentiation with respect to η alone, $Re_L = \frac{u_\infty L}{\nu}$ is the Reynolds number, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $\mathcal{J} = \frac{L^2}{jRe_L}$ is the micro-inertia density, $\lambda = \frac{\gamma}{j\rho\nu}$ is the spin- gradient viscosity and $N =$

$\dfrac{\kappa}{\mu+\kappa}$ $(0 \le N < 1)$ is the Coupling number. $Ri = \dfrac{Gr_L}{Re_L^{5/2}}$ is the mixed convection parameter, which represents the ratio of buoyancy forces to the inertia forces and is used to delineate the free, forced and mixed convection regimes. $Ri \ll 1$ corresponds to pure forced convection, whereas $Ri \gg 1$ corresponds to pure free convection. $Gr_L = \dfrac{g^* \beta_T q_w L^4}{k\nu^2}$ is the thermal Grashof number and $\mathcal{B} = \dfrac{\beta_C q_m k}{\beta_T q_w D}$ is the buoyancy ratio. When $\mathcal{B} = 0$, the flow is driven by thermal buoyancy alone. $\varepsilon_1 = \dfrac{Ak}{q_w} Re_L^{1/2}$ and $\varepsilon_2 = \dfrac{BD}{q_m} Re_L^{1/2}$ are the thermal and solutal stratification parameters.

The boundary conditions (6) in terms of f, g, θ, ϕ become

$$\eta = 0: \ f'(\xi,0) = 0,\ f(\xi,0) = -2\xi \left(\frac{\partial f}{\partial \xi}\right)_{\eta=0},\ g(\xi,0) = 0,\ \theta'(\xi,0) = -1,\ \phi'(\xi,0) = -1 \tag{13a}$$

$$\eta \to \infty: \ f'(\xi,\infty) = 1,\ g(\xi,\infty) = \theta(\xi,\infty) = \phi(\xi,\infty) = 0 \tag{13b}$$

If $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$, the problem reduces to mixed convection heat and mass transfer on a vertical plate with uniform and constant heat and mass fluxes in an unstratified micropolar fluid and for the stratification, we have $\varepsilon_1 > 0$ and $\varepsilon_2 \neq 0$. We notice that for $N = 0$, where the flow, temperature and concentration fields are unaffected by the microstructure of the fluid and the microrotation component is a passive quantity. Hence, in the limit $N \to 0$, the governing Eqs. (1)-(5) reduce to the corresponding equations for a viscous fluid. Furthermore, the case of the pure forced convection along a flat plate of Lee *et al.* [24] can be obtained by taking $N = 0$, $Ri = 0$, $\mathcal{B} = 0$, $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$, who investigated the mixed convection heat transfer in an unstratified Newtonian fluid flow along a cylinder.

The wall shear stress and the wall couple stress are

$$\tau_w = \left[(\mu+\kappa)\frac{\partial u}{\partial y} + \kappa\omega\right]_{y=0}, \qquad m_w = \gamma \left[\frac{\partial \omega}{\partial y}\right]_{y=0} \tag{14}$$

and the heat and mass transfers from the plate respectively are given by

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}, \qquad q_m = -D\left[\frac{\partial C}{\partial y}\right]_{y=0}. \tag{15}$$

The dimensionless wall shear stress $C_f = \dfrac{2\tau_w}{\rho U_*^2}$, wall couple stress $M_w = \dfrac{m_w}{\rho U_*^2 L}$, the local Nusselt number $Nu_x = \dfrac{q_w x}{k(T_w - T_{\infty,0})}$ and local Sherwood number $Sh_x = \dfrac{q_m x}{D(C_w - C_{\infty,0})}$, where $U_* = u_\infty$ is the characteristic velocity, are given by

$$C_f Re_x^{1/2} = \left(\frac{2}{1-N}\right) f''(\xi,0), \qquad M_w Re_x = \left(\frac{\lambda}{\mathcal{J}}\right) g'(\xi,0) \tag{16a}$$

$$\frac{Nu_x}{Re_x^{1/2}} = \frac{1}{\varepsilon_1 \xi^{1/2} + \theta(\xi,0)}, \qquad \frac{Sh_x}{Re_x^{1/2}} = \frac{1}{\varepsilon_1 \xi^{1/2} + \phi(\xi,0)}, \tag{16b}$$

where $Re_x = \dfrac{u_\infty x}{\nu}$ is the local Reynolds number.

