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Swirling Jets of Conducting Fluids in the Presence of a Transverse Magnetic Field

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Abstract The effect of an axial magnetic field, which varies inversely as the radius vector, on the velocity and temperature distribution in a swirling jet of a viscous incompressible electrically conducting fluid are studied. It is noted that the radial as well as tangential velocity decreases near the slit of the jet with the increase in the value of the magnetic interaction parameter. It is also observed that the effect of magnetic field is to increase the temperature for metal fluids whereas the reverse phenomenon happens for other fluids. These decelerated fluid particles move in the positive axial direction and the points where they exactly balance the motion of the incoming fluid have been calculated for different values of the magnetic interaction parameter.

Keywords Swirling jet; boundary layer; volume flux; transverse magnetic field.

2010 Mathematics Subject Classification 76D25;76W05

1 Introduction

The flow of a laminar from a slit and a jet ensuing from a circular orifice without swirl, of incompressible viscous fluid were investigated by Schlichting [1] and Bickley [2]. The problem of the circular jet, of a viscous, incompressible, electrically non-conducting fluid, with an axially symmetrical swirling component of velocity has been studied by Loitsianski [3] using an iterative procedure. Gortler [4] has considered the same problem, on the assumption that the swirling velocity is small compared with the velocity component along the jet and he expresses the swirling component of velocity as an eigen function expansion. The study of swirling circular jet have also been made by Shtem and Hussain [5], Gallaire *et al.* [6] and Facciolo *et al.* [7]. Riley [8] has studied the radial free jet, radial wall jet and the radial liquid jet, with swirl, for both incompressible and compressible fluids. In all the three cases considered by Riley, an investigation was made on the effects of departures from similarity in the swirling component of velocity to find out how rapidly the final similarity form is attained. It was found that the swirling component in the radial liquid jet attains its final similarity form very rapidly indeed, a fact which is probably accounted for by the absence of the outer mixing region common to the other two jets. The interest in the hydromagnetic swirling jets is of more recent origin. The efforts made by us in this direction reveal that in MHD case such a compact solution is not possible and therefore a perturbation on the Loitsianski model is applied and the solutions are obtained. Mishra *et al.* [9] studied the effect of an axial magnetic field, which varies inversely as the radius vector, on the velocity distribution in a swirling jet of a viscous incompressible electrically conducting fluid which originates from a circular slit. They found that the radial as well as tangential velocity decreases near the slit of the jet with the increase in magnetic interaction parameter. In the present paper we have studied the effects of an axial magnetic field, which varies inversely as the radius vector, on the velocity and temperature distribution in a swirling jet of a viscous incompressible electrically conducting fluid, which originates from a circular slit. A perturbation on the Loitsianski [3] model is applied both for velocity and temperature distribution.

2 Formulation of The Problem

Let a rotating incompressible, viscous, electrically conducting fluid be discharged through a circular slit formed by two circular discs of negligible radii and negligible distance apart in the presence of variable axial magnetic field $Bz(0, 0, B_0 r^{-1})$ and mix with the same surrounding fluid being initially at rest. Taking the origin in the slit, the governing equations in cylindrical polar coordinates for a non-relativistic fluid motion may be written as:

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rv) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial r} + \nu\frac{\partial u}{\partial z} - \frac{w^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] - \frac{\sigma_e B_0^2 u}{\rho r^2}, \tag{2}$$

$$u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right], \tag{3}$$

$$u\frac{\partial w}{\partial r} + v\frac{\partial w}{\partial z} - \frac{uw}{r} = \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{w}{r^2} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{\sigma_e B_0^2 u}{\rho r^2}, \tag{4}$$

where u, v and w are velocity component in r, z and θ directions respectively and $B_z = B_0/r$.

