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Analyses of Prior Selections for Gumbel Distribution

[1]**Mohammad Rostami and** [2]**Mohd Bakri Adam**
[1,2]Intitute for Mathematical Research,
Universiti Putra Malaysia
e-mail: [1]rostamy.mohammad@gmail.com [2]bakri@science.upm.edu.my,

Abstract In this paper, we acquaint some selections of priors for Gumbels' parameters model. Simulation studies of Gumbel Distribution for eighteen pairs of priors based on the parameters' characteristics and existing literatures were carried out. The usage of Markov Chain Monte Carlo via Metropolis-Hasting algorithm is implemented. Our findings show that the combination of Gumbel and Rayleigh are the most compromise pair of priors for Gumbel model. We successfully employed the recommendation of the best pair priors to model the Malaysia Gold prices from 2001 to 2011.

Keywords Gumbel model; Bayesian approach; simulation; MCMC; Malaysia gold prices

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1 Introduction

The Gumbel distribution is one of the special states of generalized extreme value distribution (GEV). In probability theory and statistics, the Gumbel distribution is applied to model the distribution of the maximum (or the minimum) of a number of observations. This distribution is usually applied in forecasting of natural rare events such as floods, earth quakes, volcanic eruptions and other natural disasters.

Let X be a random variable that followed Gumbel distribution with location parameter μ and scale parameter ω denoted as $\mathrm{G}(\mu,\omega)$. The cumulative function is

$$F(x;\mu,\omega) = \exp\left\{-\exp\left[-\left(\frac{x-\mu}{\omega}\right)\right]\right\}, \tag{1}$$

where

$$-\infty < x < +\infty, \qquad -\infty < \mu < +\infty, \qquad \omega > 0$$

and the probability density function is in the form

$$f(x) = \frac{1}{\omega}\exp\left\{-\left(\frac{x_i-\mu}{\omega}\right) - \exp\left[-\left(\frac{x_i-\mu}{\omega}\right)\right]\right\}, \quad -\infty < x < +\infty \tag{2}$$

as shown in Figure 1. Fiorentino and Gabriele [1] proposed some modifications to calculate maximum likelihood estimation of the Gumbel parameters, emphasizing to reduce the bias of quantile estimators and parameters. In addition, Bain [2], Engelhardt and Bain [3], and Hassanein [4] considered a few statistical inferences of Gumbel model for censored data. Furthermore, the Gumbel model, has been characterized by Nagaraja [5] using summation independence of the lower record spacings.

Englund and Rackwitz [6] estimated the Gumbel parameters by using a conjugate prior for each parameter. Moreover, Coles [7] used a Bayesian approach to estimate Gumbel

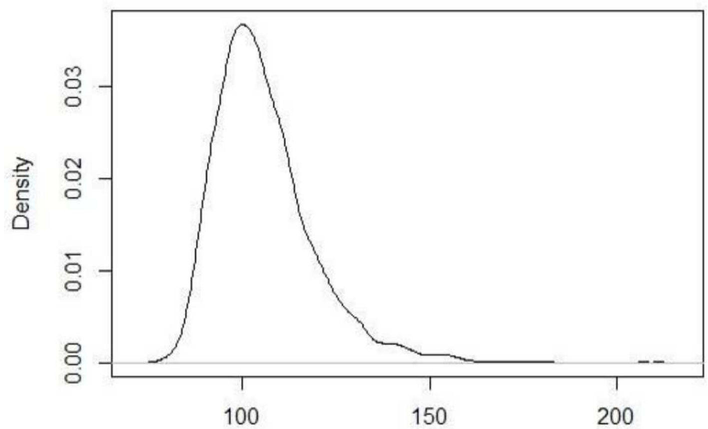

Figure 1: Probability Density of Gumbel Distribution (N=5000 and Bandwidth=1.893)

parameters by choosing normal and log normal as priors for daily rainfall recorded data in Venezuela.

Ali Mousa *et. al* [8] conducted a Bayesian method to examine characterization, prediction and estimation of Gumbel model based on records data. Actually, they proposed a Jeffreys' non-informative prior distribution for μ and a conjugate prior distribution for σ .

Since the selection of prior distributions for Gumbel model are completely subjective and it can lead to various estimation of the parameters. In this paper, we would like to compare different pair of priors and find the best priors for Gumbel model. To achieve this aim, firstly, the joint and marginal posteriors distributions for eighteen pairs of priors are considered. Then, by conducting Monte Carlo Markove Chain (MCMC) method via Metropolis-Hasting algorithm, Bayesian estimations of the parameters are obtained to get the best pair of priors.

2 Likelihood Function

Let $\{X_i = x_i\}, i = 1, 2, \ldots, n$ be an n observations from the Gumbel distribution $G(\mu, \omega)$. The likelihood function is given as follows:

$$\begin{aligned} L(\mu, \omega|\underline{x}) &= \prod_{i=1}^{n} f(x_i, \mu, \omega) \\ &= \prod_{i=1}^{n} \frac{1}{\omega} \exp\left\{-\left(\frac{x_i-\mu}{\omega}\right) - \exp\left[-\left(\frac{x_i-\mu}{\omega}\right)\right]\right\} \\ &= \frac{1}{\omega^n} \exp\left\{-\left(\sum_{i=1}^{n}\frac{x_i-\mu}{\omega}\right) - \sum_{i=1}^{n}\exp\left[-\left(\frac{x_i-\mu}{\omega}\right)\right]\right\}. \end{aligned} \tag{3}$$