3 Results and Discussions

The flow Eqs. (9) and (10) which are coupled, together with the energy and concentration Eqs. (11) and (12), constitute non-linear nonhomogeneous differential equations for which closed-form solutions cannot be obtained and hence we have to solve the problem numerically. Hence the governing boundary layer Eqs. (9) to (12) have been solved numerically using the Keller-box implicit method discussed in Cebeci and Bradshaw [23]. The method has the following four main steps:

i Reduce the system of Eqs. (9) to (12) to a first order system;

ii Write the difference equations using central differences;

iii Linearize the resulting algebraic equations by Newtons method and write them in matrix-vector form;

iv Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been proven to be adequate and give accurate results for boundary layer equations. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity approaches one and the microrotation, temperature and concentration profiles approach zero. We have taken $\eta_\infty = 8$ and a grid size of η of 0.01. We have computed the solutions for the dimensionless velocity, angular momentum, temperature and concentration function as shown graphically in Figure 2(a) to Figure 5(d). The effects of micropolar parameter N, mixed convection parameter Ri and stratification parameters, ε_1 and ε_2 have been discussed. To study the effect of N, Ri, ε_1 and ε_2, computations were carried out by taking $\mathcal{J} = 0.01$, $\lambda = 0.5$, $\mathcal{B} = 1.0$, $Pr = 0.71$, $Sc = 0.22$ and $\xi = 0.1$. These values are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [7].

In the absence of coupling number N, mixed convection parameter Ri, stratification parameters ε_1 and ε_2 with $\mathcal{J} = 0$, $\lambda = 0$ and $\mathcal{B} = 0$, the results have been compared with that of previous work Lee *et al.* [24] and it is found that they are in good agreement, as shown in Table 1.

In Figure 2(a) to Figure2(d), the effects of the coupling number N on the dimensionless velocity, microrotation, temperature and concentration profiles are presented for fixed values of mixed convection parameter, thermal and solutal stratification parameters. As N increases, it can be observed from Figure 2(a) that the maximum velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall. The velocity in case of micropolar fluid is less compared to the case of viscous fluid case ($N \to 0$ corresponds to viscous fluid). From Figure 2(b), we see that the microrotation changes in sign from negative to positive within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an increase in the value of N. As $N \to 0$, the microrotation tends to zero because in the limit $N \to 0$, the micro polarity is lost and the fluid is to behave as non-polar fluid. It is clear from Figure 2(c) that the temperature boundary layer thickness increases with the increase of coupling number N. It can be seen from Figure 2(d) that the concentration boundary layer thickness enhances with the increase of coupling number N. The temperature and concentration in case of micropolar fluids is more than that of the corresponding Newtonian fluid case.

Figure 3(a) shows the dimensionless velocity profile for various values of the mixed convection parameter Ri with fixed values of N, ε_1 and ε_2. It reveals that as the value of Ri increases, the dimensionless velocity rises. Compared with the limiting case of $Ri = 0$ (i.e., pure forced convection), an increase in the value of Ri gives rise to a higher velocity. Since a greater value of Ri indicates a greater buoyancy effects in mixed convection flow leads to an acceleration of the fluid flow. From Figure 3(b), we observe that the microrotation changes in sign from negative to positive within the boundary layer. Moreover, the magnitude of microrotation increases with an increase in the value of Ri. Figure 3(c) illustrates the dimensionless temperature for selected values of Ri. The results indicate that the dimensionless temperature decreases with increase in the value of Ri. In the limiting case of $Ri = 0$ (i.e., pure forced convection), an increase in the value of Ri gives rise to a reduced temperature. The reason is that a greater value of Ri indicates a greater buoyancy effects, which increases the convection cooling effect and hence reduces the temperature. The effect of mixed convection parameter Ri on the dimensionless concentration is depicted in Figure 3(d). It is clear that the concentration boundary layer thickness reduces with the enhance of mixed convection parameter Ri.