Now applying Prandtl boundary layer assumptions to the equations (1)-(4) and taking the pressure gradient to be zero, because the constant pressure in the surrounding fluid impress itself on the jet, the boundary layer equations governing the flow are

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rv) = 0, \tag{5}$$

$$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} - \frac{w^2}{r} = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma_e B_0^2 u}{\rho r^2}, \tag{6}$$

$$u\frac{\partial w}{\partial r} + v\frac{\partial w}{\partial z} + \frac{uw}{r} = \nu\frac{\partial^2 w}{\partial z^2} - \frac{\sigma_e B_0^2 w}{\rho r^2}, \tag{7}$$

$$u\frac{\partial \theta}{\partial r} + v\frac{\partial \theta}{\partial z} = \frac{\nu}{PrT_\infty}\frac{\partial^2 \theta}{\partial z^2} + \frac{\nu}{C_p}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \frac{\sigma_e B_0^2}{\rho C_p}\left(\frac{u^2 + w^2}{r^2}\right), \tag{8}$$

$$\theta = \frac{T - T_\infty}{T_\infty}. \tag{9}$$

The boundary conditions are

$$z = 0 : \frac{\partial u}{\partial z} = 0, v = 0, \frac{\partial w}{\partial z} = 0, \frac{\partial \theta}{\partial z} = 0, \tag{10}$$

$$z \to \infty : u \to \frac{m}{r}, w \to \frac{\sigma_e B_0^2}{\rho r^2}, \theta \to 0, \tag{11}$$

where compatibility conditions are used in obtaining the conditions at infinity. Besides these boundary conditions following integral conditions should also be satisfy

$$\lim_{\mathrm{r}\to\infty} 2\pi\rho r \int_{-\infty}^{\infty} u_0^2 dz = J_0, \tag{12}$$

$$\lim_{\mathrm{r}\to\infty} 2\pi\rho r^2 \int_{-\infty}^{\infty} u_0 w_0 dz = L_0, \tag{13}$$

$$\frac{d}{dr}\int_{-\infty}^{\infty} \theta_0 u_0 dz = \frac{v}{C_p} K_0 \int_{-\infty}^{\infty} \left\{ \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 \right\} dz + \frac{\sigma_e B_0}{\rho C_p} \int_{-\infty}^{\infty} \left\{ \frac{u^2 + w^2}{r^2} \right\} dz, \tag{14}$$

where u_0 and w_0 are the velocity distributions and θ_0 is the temperature distribution in the corresponding non magnetic case i.e. when the magnetic field is zero ($m = 0$) and

$$m = \frac{\sigma_e B_0^2}{\rho}. \tag{15}$$

These integral conditions have been obtained by integrating equations (5), (6) and (8) with respect to z between $-\infty$ to ∞, taking into account the continuity equation (5) or the boundary conditions (10) and (11) respectively.

3 Numerical Solution

3.1 Analysis of the Velocity Boundary Layer

Introducing the stream function Ψ_m, such that,

$$ru = \frac{\partial \Psi_m}{\partial z}, rv = -\frac{\partial \Psi_m}{\partial r}. \tag{16}$$

The equation of continuity (5) is identically satisfied and the momentum equations (6) and (7) can be reduced to a set of ordinary differential equations if the following series expansions, which are perturbations on the Loitsianski model [3], are satisfied:

$$\Psi_m = \beta^{\frac{1}{2}} \alpha \nu^{\frac{1}{2}} r \left[F_0 + \left(\frac{m}{\alpha^2 r}\right) F_1 + \left(\frac{m}{\alpha^2 r}\right)^2 F_2 + \left(\frac{m}{\alpha^2 r}\right)^3 F_3 + ... \right], \tag{17}$$

$$w = \frac{\beta \alpha^2}{r} \left[G_0 + \left(\frac{m}{\alpha^2 r}\right) G_1 + \left(\frac{m}{\alpha^2 r}\right)^2 G_2 + \left(\frac{m}{\alpha^2 r}\right)^3 G_3 + ... \right], \tag{18}$$

where $F_0, F_1, F_2...., G_0, G_1, G_2, ..., H_0, H_1, H_2, ...$ are functions of ξ to be determined and

$$\xi = \left(\frac{\beta}{\nu}\right)^{\frac{1}{2}} \frac{\alpha z}{r}. \tag{19}$$