3 Posterior Distribution

In this section, posterior distributions are obtained for 18 pairs of priors from Normal (N), Weibull (W), Gumbel (G), Logistic (Lo), Gamma (Ga), Rayleigh (R), Exponential (exp), Log Normal (LN) and Inverse Gaussian (IG) distributions. From equation 1, it is clearly that μ (the location parameter) accepts only real values while ω (the scale parameter) only accepts positive real values. In this paper, priors are selected to follow their domains similar to Gumbel parameters' domains. Additionally for computationally and theoretical easiness, we assumed that μ and ω are independent. The posterior and marginal posterior are presented in Table 1 and Table 2 with A given as

$$A = -\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\omega}\right) - \sum_{i=1}^{n}\exp\left(-\frac{x_i-\mu}{\omega}\right).$$

4 Metropolis-Hasting Algorithm

The Metropolis-Hasting algorithm is a Markov chain Monte Carlo (MCMC) method for collecting a sequence of random samples from a probability distribution where direct sampling is difficult (Gilks *et.al.* [9]). Consider the current values of the chain as $\theta_1^{(j)}, \ldots, \theta_d^{(j)}$ and we need to estimate $\theta_1^{(j+1)}$. This means that we should update $\theta_1^{(j)}$ to $\theta_1^{(j+1)}$ according to conditional distribution $\pi(\theta_1 \mid \theta_2^{(j)}, \ldots, \theta_d^{(j)})$. Metropolis-Hasting updating mechanism is carried out as follows:

- Propose θ_1^{can} as a candidate value *can*, it is drawn from an arbitrary distribution with density $q(\theta_1^{can} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})$.
- Replace the next value of $\theta_1^{(j)}$ in the chain

$$\theta_1^{(j+1)} = \begin{cases} \theta_1^{can} & \text{with probability}\, p, \\ \theta_1^{(j)} & \text{with probability}\, 1-p, \end{cases}$$

where the acceptance probability is

$$p = \min\left\{1, \frac{\pi(\theta_1^{can} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})}{\pi(\theta_1^{j} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})}\frac{q(\theta_1^{(j)} \mid \theta_1^{can}, \theta_2^{(j)}, ..., \theta_d^{(j)})}{q(\theta_1^{can} \mid \theta_1^{j}, \theta_2^{(j)}, ..., \theta_d^{(j)})}\right\}$$

with $\pi(\theta_1^{can} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})$ denoting the density corresponding to the conditional posterior distribution of θ_1 evaluated at $\theta_1 = \theta_1^{can}$ and similarly for $\pi(\theta_1^{j} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})$.

In this paper, the candidate generator $q(\theta_1^{can} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})$produce the normal distribution for θ_1^{can} with mean $= \theta_1^{j}$ and variance $= v$ for programming easiness. Then the acceptance probability becomes

$$p = \min\left\{1, \frac{\pi(\theta_1^{can} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})}{\pi(\theta_1^{j} \mid \theta_2^{(j)}, ..., \theta_d^{(j)})}\right\}.$$

Table 1: Posterior Distributions for Different Pairs of Prior

no	Parameter	Prior	Posterior
1	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$\exp\left[A - \left(\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \lambda_0\omega\right] \times \omega^{(\alpha_0-n-1)}$
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	
2	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$\exp\left[A - \lambda_0\omega^{\alpha_0}\right] \times \frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}\omega^{(\alpha_0-n-1)}$
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	
3	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$\exp\left\{A - \left[\frac{\kappa_0}{2}(\mu-\mu_0)^2 - \lambda_0\omega^{\alpha_0}\right]\right\} \times \omega^{(\alpha_0-n-1)}$
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	
4	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$\exp\left[A - \left(\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \lambda_0\omega\right]$
	ω	$\mathrm{Exp}(\lambda_0)$	
5	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$\exp\left\{A - \left(\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left[\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \frac{1}{2\lambda_0}(\log\omega - \alpha_0)^2\right]\right\} \times \omega^{(-n-1)}$
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	
6	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$\exp\left\{A - \left[\left(\frac{\mu-\mu_0}{\kappa_0}\right) + \left(\frac{\lambda_0(\omega-\alpha_0)^2}{2\alpha_0^2\omega}\right)\right] - \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right\} \times \omega^{\left(-n-\frac{3}{2}\right)}$
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	
7	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$\exp\left[A - \left(\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \lambda_0\omega\right] \times \omega$
	ω	$\mathrm{R}(2, \lambda_0)$	
8	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$\exp\left[A - \left(\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \lambda_0\omega^{\alpha_0}\right] \times \omega^{(\alpha_0-1)}$
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	
9	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$\exp\left\{A - \left[\frac{\kappa_0}{2}(\mu-\mu_0)^2 - \lambda_0\omega\right]\right\} \times \omega^{(-n)}$
	ω	$\mathrm{Exp}(\lambda_0)$	
10	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$\exp\left\{A - \left[\frac{\kappa_0}{2}(\mu-\mu_0)^2 - \frac{\lambda_0(\omega-\alpha_0)^2}{2\alpha_0^2\omega}\right]\right\} \times \omega^{\left(-n-\frac{3}{2}\right)}$
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	
11	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$\exp\left\{A - \left[\frac{\kappa_0}{2}(\mu-\mu_0)^2 - \frac{1}{2\lambda_0}(\log\omega-\alpha_0)^2\right]\right\} \times \omega^{(-n-1)}$
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	
12	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$\exp\left\{A - \left[\frac{\kappa_0}{2}(\mu-\mu_0)^2 - \lambda_0\omega^2\right]\right\} \times \omega^{(-n-1)}$
	ω	$\mathrm{R}(2, \lambda_0)$	
13	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$\exp\left\{A - \left[\frac{\kappa_0}{2}(\mu-\mu_0)^2 - \lambda_0\omega\right]\right\} \times \omega^{(\alpha_0-n-1)}$
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	
14	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$\exp\left[A - \lambda_0\omega\right] \times \frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}\omega^{(-n)}$
	ω	$\mathrm{Exp}(\lambda_0)$	
15	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$\exp\left[A - \lambda_0\omega\right] \times \frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}\omega^{(\alpha_0-n-1)}$
	ω	$\mathrm{Ga}\ (\alpha_0, \lambda_0)$	
16	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$\exp\left[A - \lambda_0\omega^2\right] \times \frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}\omega^{(-n-1)}$
	ω	$\mathrm{R}(2, \lambda_0)$	
17	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$\exp\left[A - \frac{\lambda_0(\omega-\alpha_0)^2}{2\alpha_0^2\omega}\right] \times \frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}\omega^{-n-\frac{3}{2}}$
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	
18	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$\exp\left[A - \frac{1}{2\lambda_0}(\log\omega-\alpha_0)^2\right] \times \frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}\omega^{-n-1}$
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	