Figure 4(a) displays the non-dimensional velocity for different values of thermal stratification parameter ε_1 for fixed values of N, Ri and ε_2. Also, it can be noted that the velocity of the fluid decreases with the increase of thermal stratification parameter. This is because of thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. From Figure 4(b), we observe that the microrotation changes in sign from negative to positive within the boundary layer. The dimensionless temperature for different values of thermal stratification parameter for $N = 0.3$, $Ri = 0.5$ and $\varepsilon_2 = 0.1$, is shown in Figure 4(c). It is evident that the temperature of the fluid decreases with the increase of thermal stratification parameter. When the thermal stratification effect is taken into consideration, the effective temperature difference between the plate and the ambient fluid will decrease; therefore, the thermal boundary layer is thickened and the temperature is reduced. Figure 4(d) depicts the dimensionless concentration for different values of thermal stratification parameter for $N = 0.3$, $Ri = 0.5$ and $\varepsilon_2 = 0.1$. It is seen that the concentration of the fluid increases with the increase of thermal stratification parameter. It can be noted that the effect of the stratification on the temperature is the formation of a region with a temperature deficit (i.e. a negative dimensionless temperature). This is in tune with the observation made in Refs [Gebhart *et al.* [14]; Prandtl [15]; Jaluria and Himasekhar [16]; Murthy *et al.* [17]; Lakshmi Narayana and Murthy [19], [20]].

The dimensionless velocity component for different values of solutal stratification parameter ε_2 with constant N, Ri and ε_1, is depicted in Figure 5(a). It is observed that the velocity of the fluid decreases with the increase of solutal stratification parameter. From Figure 5(b), we notice that the microrotation changes in sign from negative to positive within the boundary layer. The dimensionless temperature for different values of solutal stratification parameter for $N = 0.3$, $Ri = 0.5$ and $\varepsilon_1 = 0.1$, is displayed in Figure 5(c). It is evident that the temperature of the fluid increases with the increase of solutal stratification parameter. Figure 5(d) demonstrates the dimensionless concentration for different values of solutal stratification parameter with $N = 0.3$, $Ri = 0.5$ and $\varepsilon_1 = 0.1$. It is clear that the concentration of the fluid decreases with the increase of thermal stratification parameter. Hence, comparing with the unstratified fluid (i.e. $\varepsilon_1 = \varepsilon_2 = 0$), an increase in the stratification parameters, ε_1 and ε_2, causes a reduction in the velocity boundary layer.

Table 2 shows the effects of the coupling number N on the skin friction parameter $f''(\xi, 0)$ and the dimensionless wall couple stress $g'(\xi, 0)$. It shows that the skin friction factor is lower for micropolar fluid than the Newtonian fluids ($N = 0$). Since micropolar fluids offer a great resistance(resulting from vortex viscosity) to the fluid motion and causes larger skin friction factor compared to Newtonian fluid. The results also indicate that the large values of coupling number N, lower wall couple stresses. Table 2 illustrates the effects of the mixed convection parameter Ri on the skin friction parameter $f''(\xi, 0)$ and the dimensionless wall couple stress $g'(\xi, 0)$. It is observed that the local skin friction factor increases as Ri increases. The reason is that an increase in the buoyancy effect in mixed convection flow leads to an acceleration of the fluid flow, which increases the local skin friction factor. Also, it is found that the wall couple stress decrease with increase of the mixed convection parameter Ri. This observation is consistent with the velocity, microrotation, temperature and concentration distributions presented in Figure 3(a) to Figure 3(d). The effects of the thermal stratification parameter ε_1, on the skin friction parameter $f''(\xi, 0)$ and the dimensionless wall couple stress $g'(\xi, 0)$, are presented in Table 2. It demonstrates that the skin friction parameter decreases while the wall couple stress increases as ε_1 increases. The results of Table 2 describe the effect of solutal stratification parameter ε_2, on the skin friction parameter $f''(\xi, 0)$ and the dimensionless wall couple stress $g'(\xi, 0)$. It is clear that the skin friction parameter decreases but the wall couple stress increases as ε_2 increases. Furthermore, the skin friction parameter is higher and wall couple stress parameter is lower for the unstratified fluid (i.e. $\varepsilon_1 = \varepsilon_2 = 0$) than for the stratified fluid (i.e. $\varepsilon_1 > 0$ and $\varepsilon_2 \neq 0$).