Now, using (16) the expressions for u, v and w are obtained as follows:

$$u = \frac{\beta\alpha^2}{r}\left[F_0' + \left(\frac{m}{\alpha^2 r}\right)F_1' + \left(\frac{m}{\alpha^2 r}\right)^2 F_2' + \left(\frac{m}{\alpha^2 r}\right)^3 F_3' + ...\right], \tag{20}$$

$$\begin{aligned} v = -\frac{\alpha\sqrt{\beta}\nu}{r}[F_0 - \xi F_0' - \left(\frac{m}{\alpha^2 r}\right)\xi F_1' - \left(\frac{m}{\alpha^2 r}\right)^2 (F_2 + \xi F_2') \\ - \left(\frac{m}{\alpha^2 r}\right)^3 (2F_3 + \xi F_3') + ...], \end{aligned} \tag{21}$$

$$w = \frac{\beta\alpha^2}{r}\left[G_0 + \left(\frac{m}{\alpha^2 r}\right)G_1 + \left(\frac{m}{\alpha^2 r}\right)^2 G_2 + \left(\frac{m}{\alpha^2 r}\right)^3 G_3 + ...\right], \tag{22}$$

where prime denotes differentiation with respect to ξ.

Substituting (21)-(22) in equations (6) and (7) and equating the coefficients of the same powers of $(m/\alpha^2 r)$ we get the following set of ordinary differential equations:

I Terms independent of $(m/\alpha^2 r)$ are

$$F_0''' + F_0F_0'' + F_0'^2 = -G_0^2, \tag{23}$$
$$G_0'' + F_0G_0' = 0. \tag{24}$$

Boundary conditions are

$$\begin{aligned} &\xi = 0 : F_0'' = 0, F_0 = 0, G_0' = 0, \\ &\xi \to \pm\infty : F_0', = 0, G_0 = 0. \end{aligned} \tag{25}$$

II Terms containing $(m/\alpha^2 r)$ are

$$F_1''' + F_0F_1'' + 3F_0'F_1' = F_0' - 2G_0G_1, \tag{26}$$
$$G_1'' + F_0G_1' + F_0'G_1 = G_0. \tag{27}$$

Boundary conditions are

$$\begin{aligned} &\xi = 0 : F_1'' = 0, F_1 = 0, G_1' = 0, \\ &\xi \to \pm\infty : F_1', = 1, G_1 = 1. \end{aligned} \tag{28}$$

III Terms containing $(m/\alpha^2 r)^2$ are

$$F_2''' + F_0F_2'' + 4F_0'F_2' - F_0''F_2 = F_1' - 2F_1'^2 - G_1^2 - 2G_0G_2, \tag{29}$$
$$G_2'' + F_0G_2' + 2F_0'G_2 = G_1(1 - F_1') + F_2G_0'. \tag{30}$$

Boundary conditions are

$$\begin{aligned} &\xi = 0 : F_2'' = 0, F_2 = 0, G_2' = 0, \\ &\xi \to \pm\infty : F_2' = 0, G_2 = 0. \end{aligned} \tag{31}$$

IV Terms containing $(m/\alpha^2 r)^3$ are

$$F_3''' + F_0F_3'' + 5F_0'F_3' - 2F_0''F_3 = F_2'(1-5F_1') + F_1''F_2 - 2G_0G_3 - 2G_1G_2, \quad (32)$$
$$G_3'' + F_0G_3' + 3F_0'G_3 = G_2(1-2F_1') + F_2G_1' - F_2'G_1 + 2F_3G_0' - F_3'G_0. \quad (33)$$

Boundary conditions are

$$\begin{aligned}&\xi = 0 : F_3'' = 0, F_3 = 0, G_3' = 0, H_3' = 0,\\ &\xi \to \pm\infty : F_3', = 0, G_3 = 0.\end{aligned} \quad (34)$$

Similar equations corresponding to higher order perturbation term may be obtained. However, here we shall confine ourselves to third order perturbation equations. Equations (23) and (24) with boundary conditions (25) are the known equations of the Loitsianski model [3] for the non magnetic case and having the solution (Mishra and Bansal [9])

$$F_0 = \tanh\left(\frac{\xi}{2}\right) \quad (35)$$

and

$$G_0 = 0. \quad (36)$$

The remaining equations (26) to (34) have been solved numerically by known techniques for two point boundary value problems of non-homogeneous linear ordinary differential equations.