Table 2: Marginal Posterior Distributions for Different Pair of Priors

no	Parameter	Prior	Posterior
1	μ	$\mathrm{G}(\mu_0,\kappa_0)$	$\exp\left[A-\left(\frac{\mu-\mu_0}{\kappa_0}\right)-\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]$
	ω	$\mathrm{Ga}\ (\alpha_0,\lambda_0)$	$\exp\left[A-\lambda_0\omega\right]\omega^{(\alpha_0-n-1)}$
2	μ	$\mathrm{Lo}(\mu_0,\kappa_0)$	$\exp[A]\times\frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}$
	ω	$\mathrm{W}(\alpha_0,\lambda_0)$	$\exp\left[A-\lambda_0\omega^{\alpha_0}\right]\omega^{(\alpha_0-n-1)}$
3	μ	$\mathrm{N}(\mu_0,\frac{1}{\kappa_0})$	$\exp\left[A-\frac{\kappa_0}{2}(\mu-\mu_0)^2\right]$
	ω	$\mathrm{W}(\alpha_0,\lambda_0)$	$\exp\left[A-\lambda_0\omega^{\alpha_0}\right]\omega^{(\alpha_0-n-1)}$
4	μ	$\mathrm{G}(\mu_0,\kappa_0)$	$\exp\left[A-\left(\frac{\mu-\mu_0}{\kappa_0}\right)-\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]$
	ω	$\mathrm{Exp}\ (\lambda_0)$	$\exp\left[A-\lambda_0\omega\right]\omega^{(-n)}$
5	μ	$\mathrm{G}(\mu_0,\kappa_0)$	$\exp\left[A-\left(\frac{\mu-\mu_0}{\kappa_0}\right)-\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]$
	ω	$\mathrm{LN}(\alpha_0,\lambda_0)$	$\exp\left[A-\frac{1}{2\lambda_0}(\log\omega-\alpha_0)^2\right]\omega^{(-n-1)}$
6	μ	$\mathrm{G}(\mu_0,\kappa_0)$	$\exp\left[A-\left(\frac{\mu-\mu_0}{\kappa_0}\right)-\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]$
	ω	$\mathrm{IG}(\alpha_0,\lambda_0)$	$\exp\left[A-\frac{\lambda_0(\omega-\alpha_0)^2}{2\alpha_0^2\omega}\right]\omega^{\left(-n-\frac{3}{2}\right)}$
7	μ	$\mathrm{G}(\mu_0,\kappa_0)$	$\exp\left[A-\left(\frac{\mu-\mu_0}{\kappa_0}\right)-\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]$
	ω	$\mathrm{R}(2,\lambda_0)$	$\exp\left[A-\lambda_0\omega^2\right]\omega^{(-n-1)}$
8	μ	$\mathrm{G}(\mu_0,\kappa_0)$	$\exp\left[A-\left(\frac{\mu-\mu_0}{\kappa_0}\right)-\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]$
	ω	$\mathrm{W}(\alpha_0,\lambda_0)$	$\exp\left[A-\lambda_0\omega^{\alpha_0}\right]\omega^{(\alpha_0-n-1)}$
9	μ	$\mathrm{N}(\mu_0,\frac{1}{\kappa_0})$	$\exp\left[A-\frac{\kappa_0}{2}(\mu-\mu_0)^2\right]$
	ω	$\mathrm{Exp}\ (\lambda_0)$	$\exp\left[A-\lambda_0\omega\right]\omega^{(-n)}$
10	μ	$\mathrm{N}(\mu_0,\frac{1}{\kappa_0})$	$\exp\left[B-\frac{\kappa_0}{2}(\mu-\mu_0)^2\right]$
	ω	$\mathrm{IG}\ (\alpha_0,\lambda_0)$	$\exp\left[A-\frac{\lambda_0(\omega-\alpha_0)^2}{2\alpha_0^2\omega}\right]\omega^{\left(-n-\frac{3}{2}\right)}$
11	μ	$\mathrm{N}\ (\mu_0,\frac{1}{\kappa_0})$	$\exp\left[A-\frac{\kappa_0}{2}(\mu-\mu_0)^2\right]$
	ω	$\mathrm{LN}\ (\alpha_0,\lambda_0)$	$\exp\left[A-\frac{1}{2\lambda_0}(\log\omega-\alpha_0)^2\right]\omega^{-n-1}$
12	μ	$\mathrm{N}\ (\mu_0,\frac{1}{\kappa_0})$	$\exp\left[A-\frac{\kappa_0}{2}(\mu-\mu_0)^2\right]$
	ω	$\mathrm{R}\ (2,\lambda_0)$	$\exp\left[A-\lambda_0\omega^2\right]\omega^{(-n-1)}$
13	μ	$\mathrm{N}\ (\mu_0,\frac{1}{\kappa_0})$	$\exp\left[A-\frac{\kappa_0}{2}(\mu-\mu_0)^2\right]$
	ω	$\mathrm{Ga}(\alpha_0,\lambda_0)$	$\exp\left[A-\lambda_0\omega\right]\omega^{(\alpha_0-n-1)}$
14	μ	$\mathrm{Lo}\ (\mu_0,\kappa_0)$	$\exp[A]\times\frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}$
	ω	$\mathrm{Exp}\ (\lambda_0)$	$\exp\left[A-\lambda_0\omega\right]\omega^{(-n)}$
15	μ	$\mathrm{Lo}(\mu_0,\kappa_0)$	$\exp[A]\times\frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}$
	ω	$\mathrm{Ga}(\alpha_0,\lambda_0)$	$\exp\left[A-\lambda_0\omega\right]\omega^{(\alpha_0-n-1)}$
16	μ	$\mathrm{Lo}(\mu_0,\kappa_0)$	$\exp[A]\times\frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}$
	ω	$\mathrm{R}\ (2,\lambda_0)$	$\exp\left[A-\lambda_0\omega^2\right]\omega^{(-n-1)}$
17	μ	$\mathrm{Lo}(\mu_0,\kappa_0)$	$\exp[A]\times\frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}$
	ω	$\mathrm{IG}\ (\alpha_0,\lambda_0)$	$\exp\left[A-\frac{\lambda_0(\omega-\alpha_0)^2}{2\alpha_0^2\omega}\right]\omega^{\left(-n-\frac{3}{2}\right)}$
18	μ	$\mathrm{Lo}(\mu_0,\kappa_0)$	$\exp[A]\times\frac{\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)}{\kappa_0\left[1+\exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right)\right]^2}$
	ω	$\mathrm{LN}(\alpha_0,\lambda_0)$	$\exp\left[\left(-\sum_{i=1}^{n}\frac{x_i-\mu}{\omega}\right)-\sum_{i=1}^{n}\exp\left(-\frac{x_i-\mu}{\omega}\right)-\frac{1}{2\lambda_0}(\log\omega-\alpha_0)^2\right]\omega^{-n-1}$