4 Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in a doubly stratified micropolar fluid over a vertical plate with uniform and constant heat and mass flux conditions is presented. Using the pseudo-similarity variables, the governing equations are transformed into a set of nonsimilar parabolic equations where numerical solution has been presented for a wide range of parameters. The higher values of the coupling number N (i.e., the effect of microstructure becomes significant) result in lower velocity, but higher wall temperature, wall concentration distributions in the boundary layer compared the Newtonian fluid case($N = 0$). The numerical results indicate that the micropolar fluids reduce the skin friction and wall couple stresses. An increase in the mixed convection parameter Ri, enhance the velocity and skin friction parameter but reduces temperature and concentration distributions and the wall couple stress in the boundary layer. An increase in the thermal stratification parameter ε_1, reduce the velocity, temperature distributions and skin friction parameter but increase the non-dimensional concentration distribution and the wall couple stress in the boundary layer. An increase in the solutal stratification parameter ε_2, reduce the velocity, concentration distributions and skin friction parameter but increase the temperature distribution and the wall couple stress in the boundary layer. We observe that the microrotation changes in sign from negative to positive values within the boundary layer in the presence of stratification. The results indicate that the skin friction and wall couple stresses are reduced in the case of micropolar fluid when compared with the case of Newtonian fluid.

Nomenclature

A	Slope of ambient temperature
B	Slope of ambient concentration
$\mathcal{B}$	Buoyancy ratio
C	Concentration
C_f	Skin friction coefficient
$C_{\infty,0}$	Ambient concentration
D	Solutal diffusivity
f	Reduced stream function
g^*	Gravitational acceleration
g	Dimensionless microrotation
Gr_L	Thermal Grashof number
j	Micro-inertia density
$\mathcal{J}$	Dimensionless micro-inertia density
k	Thermal conductivity
L	Length of the plate
M_w	Dimensionless wall couple stress
m_w	Wall couple stress
Nu_x	Local Nusselt number
N	Coupling number
Pr	Prandtl number
q_w, q_m	Heat, Mass fluxes
Re_L	Reynolds number
Re_x	Local Reynolds number
Ri	Mixed convection parameter
Sc	Schmidt number
T	Temperature
$T_{\infty,0}$	Ambient temperature
U_*	Characteristic velocity
u, v	Velocity components in x and y directions
u_∞	Free stream velocity
x, y	Coordinates along and normal to the plate
α	Thermal diffusivity
β_T, β_C	Coefficients of thermal and solutal expansion
γ	Spin-gradient viscosity
η	Pseudo-similarity variable
θ	Dimensionless temperature
ϕ	Dimensionless concentration
κ	Vortex viscosity
λ	Dimensionless spin-gradient viscosity
μ	Dynamic viscosity
ν	Kinematic viscosity
ξ	Dimensionless streamwise coordinate
ρ	Density of the fluid
τ_w	Wall shear stress
ψ	Stream function
ω	Component of microrotation
$\varepsilon_1, \varepsilon_2$	Thermal and Solutal stratification parameters

