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Certain Coding Theorems Based on Generalized Inaccuracy Measure of Order α and Type β and 1:1 Coding

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Abstract In this paper, A new mean codeword length $L^t_\beta(U)$ is defined. We have established some noiseless coding theorems based on generalized inaccuracy measure of order α and type β. Further, we have defined mean codeword length $L^t_{\beta,1:1}(U)$ for the best one-to-one code. Also we have shown that the mean codeword lengths $L^t_{\beta,1:1}(U)$ for the best one-to-one code (not necessarily uniquely decodable) are shorter than the mean codeword length $L^t_\beta(U)$. Moreover, we have studied tighter bounds of $L^t_\beta(U)$.

Keywords Generalized inaccuracy measures; Codeword; mean codeword length; Kraft's inequality; Holder's inequality.

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1 Introduction

It is well known fact that information measures are important for practical applications of information processing. For measuring information, a general approach is provided in a statistical framework based on information entropy introduced by Shannon [1]. As a measure of information, the Shannon entropy satisfies some desirable axiomatic requirements and also it can be assigned operational significance in important practical problems, for instance, in coding and telecommunication. In coding theory, usually we come across the problem of efficient coding of messages to be sent over a noiseless channel where our concern is to maximize the number of messages that can be sent through a channel in a given time. Thus, we find the minimum value of a mean codeword length subject to a given constraint on codeword lengths. However, since the codeword lengths are integers, the minimum value will lie between two bounds and a noiseless coding theorem seeks to find these two lower bounds which are in terms of some measure of entropy for a given mean and a given constraint. For uniquely decipherable codes, Shannon [1] found the lower bounds for the arithmetic mean by using his own entropy. Campbell [2] defined his own exponentiated mean and by applying Kraft's [3] inequality, found lower bounds for his mean in terms of Renyi's [4] measure of entropy. Guiasu and Picard [5], Longo [6], Gurdial and Pessoa [7], Taneja and Tuteja [8], Autar and Khan [9], Jain and Tuteja [10], Hooda and Bhaker [11], Bhatia [12] and Singh, Kumar and Tuteja [13] considered the problem of 'useful' information measures and used it studying the noiseless coding theorems for sources involving utilities.

Chapeau-Blondeau *et al.* [14] have presented an extension to source coding theorem traditionally based upon Shannon's entropy and later generalized to Renyi's entropy. Chapeau-Blondeau *et al.* [15] have described a practical problem of source coding and investigated an important relation stressing that Renyi's entropy emerges at an order α differing from the traditional Shannon's entropy.

Ramamoorthy [16] considered the problem of transmitting multiple compressible sources over a network at minimum cost with the objective to find the optimal rates at which the sources should be compressed. Tu *et al.* [17] have presented a new scheme based on variable length coding, capable of providing reliable resolutions for flow media data transmission in spatial communication. Some interesting work for the construction of information theoretic source network coding in the presence of eavesdroppers has been presented by Luo *et al.* [18]. Wu *et al.* [19] have constructed a space trellis and design a low-complexity joint decoding algorithm with a variable length symbol-a posteriori probability algorithm in resource constrained deep space communication networks.

The mean length of a noiseless uniquely decodable code for a discrete random variable X satisfies

$$H\left(X\right) \leq L_{UD} < H\left(X\right) + 1 \tag{1}$$

Where

$$H\left(X\right) = -\sum_{i=1}^{N} p_i \log p_i \tag{2}$$

is the Shannon's entropy [1] of the random variable X and

$$L_{UD} = \sum_{i=1}^{N} p_i n_i \tag{3}$$

be the unique and decodable mean codeword length studied by Shannon [1]. Shannon's restriction of coding of X to prefix codes is highly justified by the implicit assumption that the description will be concatenated and thus must be uniquely decodable and instantaneous codes, cf. [20], [21], the expected codeword length is the same for both the set of codes.

There are some communication situations in which a random variable X is being transmitted rather than a sequence of random variables. For this context Leung-Yang-Cheong and Cover [22] considered one to one codes i.e., codes which assign a distinct binary code to each outcome of the random variable X without regard to the condition that concatenations of the descriptions must be uniquely decipherable.

Bhatia [23], [24], have extended the idea of the one to one code to the Kerridge's inaccuracy [25] and also derived lower bounds to the exponential mean codeword length for the best one to one codes in terms of generalized inaccuracy of order α.