Table 3: Acceptance Probability of Metropolis-Hasting algorithm

No	Parameter	Prior	log	Value of log
1	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$+$	$B + \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \exp(-\frac{\mu^{can}-\mu_0}{\kappa_0})$
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	$++$	$C + [(\alpha_0 - n - 1)(\log\omega^{can} - \log\omega)] + \lambda_0(\omega - \omega^{can})$
2	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$+$	$B + 2\left\{\log\left[1 + \exp\left(\frac{\mu_0-\mu}{\kappa_0}\right)\right] - \log\left[1 + \exp\left(\frac{\mu_0-\mu^{can}}{\kappa_0}\right)\right]\right\}$
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	$++$	$C + (\alpha_0 - n - 1)(\log\omega^{can} - \log\omega) + \lambda_0[\omega^{\alpha_0} - (\omega^{can})^{\alpha_0}]$
3	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$+$	$B + \frac{\kappa_0}{2}\left[(\mu - \mu_0)^2 - (\mu^{can} - \mu_0)^2\right]$
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	$++$	$C + [(\alpha_0 - n - 1)(\log\omega^{can} - \log\omega)] + \lambda_0[\omega - (\omega^{can})^{\alpha_0}]$
4	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$+$	$B + \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu^{can}-\mu_0}{\kappa_0}\right)$
	ω	$\mathrm{Exp}(\lambda_0)$	$++$	$C + [(-n)(\log\omega^{can} - \log\omega)] + \lambda_0[\omega - (\omega^{can})]$
5	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$+$	$B + \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu^{can}-\mu_0}{\kappa_0}\right)$
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	$++$	$C + [(-n - 1)(\log\omega^{can} - \log\omega)] + \frac{1}{2\lambda_0}\left[(\log\omega - \alpha_0)^2 - (\log\omega^{can} - \alpha_0)^2\right]$
6	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$+$	$B + \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu^{can}-\mu_0}{\kappa_0}\right)$
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	$++$	$C + \left[\left(-n - \frac{3}{2}\right)(\log\omega^{can} - \log\omega)\right] + \frac{\lambda_0}{2\alpha_0^2}\left[\frac{(\omega-\alpha_0)^2}{\omega} - \frac{(\omega^{can}-\alpha_0)^2}{\omega^{can}}\right]$
7	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$+$	$B + \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu^{can}-\mu_0}{\kappa_0}\right)$
	ω	$\mathrm{R}(2, \lambda_0)$	$++$	$C + [(-n - 1)(\log\omega^{can} - \log\omega)] + \lambda_0\left[\omega^2 - (\omega^{can})^2\right]$
8	μ	$\mathrm{G}(\mu_0, \kappa_0)$	$+$	$B + \exp\left(-\frac{\mu-\mu_0}{\kappa_0}\right) - \exp\left(-\frac{\mu^{can}-\mu_0}{\kappa_0}\right)$
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	$++$	$C + [(\alpha_0 - n - 1)(\log\omega^{can} - \log\omega)] + \lambda_0[\omega^{\alpha_0} - (\omega^{can})^{\alpha_0}]$
9	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$+$	$B + \frac{\kappa_0}{2}\left[(\mu - \mu_0)^2 - (\mu^{can} - \mu_0)^2\right]$
	ω	$\mathrm{Exp}(\lambda_0)$	$++$	$C + [(-n)(\log\omega^{can} - \log\omega)] + \lambda_0[\omega - (\omega^{can})]$
10	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$+$	$B + \frac{\kappa_0}{2}\left[(\mu - \mu_0)^2 - (\mu^{can} - \mu_0)^2\right]$