Subscripts

w	Wall condition
∞	Ambient condition
C	Concentration
T	Temperature

Superscript

$'$	Differentiation with respect to η

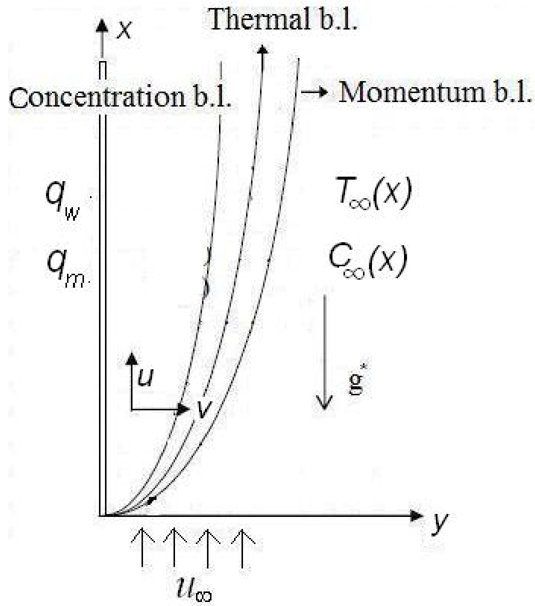
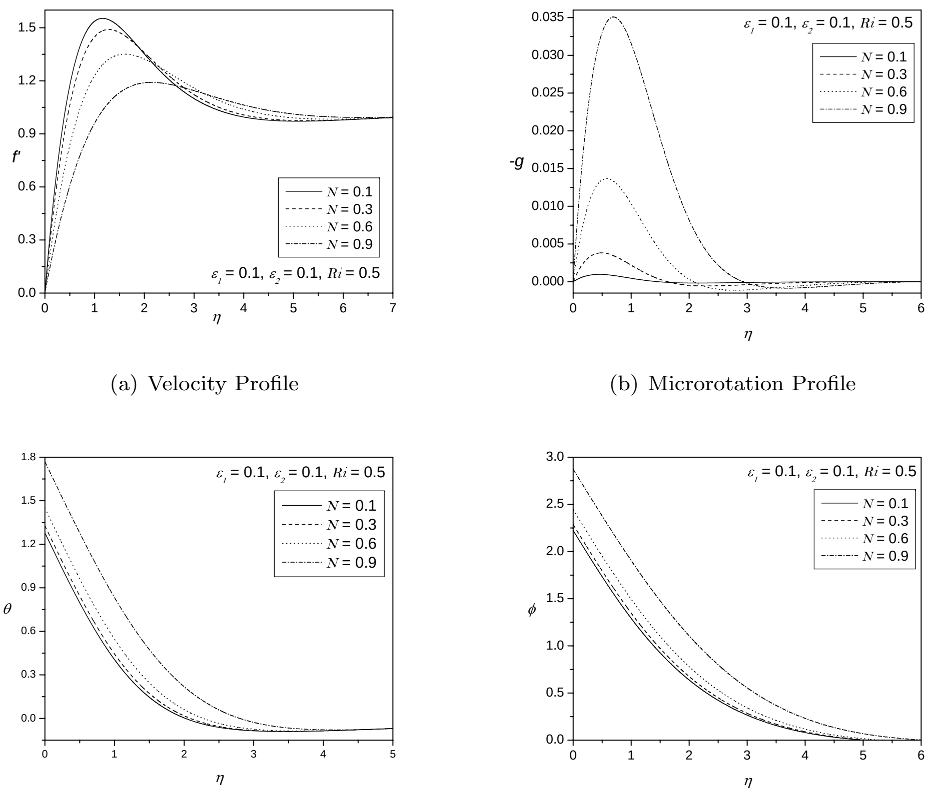