In Section 2, we have established a generalized coding theorem for personal probability codes by considering useful inaccuracy of order α and type β. In Section 3, we generalized the idea of the best 1:1 code to useful inaccuracy of order α and type β.

Throughout the paper $\mathbb{N}$ denotes the set of the natural numbers and for $N \in \mathbb{N}$ we set

$$\Delta_N = \left\{ \left(p_1, ..., p_N\right) / p_i \geq 0, i = 1, ..., N, \sum_{i=1}^{N} p_i = 1 \right\}.$$

In case there is no rise to misunderstanding we write $P \in \Delta_N$ instead of $(p_1, ..., p_N) \in \Delta_N$. Throughout this paper, $\sum$ will stand for $\sum_{i=1}^{N}$ unless otherwise stated and logarithms are taken to the base $D\,(D > 1)$.
Consider the model given below for a finite random experiment scheme having $(x_1, x_2, ..., x_N)$ as the complete system of events, happening with respective probabilities $P = (p_1, p_2, ..., p_N)$,

$p_i \geq 0,\ \sum_{i=1}^{N} p_i = 1$ and credited with utilities $U = (u_1, u_2, ..., u_N)$, $u_i > 0$, $i = 1, 2, ..., N$. Denote

$$S = \begin{bmatrix} x_1\ x_2\ ...\ x_N \\ p_1\ p_2\ ...\ p_N \\ u_1\ u_2\ ...\ u_N \end{bmatrix}. \tag{4}$$

We call (4) the utility information scheme.
Let $Q = (q_1, q_2, ..., q_N)$ be the predicted distribution having the utility distribution $U = (u_1, u_2, ..., u_N)$, $u_i > 0$, $i = 1, 2, ..., N$. Taneja and Tuteja [8] have suggested and characterized the 'useful' inaccuracy measure

$$I(P,Q;U) = -\sum_{i=1}^{N} u_i p_i \log q_i. \tag{5}$$

By considering the weighted mean code word length [5]

$$L(U) = \frac{\sum_{i=1}^{N} u_i p_i n_i}{\sum_{i=1}^{N} u_i p_i} \tag{6}$$

where $n_1, n_2, ..., n_N$ are the code lengths of $x_1, x_2, ..., x_N$ respectively.
Taneja and Tuteja [8] derived the lower and upper bounds on L(U) in terms of I(P,Q;U).
Bhatia [23] defined the 'useful' average code length of order t as

$$L^t(U) = \frac{1}{t}\log\left[\frac{\sum_{i=1}^{N} u_i^{t+1} p_i D^{tn_i}}{\left(\sum_{i=1}^{N} u_i p_i\right)^{t+1}}\right], \quad -1 < t < \infty \tag{7}$$

where D is the size of the code alphabet. He also derived the bounds for the 'useful' average code length of order t in terms of generalized 'useful' in accuracy measure given by

$$I_\alpha(P,Q;U) = \frac{1}{1-\alpha}\log\left[\frac{\sum_{i=1}^{N} u_i p_i q_i^{\alpha-1}}{\sum_{i=1}^{N} u_i p_i}\right], \quad \alpha > 0 (\neq 1) \tag{8}$$

under the condition

$$\sum_{i=1}^{N} p_i q_i^{-1} D^{-n_i} \leq 1 \tag{9}$$

where D is the size of the code alphabet.

In this paper, we study some coding theorems by considering a new function depending on the parameters α and β and a utility function. Our motivation for studying this function is that it generalizes some information measures already existing in the literature.

2 Coding Theorems

Consider a function

$$I_\alpha^\beta(P,Q;U) = \frac{1}{1-\alpha}\log\left[\frac{\sum_{i=1}^{N} u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^\beta}\right], \quad \alpha > 0 (\neq 1),\ \beta \geq 1. \tag{10}$$

(i) When $\beta = 1$, (10) reduces to a measure of 'useful' information measure of order α due to Bhatia [23].

(ii) When $\beta = 1,\ u_i = 1,\ \forall\, i = 1, 2, ..., N.$ (10) reduces to the inaccuracy measure given by Nath [26], further it reduces to Renyi's [4] entropy by taking $p_i = q_i,\ \forall\, i = 1, 2, ..., N.$

(iii) When $\beta = 1,\ u_i = 1,\ \forall\, i = 1, 2, ..., N$ *and* $\alpha \to 1$. (10) reduces to the measure due to Kerridge [25].