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	$++$	$C + \left[\left(-n - \frac{3}{2}\right)(\log\omega^{can} - \log\omega)\right] + \frac{\lambda_0}{2\alpha_0^2}\left[\frac{(\omega-\alpha_0)^2}{\omega} - \frac{(\omega^{can}-\alpha_0)^2}{\omega^{can}}\right]$
11	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$+$	$B + \frac{\kappa_0}{2}\left[(\mu - \mu_0)^2 - (\mu^{can} - \mu_0)^2\right]$
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	$++$	$C + [(-n - 1)(\log\omega^{can} - \log\omega)] + \frac{1}{2\lambda_0}\left[(\log\omega - \alpha_0)^2 - (\log\omega^{can} - \alpha_0)^2\right]$
12	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$+$	$B + \frac{\kappa_0}{2}\left[(\mu - \mu_0)^2 - (\mu^{can} - \mu_0)^2\right]$
	ω	$\mathrm{R}(2, \lambda_0)$	$++$	$C + [(-n - 1)(\log\omega^{can} - \log\omega)] + \lambda_0\left[\omega - (\omega^{can})^2\right]$
13	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	$+$	$B + \frac{\kappa_0}{2}\left[(\mu - \mu_0)^2 - (\mu^{can} - \mu_0)^2\right]$
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	$++$	$C + (\alpha_0 - n - 1)(\log\omega^{can} - \log\omega) + \lambda_0(\omega - \omega^{can})$
14	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$+$	$B + 2\left\{\log\left[1 + \exp\left(\frac{\mu_0-\mu}{\kappa_0}\right)\right] - \log\left[1 + \exp\left(\frac{\mu_0-\mu^{can}}{\kappa_0}\right)\right]\right\}$
	ω	$\mathrm{Exp}(\lambda_0)$	$++$	$C + (-n)(\log\omega^{can} - \log\omega) + \lambda_0(\omega - \omega^{can})$
15	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$+$	$B + 2\left\{\log\left[1 + \exp\left(\frac{\mu_0-\mu}{\kappa_0}\right)\right] - \log\left[1 + \exp\left(\frac{\mu_0-\mu^{can}}{\kappa_0}\right)\right]\right\}$
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	$++$	$C + (\alpha_0 - n - 1)(\log\omega^{can} - \log\omega) + \lambda_0(\omega - \omega^{can})$
16	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$+$	$B + 2\left\{\log\left[1 + \exp\left(\frac{\mu_0-\mu}{\kappa_0}\right)\right] - \log\left[1 + \exp\left(\frac{\mu_0-\mu^{can}}{\kappa_0}\right)\right]\right\}$
	ω	$\mathrm{R}(2, \lambda_0)$	$++$	$C + (-n - 1)(\log\omega^{can} - \log\omega) + \lambda_0\left[\omega^2 - (\omega^{can})^2\right]$
17	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$+$	$B + 2\left\{\log\left[1 + \exp\left(\frac{\mu_0-\mu}{\kappa_0}\right)\right] - \log\left[1 + \exp(\frac{\mu_0-\mu^{can}}{\kappa_0})\right]\right\}$
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	$++$	$C + \left[\left(-n - \frac{3}{2}\right)(\log\omega^{can} - \log\omega)\right] + \frac{\lambda_0}{2\alpha_0^2}\left[\frac{(\omega-\alpha_0)^2}{\omega} - \frac{(\omega^{can}-\alpha_0)^2}{\omega^{can}}\right]$
18	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	$+$	$B + 2\left\{\log\left[1 + \exp\left(\frac{\mu_0-\mu}{\kappa_0}\right)\right] - \log\left[1 + \exp\left(\frac{\mu_0-\mu^{can}}{\kappa_0}\right)\right]\right\}$
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	$++$	$C + [(-n - 1)(\log\omega^{can} - \log\omega)] + \frac{1}{2\lambda_0}\left[(\log\omega - \alpha_0)^2 - (\log\omega^{can} - \alpha_0)^2\right]$

Notation: $\log(p_1) = +$ and $\log(p_2) = ++$.