Figure 1: Physical Model and Coordinate System



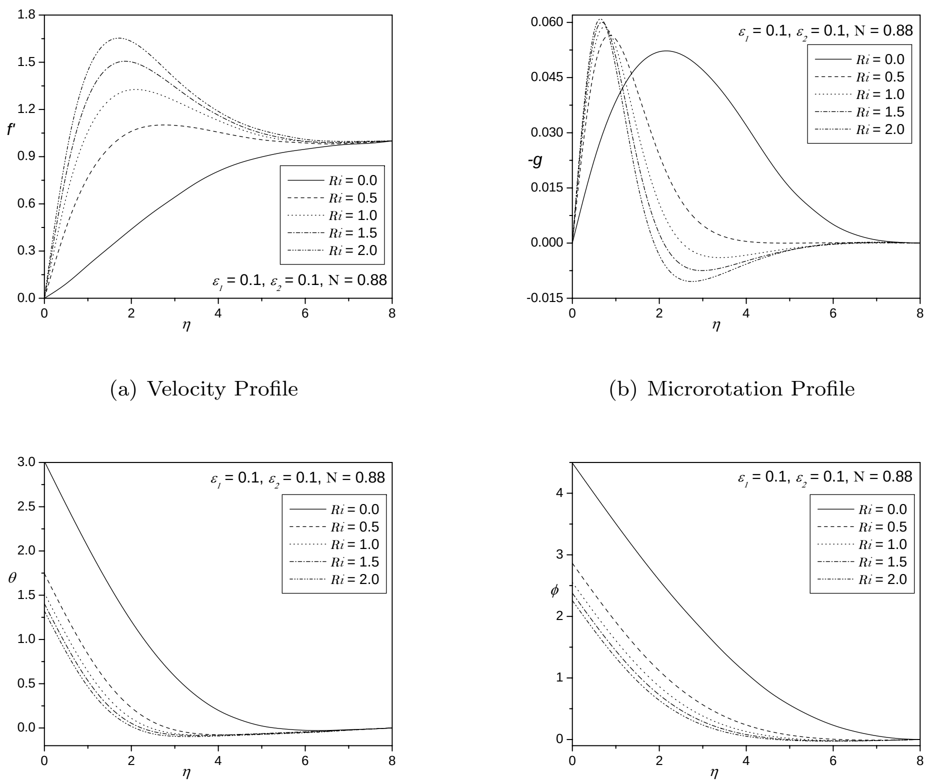
(a) Velocity Profile

(b) Microrotation Profile

(c) Temperature Profile

(d) Concentration Profile

Figure 2: (a)Velocity, (b)Microrotation, (c)Temperature and (d)Concentration Profiles for Various Values of N



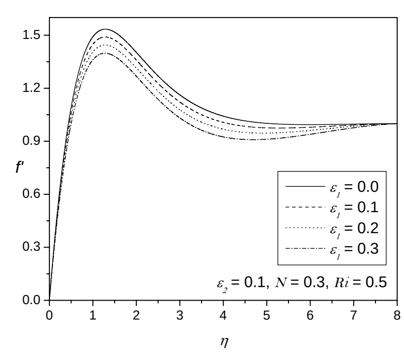
(a) Velocity Profile

(b) Microrotation Profile

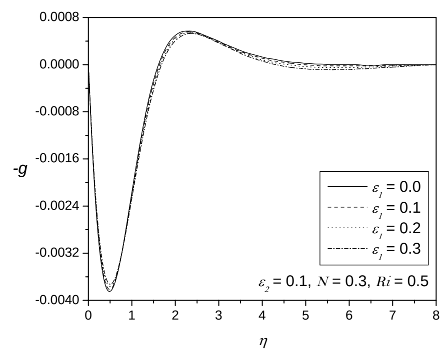
(c) Temperature Profile

(d) Concentration Profile

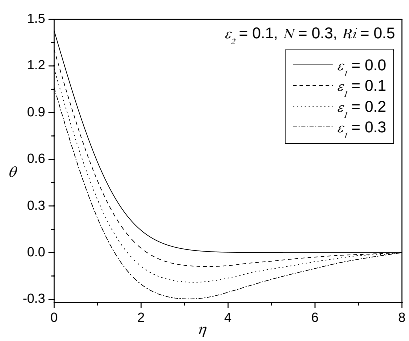
Figure 3: (a)Velocity, (b)Microrotation, (c)Temperature and (d)Concentration Profiles for Various Values of Ri