The maximum velocity exists at the axis of the jet and is given by

$$(u_{\max})_{\mathrm{at}\xi=0} = \frac{\beta\alpha^2}{r}[F_0{}'(0) + \left(\frac{m}{\alpha^2 r}\right)F_1{}'(0) + \left(\frac{m}{\alpha^2 r}\right)^2 F_2{}'(0) + \left(\frac{m}{\alpha^2 r}\right)^3 F_3{}'(0) + ...]. \quad (37)$$

Now, the expressions for the dimensionless velocity distribution are given by

$$\frac{u}{(u_0)_{\max}} = 2\left[F_0' + \left(\frac{m}{\alpha^2 r}\right)F_1' + \left(\frac{m}{\alpha^2 r}\right)^2 F_2' + \left(\frac{m}{\alpha^2 r}\right)^3 F_3' + ...\right], \quad (38)$$

$$\begin{aligned}\frac{v}{\frac{\alpha\sqrt{\beta\nu}}{r}} &= -[F_0 - \xi F_0' - \left(\frac{m}{\alpha^2 r}\right)\xi F_1' - \left(\frac{m}{\alpha^2 r}\right)^2 (F_2 + \xi F_2')\\ &\quad - \left(\frac{m}{\alpha^2 r}\right)^3 (2F_3 + \xi F_3') + ...],\end{aligned} \quad (39)$$

$$\frac{w}{\left(\frac{\beta\alpha^2}{r}\right)} = G_0 + \left(\frac{m}{\alpha^2 r}\right)G_1 + \left(\frac{m}{\alpha^2 r}\right)^2 G_2 + \left(\frac{m}{\alpha^2 r}\right)^3 G_3 + ..., \quad (40)$$

where $(u_0)_{\max}$, $(v_0)_{\max}$ and $(w_0)_{\max}$ are the maximum velocity exists at the axis of the jet for non-magnetic field ($m = 0$).

The volume flux Q in the radial direction at a distance r from the slit is given by

$$\begin{aligned}Q = 2\pi r\int_{-\infty}^{\infty} u dz &= 4\pi r\alpha\sqrt{\beta\nu}[F_0(\infty) + \left(\frac{m}{\alpha^2 r}\right)F_1(\infty)\\ &\quad + \left(\frac{m}{\alpha^2 r}\right)^2 F_2(\infty) + \left(\frac{m}{\alpha^2 r}\right)^3 F_3(\infty) + ...].\end{aligned} \quad (41)$$

3.2 Analysis of the Thermal Boundary Layer

Since the thermal boundary layer equation (8) is a linear differential equation it is convenient to obtain the complete solution of it by the superposition of two solutions of form

$$\theta = \theta_1 + \theta_2, \tag{42}$$

where θ_1 is the solution, neglecting the Joule heating and dissipation, of the homogeneous equation and θ_2 is the particular solution of the non-homogeneous equation.

(a) Initial Heating

In this case, energy equation (7) becomes

$$u\frac{\partial \theta_1}{\partial r} + v\frac{\partial \theta_1}{\partial Z} = \frac{\nu}{Pr}\frac{\partial^2 \theta_1}{\partial z^2} \tag{43}$$

with the boundary conditions

$$\begin{aligned} z = 0 &: \frac{\partial \theta_1}{\partial z} = 0, \\ z \to \pm\infty &: \theta_1 \to 0, \end{aligned} \tag{44}$$

and the integral condition

$$r^2 \int_{-\infty}^{\infty} \theta_0 u_0 dz = K_0. \tag{45}$$

The solution of the above equation has already been obtained by Yih [10] the details of which may be found in Loitsianski [3], and the result is as follow:

$$\begin{aligned} &\theta_1 = C\,\mathrm{sech}^{2Pr}\frac{\xi}{2}, \\ &\text{where} \\ &C = \frac{k_0}{2}\left(\int_{-\infty}^{\infty} \mathrm{sech}^{2+2Pr}\xi d\xi\right)^{-1}. \end{aligned} \tag{46}$$

(b) Viscous Heating

In this case an incompressible fluid having a temperature T_∞ is issued through a narrow slit and mix with the same surrounding fluid at rest and at the same temperature T_∞. The thermal boundary is formed due to viscous dissipation. Then, the enthalpy distribution θ_2 satisfies the following differential equation:

$$u\frac{\partial \theta_2}{\partial r} + v\frac{\partial \theta_2}{\partial z} = \frac{\nu}{PrT_\infty}\frac{\partial^2 \theta_2}{\partial z^2} + \frac{\nu}{C_p}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \frac{\sigma_e B_0^2}{\rho C_p}\left(\frac{u^2 + w^2}{r}\right). \tag{47}$$