To implement this algorithm in the program, we use $\log(p)$ instead of p for simplicity i.e. $\log(p_1)$ and $\log(p_2)$, in order to indicate the acceptance probability for μ and ω. The results are presented in Table 3 with C and D as follows

$$B = \frac{n}{\omega}\left(\mu^{can} - \mu\right) + \frac{\mu - \mu^{can}}{\kappa_0} + \sum_{i=1}^{n}\left[\exp\left(-\frac{x_i - \mu}{\omega}\right) - \exp\left(-\frac{x_i - \mu^{can}}{\omega}\right)\right],$$

and

$$C = \sum_{i=1}^{n}\left[(x_i - \mu)\left(\frac{1}{\omega} - \frac{1}{\omega^{can}}\right)\right] + \sum_{i=1}^{n}\left[\exp\left(-\frac{x_i - \mu}{\omega}\right) - \exp\left(-\frac{x_i - \mu}{\omega^{can}}\right)\right].$$

5 Simulation Studies, Comparison Criteria and Discussion

By applying a simulation study, Bayesian estimations of Gumbel ($\mu = 100, \omega = 10$)[1] for 18 pairs of priors with 100 simulated series for each estimation, are considered. Mean and Bootstraps' confidence interval for each pair of priors are calculated as shown in Table 4.

Table 5 gives the value of Bias, SD and RMSE of each estimation. Table 5 also shows that the minimum three of ranks means of RMSE are the pair of Gumbel and Rayleigh (mean of rank = 2.5), Gumbel and Exponential (mean of rank = 4) and Gumbel and Log Normal (mean of rank = 4), respectively.

Table 6 gives the values of coefficient variation according to the value mean and median. The formula of CV and CV* are

$$CV = \frac{\sqrt{\frac{\sum_{i=1}^{n}(x_i - mean)^2}{n}}}{mean}, \qquad CV^* = \frac{\sqrt{\frac{\sum_{i=1}^{n}(x_i - med)^2}{n}}}{med}, \tag{4}$$

where *med* refers to median. Based on the ranks mean for CV and CV*, the minimum three are belong to Gumbel and Rayleigh (mean of rank for CV=1.5, mean of rank for CV*=2), Normal and Inverse Gaussian(mean of rank for CV=4.5, mean of rank for CV*=5) and Normal and Gamma (mean of rank for CV=4.5 ,mean of rank for CV*=5).

From the criteria, the best compromise priors for Gumbel model are Gumbel and Rayleigh i.e. Gumbel distribution as a prior for μ and Rayleigh distribution as a prior for ω.

6 Application

In this section, we apply Gumbel and Rayleigh distributions as the best compromise priors for Malaysia extreme Gold price from 2000 to 2011 (12 years). Although we knew that the data suit well with Frechet Model, we will use Gumbel model instead for demonstration purposes. According to Galambos *et.al.* [10], in practice, Gumbel distribution still can be approximated as closely desired, by Weibull or Frechet distributions. So a Gumbel model implemented for Malaysia extreme gold price, from 2000 to 2011 for estimating the parameters is justified.

[1] μ accepts any real value but ω only takes positive value far away zero.

Table 4: Mean and Bootstraps' Confidence Interval of Bayesian Estimation for Different Pairs of Prior

No.	Parameter	Prior	Mean	Bootstraps' CI
1	μ	G(μ_0, κ_0)	99.93	(99.26 100.28)
	ω	Ga(α_0, λ_0)	9.98	(9.65 10.30)
2	μ	Lo (μ_0, κ_0)	99.97	(99.38 100.51)
	ω	W(α_0, λ_0)	9.99	(9.58 10.32)
3	μ	N$(\mu_0, \frac{1}{\kappa_0})$	97.83	(97.28 98.23)
	ω	W(α_0, λ_0)	9.73	(9.28 10.05)
4	μ	G(μ_0, κ_0)	99.94	(99.37 100.23)
	ω	Exp(λ_0)	10.01	(9.58 10.24)
5	μ	G(μ_0, κ_0)	99.97	(99.68 100.34)
	ω	LN(α_0, λ_0)	9.98	(9.66 10.34)
6	μ	G(μ_0, κ_0)	100.06	(99.55 100.37)
	ω	IG(α_0, λ_0)	10.00	(9.56 10.35)
7	μ	G(μ_0, κ_0)	99.90	(99.55 100.27)
	ω	R$(2, \lambda_0)$	9.75	(9.50 10.06)
8	μ	G(μ_0, κ_0)	99.98	(99.39 100.44)
	ω	W(α_0, λ_0)	10.00	(9.59 10.28)
9	μ	N$(\mu_0, \frac{1}{\kappa_0})$	97.83	(97.42 98.24)
	ω	Exp(λ_0)	9.77	(9.43 9.99)
10	μ	N$(\mu_0, \frac{1}{\kappa_0})$	97.86	(97.36 97.91)
	ω	IG(α_0, λ_0)	9.72	(9.48 9.76)
11	μ	N$(\mu_0, \frac{1}{\kappa_0})$	97.88	(97.43 98.19)
	ω	LN(α_0, λ_0)	9.76	(9.24 10.07)
12	μ	N$(\mu_0, \frac{1}{\kappa_0})$	97.79	(97.26 98.14)
	ω	R $(2, \lambda_0)$	9.51	(9.20 9.76)
13	μ	N$(\mu_0, \frac{1}{\kappa_0})$	97.83	(97.38 98.14)
	ω	Ga(α_0, λ_0)	9.73	(9.41 10.09)
14	μ	Lo(μ_0, κ_0)	99.90	(99.36 100.41)
	ω	Exp(λ_0)	10.00	(9.64 10.41)
15	μ	Lo(μ_0, κ_0)	100.01	(99.42 100.39)
	ω	Ga(α_0, λ_0)	9.96	(9.53 10.29)
16	μ	Lo(μ_0, κ_0)	99.90	(99.39 100.28)
	ω	R $(2, \lambda_0)$	9.81	(9.50 10.12)
17	μ	Lo(μ_0, κ_0)	100.04	(99.46 100.49)
	ω	IG(α_0, λ_0)	9.99	(9.57 10.22)
18	μ	Lo(μ_0, κ_0)	99.94	(99.48 100.44)
	ω	LN(α_0, λ_0)	9.95	(9.64 10.22)