(iv) When $u_i = 1,\ \forall\, i = 1, 2, ..., N$ *and* $p_i = q_i,\ \forall\, i = 1, 2, ..., N$ the measure (10) becomes Aczel and Daroczy [27] and Kapur [28] entropy.

We call this $I_\alpha^\beta(P, Q; U)$ in (10) the generalized 'useful' inaccuracy measure of order α and type β.

Further consider

$$L_\beta^t(U) = \frac{1}{t} \log \left[\frac{\sum_{i=1}^N u_i p_i^\beta D^{n_i t}}{\sum_{i=1}^N u_i p_i^\beta} \right], \quad -1 < t < \infty. \tag{11}$$

(i) For $\beta = 1,\ L_\beta^t(U)$ in (11) reduces to the 'useful' mean length $L^t(U)$ of the code given by Hooda and Bhaker [11].

(ii) When $\beta = 1,\ u_i = 1,\ \forall\, i = 1, 2, ..., N,\ L_\beta^t(U)$ in (11) reduces to the mean length given by Campbell [2].

(iii) When $\beta = 1,\ u_i = 1,\ \forall\, i = 1, 2, ..., N$ *and* $\alpha \to 1,\ L_\beta^t(U)$ in (11) reduces to the optimal code length identical to Shannon [1].

(iv) For $u_i = 1,\ \forall\, i = 1, 2, ..., N,\ L_\beta^t(U)$ in (11) reduces to the mean length given by Khan and Ahmed [29].

Now we find the lower bounds of $L_\beta^t(U)$ in terms of $I_\alpha^\beta(P, Q; U)$ under the condition

$$\sum_{i=1}^N u_i p_i^\beta q_i^{-1} D^{-n_i} \leq \sum_{i=1}^N u_i p_i^\beta;\ \beta \geq 1, \tag{12}$$

where D is the size of the code alphabet. Inequalities (9) and (12) are the generalization of Kraft's inequality [30]. A code satisfying generalized Kraft's inequalities (9) and (12) would be termed as personal probability code.

Theorem 1 *For every code whose lengths $n_1, n_2, ..., n_N$ satisfies (12), then the average code length satisfies*

$$L_\beta^t(U) \geq I_\alpha^\beta(P, Q; U), \tag{13}$$

where $\alpha = \frac{1}{1+t}$, the equality occurs if and only if

$$n_i = -\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}}. \tag{14}$$

Proof By Holder's inequality [31]

$$\left(\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{n} y_i^q\right)^{\frac{1}{q}} \leq \sum_{i=1}^{n} x_i y_i, \tag{15}$$

for all $x_i, y_i > 0, \;\; i = 1, 2, ..., N$ and $\frac{1}{p} + \frac{1}{q} = 1, \; p < 1\,(\neq 0)\,, \; q < 0$ *or* $q < 1\,(\neq 0)\,, \; p < 0$. We see that equality holds if and only if there exists a positive constant c such that

$$x_i^p = c y_i^q. \tag{16}$$

Making the substitutions

$$p = -t, \quad q = \frac{t}{t+1}$$

$$x_i = \left(\frac{u_i p_i^\beta}{\sum_{i=1}^N u_i p_i^\beta}\right)^{-\frac{1}{t}} D^{-n_i}, \quad y_i = p_i^{\beta\left(\frac{1+t}{t}\right)} \left(\frac{u_i}{\sum_{i=1}^N u_i p_i^\beta}\right)^{\left(\frac{1+t}{t}\right)} q_i^{-1}$$

in (15) and using (12), we get

$$\left[\frac{\sum_{i=1}^N u_i p_i^\beta D^{n_i t}}{\sum_{i=1}^N u_i p_i^\beta}\right]^{\frac{1}{t}} \geq \left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}\right]^{\frac{1+t}{t}}$$

Taking logarithm of both the sides with base D, we obtain (13). □

Next, we obtain a result giving an upper bound to the generalized average 'useful' codeword length.