Boundary conditions are:

$$\begin{aligned} &\xi = 0 : \frac{\partial u}{\partial \xi} = 0, v = 0, \frac{\partial w}{\partial \xi} = 0, \frac{\partial \theta_2}{\partial \xi} = 0, \\ &\xi \to \infty : u \to \frac{m}{r}, w \to \frac{\sigma_e B_0^2}{\rho r^2}, \theta_2 \to 0. \end{aligned} \tag{48}$$

The integral condition (14), with $\theta = \theta_2$ be indentically satisfied. Now write the expansion for θ_2 as:

$$\begin{aligned} \theta_2 &= \frac{\beta^2\alpha^4}{C_p T_\infty r^2}\left[H_0 + \left(\frac{m}{\alpha^2 r}\right)H_1 + \left(\frac{m}{\alpha^2 r}\right)^2 H_2 + \left(\frac{m}{\alpha^2 r}\right)^3 H_3 + ...\right], \\ \xi &= \left(\frac{\beta}{\nu}\right)^{\frac{1}{2}} \frac{\alpha z}{r}, \end{aligned} \tag{49}$$

where H_0, H_1, H_2,.... are functions of ξ.

Substituting the equations (20), (21) and (49) in equation (47) and equating the coefficients of like powers, we get the following set of equations:

I Terms independent of $(\frac{m}{\alpha^2 r})^0$ is

$$H_0'' + Pr(F_0 H_0' + 2F_0' H_0) = -Pr\left(G_0'^2 + F_0''^2\right). \tag{50}$$

Boundary conditions are

$$\begin{aligned} &\xi = 0 : F_0'' = 0, F_0 = 0, G_0' = 0, H_0' = 0, \\ &\xi \to \pm\infty : F_0',= 0, G_0 = 0, H_0 = 0. \end{aligned} \tag{51}$$

II Terms containing $\left(\frac{m}{\alpha^2 r}\right)$ is

$$H_1'' + Pr(F_0 H_1' + 3F_0' H_1) = -Pr(2F_0'' F_1'' + 2F_1' H_0 + F_0'^2 + 2G_0' G_1' + G_0^2). \tag{52}$$

Boundary conditions are

$$\begin{aligned} &\xi = 0 : F_1'' = 0, G_1' = 0, H_1' = 0, \\ &\xi \to \pm\infty : F_1' = 1, G_1 = 1, H_1 = 0. \end{aligned} \tag{53}$$

III Terms containing $\left(\frac{m}{\alpha^2 r}\right)^2$ is

$$\begin{aligned} H_2'' + &Pr(F_0 H_2' + 4F_0' H_2) = -Pr(3F_1' H_1 - F_2 H_0' + 2F_2' H_0) \\ &- Pr(2F_0'' F_2'' + F_1''^2 + 2F_0' F_1' + 2G_0' G_2' + G_1'^2 + 2G_0 G_1). \end{aligned} \tag{54}$$

Boundary conditions are

$$\begin{aligned} &\xi = 0 : F_2'' = 0, G_2' = 0, H_2' = 0, \\ &\xi \to \pm\infty : F_2' = 0, G_2 = 0, H_2 = 0. \end{aligned} \tag{55}$$

The solutions of equations (50), (52), (54) are obtained for different values of ξ with the boundary conditions (51), (53), (55). Also the solutions obtained for arbitrary values of the Prandtl number.

We have the non-dimensional value of θ_2 from equation (49) as

$$\frac{\theta_2}{\left(\frac{\beta^2\alpha^4}{C_p T_\infty r^2}\right)} = H_0 + \left(\frac{m}{\alpha^2 r}\right) H_1 + \left(\frac{m}{\alpha^2 r}\right)^2 H_2 + \left(\frac{m}{\alpha^2 r}\right)^3 H_3 + ... \tag{56}$$

4 Results and Discussion

The non-dimensional velocity profiles for radial, transverse and axial velocity components of the magnetic parameter $m/\alpha^2 r$, are plotted against the similarity variable ξ in Figure 1, Figure 2 and Figure 3 respectively.

It is observed that the effect of the magnetic parameter is to decrease the radial velocity u* as well as tangential velocity v* through the motion, where as the axial velocity w* increases through the motion with the increase in magnetic interaction parameter.