Table 5: Bias, SD and RMSE of Bayesian Estimation for Different Pairs of Priors

No.	Parameter	Prior	Bias	SD	RMSE	Rank of RMSE	Mean of Rank
1	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.0050	0.234	0.242	8	8
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	0.0004	0.168	0.168	8	
2	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.0008	0.284	0.282	11	11
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	0.0000	0.176	0.175	11	
3	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	4.6881	0.238	2.178	17	17.5
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	0.0720	0.154	0.309	18	
4	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.0031	0.187	0.193	2	**4**
	ω	$\mathrm{Exp}(\lambda_0)$	0.0002	0.159	0.158	6	
5	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.0006	0.182	0.182	1	**4**
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	0.0004	0.160	0.160	7	
6	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.0031	0.187	0.193	3	7.5
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	0.0000	0.182	0.180	12	
7	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.0093	0.173	0.197	4	**2.5**
	ω	$\mathrm{R}(2, \lambda_0)$	0.0640	0.140	0.064	1	
8	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.0005	0.221	0.220	5	4.5
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	0.0000	0.157	0.156	4	
9	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	4.6990	0.180	2.175	16	15
	ω	$\mathrm{Exp}(\lambda_0)$	0.0510	0.166	0.279	14	
10	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	4.5860	0.193	2.150	14	15.5
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	0.0764	0.132	0.306	17	
11	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	4.5170	0.180	2.133	13	14
	ω	$\mathrm{LN}\ (\alpha_0, \lambda_0)$	0.0560	0.167	0.288	15	
12	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	4.8940	0.197	2.221	18	10
	ω	$\mathrm{R}(2, \lambda_0)$	0.2348	0.503	0.137	2	
13	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	4.6860	0.181	2.172	15	15.5
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	0.0719	0.148	0.305	16	
14	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.0093	0.259	0.274	10	9.5
	ω	$\mathrm{Exp}(\lambda_0)$	0.0012	0.171	0.170	9	
15	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.0000	0.237	0.235	6	8
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	0.0018	0.167	0.171	10	
16	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.0096	0.222	0.240	7	10
	ω	$\mathrm{R}(2, \lambda_0)$	0.0372	0.156	0.247	13	
17	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.0017	0.284	0.284	12	8.5
	ω	$\mathrm{IG}\ (\alpha_0, \lambda_0)$	0.0001	0.158	0.157	5	
18	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.0038	0.251	0.256	9	6
	ω	$\mathrm{LN}\ (\alpha_0, \lambda_0)$	0.0024	0.157	0.153	3	

Table 6: CV and CV* of Bayesian Estimation for Different Pairs of Prior

No.	Parameter	Prior	CV	Rank	Mean of Rank	CV*	Rank	Mean of Rank
1	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.234	12	12.5	0.232	12	12
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	1.683	13		1.666	12	
2	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.284	17	17	0.281	17	16.5
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	1.762	17		1.745	16	
3	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	0.243	14	10.5	0.242	14	10.5
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	1.581	7		1.580	7	
4	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.187	7	8	0.185	6	7.5
	ω	$\mathrm{Exp}(\lambda_0)$	1.593	9		1.591	9	
5	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.182	2	6.5	0.180	2	6
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	1.606	11		1.622	10	
6	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.186	6	12	0.185	7	12
	ω	$\mathrm{IG}\ (\alpha_0, \lambda_0)$	1.818	18		1.822	17	
7	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.174	1	**1.5**	0.172	1	**2**
	ω	$\mathrm{R}\ (2, \lambda_0)$	1.435	2		1.436	3	
8	μ	$\mathrm{G}(\mu_0, \kappa_0)$	0.221	10	7.5	0.222	10	6
	ω	$\mathrm{W}(\alpha_0, \lambda_0)$	1.571	5		1.556	6	
9	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	0.184	3	8.5	0.182	3	8.5
	ω	$\mathrm{Exp}\ (\lambda_0)$	1.697	14		1.686	14	
10	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	0.197	8	**4.5**	0.196	8	**5**
	ω	$\mathrm{IG}\ (\alpha_0, \lambda_0)$	1.355	1		1.356	2	
11	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	0.184	4	9.5	0.182	4	6
	ω	$\mathrm{LN}\ (\alpha_0, \lambda_0)$	1.707	15		1.830	8	
12	μ	$\mathrm{N}\ (\mu_0, \frac{1}{\kappa_0})$	0.202	9	6	0.200	9	6.5
	ω	$\mathrm{R}(2, \lambda_0)$	1.438	3		1.440	4	
13	μ	$\mathrm{N}(\mu_0, \frac{1}{\kappa_0})$	0.185	5	**4.5**	0.184	5	**5**
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	1.517	4		1.511	5	
14	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.259	16	16	0.257	16	15.5
	ω	$\mathrm{Exp}(\lambda_0)$	1.719	16		1.703	15	
15	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.237	13	12.5	0.236	13	13
	ω	$\mathrm{Ga}(\alpha_0, \lambda_0)$	1.677	12		1.672	13	
16	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.222	11	10.5	0.223	11	11
	ω	$\mathrm{R}(2, \lambda_0)$	1.595	10		1.641	11	
17	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.284	18	13	0.281	18	13
	ω	$\mathrm{IG}(\alpha_0, \lambda_0)$	1.581	8		1.583	8	
18	μ	$\mathrm{Lo}(\mu_0, \kappa_0)$	0.251	15	10.5	0.249	15	8
	ω	$\mathrm{LN}(\alpha_0, \lambda_0)$	1.576	6		1.157	1	