Theorem 2 *By properly choosing the lengths $n_1, n_2, ..., n_N$ in the code of Theorem 1, $L_\beta^t(U)$ can be made to satisfy the inequality*

$$L_\beta^t(U) < I_\alpha^\beta(P, Q; U) + 1. \tag{17}$$

Proof Let n_i be the positive integer satisfying, the inequalities

$$-\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}} \leq n_i < -\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}} + 1. \tag{18}$$

Consider the interval

$$\delta_i = \left[-\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}}, -\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}} + 1\right]$$

of length 1. In every δ_i, there is exactly one positive integer n_i such that

$$0 < -\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}} \leq n_i < -\log \frac{q_i^\alpha}{\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{(\alpha-1)}}{\sum_{i=1}^N u_i p_i^\beta}} + 1. \tag{19}$$

It can be shown that the sequence $\{n_i\}\,, \; i = 1, 2, ..., N$ thus defined, satisfies (12).

From the right inequality of (19), we have

$$n_i < -\log \frac{q_i^{\alpha}}{\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}}} + 1$$

$$\Rightarrow D^{n_i\left(\frac{1-\alpha}{\alpha}\right)} < D^{\left(\frac{1-\alpha}{\alpha}\right)} \left[\frac{q_i^{\alpha}}{\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}}} \right]^{\left(\frac{\alpha-1}{\alpha}\right)}. \tag{20}$$

Multiplying both sides of (20) by $\frac{u_i p_i^{\beta}}{\sum_{i=1}^{N} u_i p_i^{\beta}}$, summing over $i = 1, 2, ..., N$ and after that raising to the power $\left(\frac{\alpha}{1-\alpha}\right)$, we get

$$\left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} D^{n_i\left(\frac{1-\alpha}{\alpha}\right)}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right]^{\left(\frac{\alpha}{1-\alpha}\right)} < D \left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right]^{\left(\frac{1}{1-\alpha}\right)}$$

Taking logarithms on both sides and using the relation $\alpha = \frac{1}{1+t}$, we get (17). □

Theorem 3 *For arbitrary $N \in \mathbb{N}$, $\alpha > 0 (\neq 1)$, $\beta \geq 1$ and for every code word lengths n_i, $i = 1, ..., N$ of Theorem 1, $L_{\beta}^{t}(U)$ can be made to satisfy,*

$$L_{\beta}^{t}(U) \geq I_{\alpha}^{\beta}(P, Q; U) > I_{\alpha}^{\beta}(P, Q; U) + \frac{1}{t} Log D. \tag{21}$$

Proof Suppose

$$n_i = -\log \frac{q_i^{\alpha}}{\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}}}, \ \alpha > 0(\neq 1), \ \beta \geq 1.$$

Clearly $\bar{n}_i$ and $\bar{n}_i + 1$ satisfy 'equality' in Holder's inequality (15). Moreover, $\bar{n}_i$ satisfies (12). Suppose n_i is the unique integer between $\bar{n}_i$ and $\bar{n}_i + 1$, then obviously, n_i satisfies (12).
Since $\alpha > 0(\neq 1)$, $\beta \geq 1$, we have

$$\left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} D^{n_i t}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right] \leq \left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} D^{\bar{n}_i t}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right] < D \left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} D^{\bar{n}_i t}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right] \tag{22}$$

Since $\left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} D^{\bar{n}_i t}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right] = \left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right]^{1+t}$.
Hence (22) becomes

$$\left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} D^{n_i t}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right] \leq \left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right]^{1+t} < D \left[\frac{\sum_{i=1}^{N} u_i p_i^{\beta} q_i^{(\alpha-1)}}{\sum_{i=1}^{N} u_i p_i^{\beta}} \right]^{1+t}$$

Raising to the power $\frac{1}{t}$ and taking logarithms on both sides, we get (21). □

3 Lower Bound on the Exponentiated Average 'Useful' Codeword Length for the Best 1:1 Code

Let X be a random variable taking on a finite number of values $(x_1, x_2, ..., x_N)$ with probabilities $(p_1, p_2, ..., p_N)$ and utilities $(u_1, u_2, ..., u_N)$. Let $n_i,\ i = 1, 2, ..., N$ be the lengths of the code words in the best 1:1 binary code (0, 1, 00, 10, 01, 11, 000, ...), for encoding the random variable X, n_i is the length of the codeword assigned to the output x_i. It is clear that $n_1 \leq n_2 \leq n_3 \leq ... \leq n_N$ and in general $n_i = \left[\log_2\left(\frac{i+2}{2}\right)\right]$, where $[x]$ denotes the smallest integer greater than or equal to x. Thus the average 'useful' codeword length for the best 1:1 code is given by

$$L_{1:1}(U) = \frac{\sum u_i p_i \left[\log_2\left(\frac{i+2}{2}\right)\right]}{\sum u_i p_i} \tag{23}$$