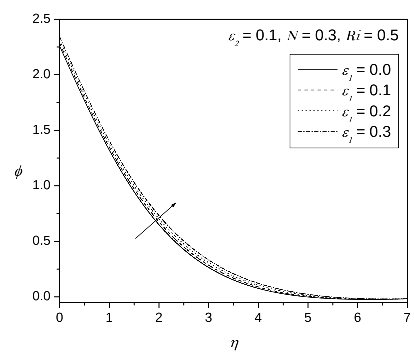
(a) Velocity Profile



(b) Microrotation Profile

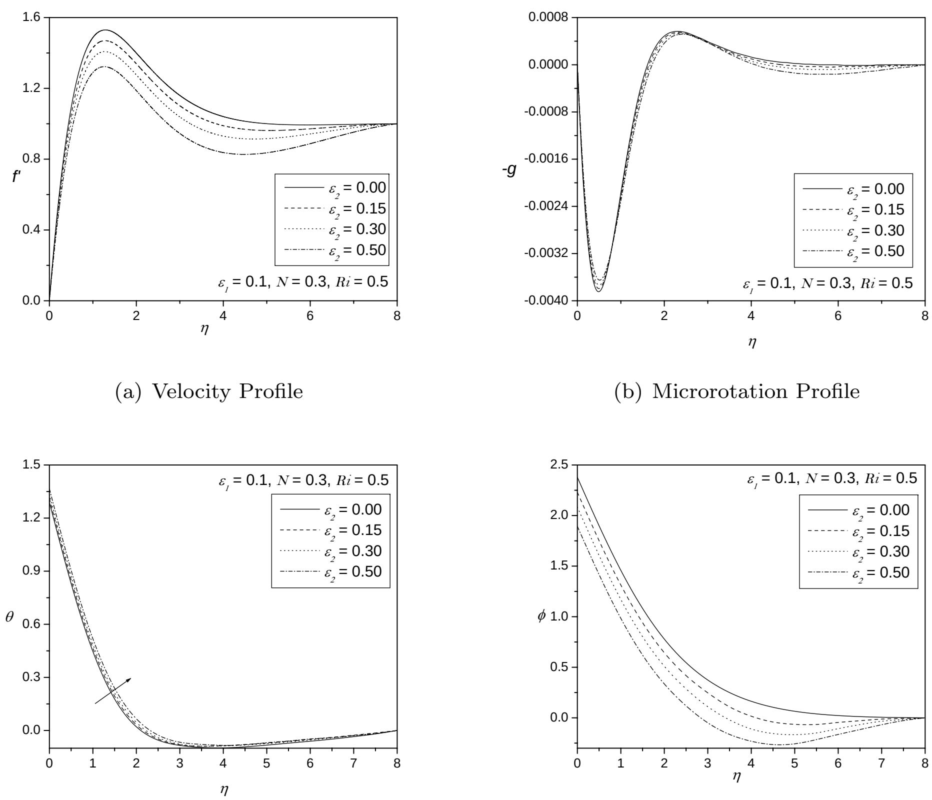


(c) Temperature Profile



(d) Concentration Profile

Figure 4: (a)Velocity, (b)Microrotation, (c)Temperature and (d)Concentration Profiles for Various Values of ε_1



(a) Velocity Profile

(b) Microrotation Profile

(c) Temperature Profile

(d) Concentration profile

Figure 5: (a)Velocity, (b)Microrotation, (c)Temperature and (d)Concentration Profiles for Various Values of ε_2

Table 1: Comparison of $\dfrac{Nu_x}{\sqrt{Re_x}} = \dfrac{1}{\theta(\xi, 0)}$ for Mixed Convection with a Vertical Flat Plate and Newtonian fluids Lee et al. [24]

Pr	Lee *et al.* [24]	Present
0.1	0.2007	0.2007
0.7	0.4059	0.4059
7.0	0.8856	0.8856
100	2.1512	2.1521

Table 2: Effect of Skin Friction and Wall Couple Stress for Various Values of N, Ri, ε_1 and ε_2.

N	Ri	ε_1	ε_2	$f''(\xi, 0)$	$-g'(\xi, 0)$
0.0	0.5	0.1	0.1	3.65187	0.00000
0.1	0.5	0.1	0.1	3.45094	0.00508
0.4	0.5	0.1	0.1	2.77192	0.02770
0.6	0.5	0.1	0.1	2.22066	0.05604
0.9	0.5	0.1	0.1	1.00201	0.20420
0.9	0.0	0.1	0.1	0.11468	0.06207
0.9	0.5	0.1	0.1	1.00201	0.20420
0.9	1.0	0.1	0.1	1.51259	0.26221
0.9	1.5	0.1	0.1	1.92504	0.30399
0.9	2.0	0.1	0.1	2.28370	0.33775
0.3	0.5	0.0	0.1	3.10190	0.01888
0.3	0.5	0.1	0.1	3.01351	0.01849
0.3	0.5	0.2	0.1	2.92529	0.01810
0.3	0.5	0.3	0.1	2.83733	0.01770
0.3	0.5	0.5	0.1	2.74974	0.01730
0.3	0.5	0.1	0.00	3.09250	0.01884
0.3	0.5	0.1	0.15	2.97406	0.01832
0.3	0.5	0.1	0.30	2.85601	0.01779
0.3	0.5	0.1	0.40	2.77764	0.01743
0.3	0.5	0.1	0.50	2.69962	0.01706

References

[1] Somers, E. V. Theoretical considerations of combined thermal and mass transfer from a flat plate. *ASME J. Appl. Mech.* 1956. 23: 295–301.