The Malaysia extreme gold prices data has non stationary characteristics in the location and scale parameters. We eliminate this trend using Box-Cox transformation [11] as for $y > 0$

$$y_i^{(\lambda)} = \begin{cases} \frac{y_i^{\lambda}-1}{\lambda}, & \text{if} \quad \lambda \neq 0, \\ \log(y_i), & \text{if} \quad \lambda = 0. \end{cases}$$

By applying a difference lag 1, the non stationary in condition has been removed. The raw and transformed daily price are shown in Figures 2 and 3, respectively. As can be seen, Figure 3 has stationaries in both mean and variance.

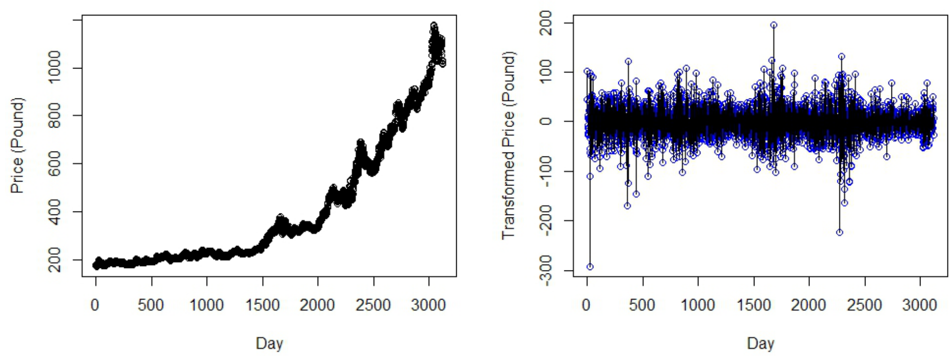

Figure 2: Raw (Left) and Transformed (Right) Daily Malaysia Gold Price (2000-2011)

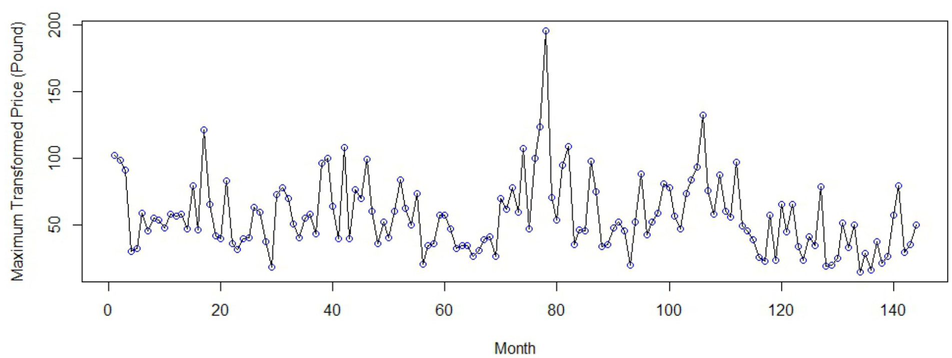

Figure 3: Transformed Monthly Malaysia Gold Price (2000-2011)

After the pre process, the maximum value for each month were chosen and used to calculate likelihood function. The estimation of parameters μ and ω for extreme Gold Price

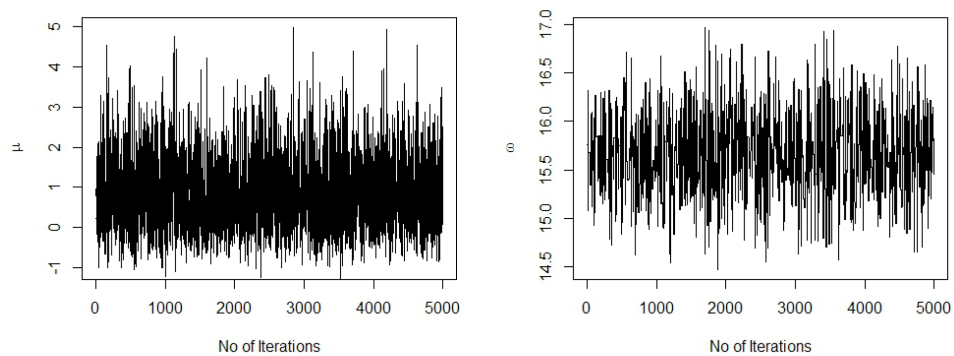

Figure 4: The 5000 Draws of μ and ω in Metropolis-Hasting Algorithm

in Malaysia were obtained. Figure 4 shows the draws that the μ and ω estimate values are converged and give the average estimates as follows:

$$\hat{\mu} = 0.225, \text{ and } \hat{\omega} = 15.633.$$

7 Conclusion

In this paper, eighteen pairs of priors according the parameters' characteristics and existing literatures [7] were selected for Gumbel distribution and compared the posterior estimations by applying Metropolis- Hosting algorithm .Based on our findings, the combination of Gumbel and Rayleigh are the most productive pair of priors for this model. We believe that the results presented here can be used for any study in Bayesian extreme modeling based on Gumbel distribution.

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