When utilities are ignored, (23) reduces to $L_{1:1}$, cf. [22].
From (11), the exponentiated average 'useful' codeword length for binary codes can be given by

$$L^t_\beta(U) = \frac{1}{t}\log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta 2^{n_i t}}{\sum_{i=1}^N u_i p_i^\beta}\right] \tag{24}$$

we may call (24) 'useful' codeword length for the best 1:1 code. Thus the exponentiated average 'useful' codeword length for the best 1:1 code is given by

$$L^t_{\beta,1:1}(U) = \frac{1}{t}\log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta 2^{t\left[\log_2\left(\frac{i+2}{2}\right)\right]}}{\sum_{i=1}^N u_i p_i^\beta}\right] \tag{25}$$

We will now prove the following theorem, which gives a lower bound on $L^t_{\beta,1:1}$.

Theorem 4 *For $I^\beta_\alpha(P,Q;U)$, $L^t_\beta(U)$ and $L^t_{\beta,1:1}$ as given in (10), (24) and (25) respectively, the following estimates hold:*

$$L^t_{\beta,1:1}(U) \geq I^\beta_\alpha(P,Q;U) - \log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{-1}\left(\frac{2}{i+2}\right)}{\sum_{i=1}^N u_i p_i^\beta}\right], \tag{26}$$

and

$$L^t_{\beta,1:1}(U) \geq L^t_\beta(U) - \log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{-1}\left(\frac{1}{i+2}\right)}{\sum_{i=1}^N u_i p_i^\beta}\right] - 2. \tag{27}$$

Proof From (25), we have

$$L^t_{\beta,1:1}(U) \geq \frac{1}{t}\log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta \left(\frac{i+2}{2}\right)^t}{\sum_{i=1}^N u_i p_i^\beta}\right] \tag{28}$$

Now

$$I_\alpha^\beta(P,Q;U) - L_{\beta,1:1}^t(U) \le \left(\frac{t+1}{t}\right)\log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{-\left(\frac{t}{t+1}\right)}}{\sum_{i=1}^N u_i p_i^\beta}\right] - \frac{1}{t}\log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta \left(\frac{i+2}{2}\right)^t}{\sum_{i=1}^N u_i p_i^\beta}\right]$$

$$= \log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{-\left(\frac{t}{t+1}\right)}}{\sum_{i=1}^N u_i p_i^\beta}\right]^{\left(\frac{t+1}{t}\right)} \left[\frac{\sum_{i=1}^N u_i p_i^\beta \left(\frac{i+2}{2}\right)^t}{\sum_{i=1}^N u_i p_i^\beta}\right]^{-\frac{1}{t}} \tag{29}$$

Applying Holder's inequality to (29), we obtain

$$I_\alpha^\beta(P,Q;U) - L_{\beta,1:1}^t(U) \le \log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{-1}\left(\frac{2}{i+2}\right)}{\sum_{i=1}^N u_i p_i^\beta}\right],$$

which gives (26).
Now from (17)

$$L_\beta^t(U) < I_\alpha^\beta(P,Q;U) + 1$$

So

$$L_\beta^t(U) - L_{\beta,1:1}^t(U) < I_\alpha^\beta(P,Q;U) - L_{\beta,1:1}^t(U) + 1$$

$$\le 2 + \log_2\left[\frac{\sum_{i=1}^N u_i p_i^\beta q_i^{-1}\left(\frac{1}{i+2}\right)}{\sum_{i=1}^N u_i p_i^\beta}\right],$$

which proves (27). □

4 Conclusion

We study some coding theorems by considering a new function depending on the parameters α and β and a utility function. Our motivation for studying this function is that it generalizes some information measures already existing in the literature. We know that optimal code is that code for which the value $L_\beta^t(U)$ is equal to its lower bound. From the result of the Theorem 1, it can be seen that the mean codeword length of the optimal code is dependent on three parameters t, U and β, while in the case of Shannon's theorem it does not depend on any parameter. So it can be reduced significantly by taking suitable values of parameters.

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