[2] Kliegel, J. R. *Laminar free and forced convection heat and mass transfer from a vertical flat plate*. University of California: Ph.D. Thesis, 1959.

[3] Szewczyk, A. A. Combined forced and free-convection laminar flow. *J. Heat Transfer*. 1964. C86: 501–507.

[4] Lloyd, J. R. and Sparrow, E. M. Combined free and forced convective flow on vertical surfaces. *Int. J. Heat Mass Transfer.* 1970. 13: 434–438.

[5] Kafoussias, N. G. Local similarity solution for combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. *Int. J. Energy Res.* 1990. 14: 305–309.

[6] Bejan, A. *Convection Heat Transfer*. New York: John Wiley. 1994.

[7] Eringen, A. C. Theory of micropolar fluids. *J. Math and Mech.* 1966. 16: 1–18.

[8] Lukaszewicz, G. *Micropolar Fluids - Theory and Applications*. Basel: Birkhauser. 1999.

[9] Ahmadi, G. Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate. *Int. J. Eng. Sci.* 1976. 14: 639–646.

[10] Takhar, H. S. and Soundalgekar, V. M. Heat transfer on a semi-infinite plate of micropolar fluid. *Rheol. Acta.* 1980. 19: 525–526.

[11] Jena, S. K. and Mathur, M. N. Mixed convection flow of a micropolar fluid from an isothermal vertical plate. *Camp. Maths. with Appls.* 1984. 10: 291–304.

[12] Gorla, R. S. R. , Lin, P. P. and An-Jen Yang. Asymptotic boundary layer solutions for mixed convection from a vertical surface in a micropolar fluid. *Int. J. Engng Sci.* 1990. 28: 525–533.

[13] Wang, T. Y. Mixed convection from a vertical plate to non-Newtonian fluids with uniform surface heat flux. *Int. Comm. Heat Mass Transfer*. 1995. 22: 369–380.

[14] Gebhart, B., Jaluria, Y., Mahajan, R. and Sammakia, B. *Buoyancy lnduced Flows and Transport*. Hemisphere Publishing Co. 1988.

[15] Prandtl. L. *Essentials of Fluid Dynamics.* , London: Blackie. 1952.

[16] Jaluria, Y. and Himasekhar, K. Buoyancy induced two dimensional vertical flows in a thermally stratified environment. *Computer and Fluids*. 1983. 11:39-49.

[17] Murthy, P. V. S. N., Srinivasacharya, D. and Krishna, P. V. S. S. S. R. Effect of double stratification on free convection in Darcian porous medium. *J. Heat Transfer*. 2004. 126: 297–300.

[18] Rathish Kumar, B. V. and Shalini. Double diffusive natural convection in a doubly stratified wavy porous enclosure. *Applied Mathematics and Computation*. 2005. 171:180–202.

[19] Lakshmi Narayana, P. A. and Murthy, P. V. S. N. Free convective heat and mass transfer in a doubly stratified non-Darcy porous medium. *J. Heat Transfer*. 2006. 128: 1204–1212.

[20] Lakshmi Narayana, P. A. and Murthy, P. V. S. N. Soret and Dufour effects on free convection heat and mass transfer in a doubly stratified Darcy porous medium. *J. Porous media*. 2007. 10: 613–623.

[21] Jaluria, Y. and Gebhart, B. Stability and transition of buoyancy induced flows in a stratified medium. *Journal of Fluid Mechanics*. 1974. 66: 593-12.

[22] Srinivasacharya, D. and RamReddy, Ch. Heat and mass transfer by natural convection in a doubly stratified non-Darcy micropolar fluid. *Int. Comm. Heat Mass Transfer*. 2010. 37: 873-880.

[23] Cebeci, T. and Bradshaw, P. *Physical and Computational Aspects of Convective Heat Transfer*. , New York: Springer-Verlin. 1984.

[24] Lee, S. L., Chen, T. S. and Armaly, B. F. Mixed convection along vertical cylinders and needles with uniform surface heat flux. *ASME J. Heat Transfer*. 1987. 109: 711-716.