Following physical explanation may be given for the result obtained. Due to the action of the centrifugal forces the fluid near the slit of the jet will be thrown outward and to compensate this a flow in axial direction towards the slit will follow. Now, the Lorentz force retards the radial and tangential motion of the fluid and therefore these decelerated fluid particles, from the law of conservation of mass, start moving in the positive axial direction and thus reducing the incoming axial velocity numerically or, in other words, increasing it algebraically. It may happen that for a prescribed value of $m/\alpha^2 r$, the incoming flow is exactly balanced by the outward flow.

The temperature profiles for various values of the magnetic parameter $m/\alpha^2 r$ for different values of the Prandtl number Pr are plotted against the similarity variable in Figure 4, Figure 5, Figure 6 and Figure 7. It is observed from the figures that the effect of the magnetic field is to increase the temperature for fluids like liquid metals, e.g. mercury, i.e. $Pr = 0.044$ whereas the reverse phenomenon happens for other fluids, i.e. in the case of increasing Prandtl number.

5 Conclusion

A Mathematical model has been presented for the Swirling jets of conducting fluid in the presence of a transverse magnetic field from the study. We observe that the radial as well as tangential velocity decreases near the slit of the jet with the increase in the value of the magnetic interaction parameter. Thus we conclude that we can control the velocity field by introducing magnetic field. it is also observed that the effect of magnetic field is to increase the temperature for fluids like fluid metals whereas reverse phenomenon happens for other fluids.

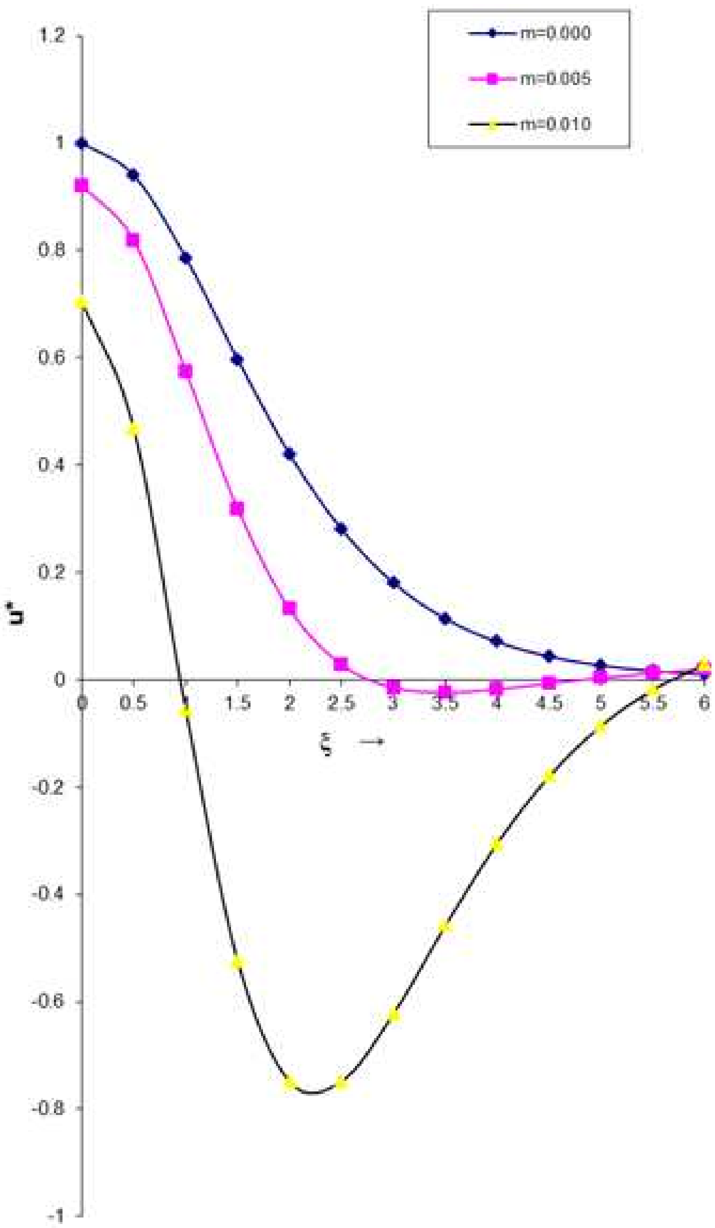

Figure 1: Radial Velocity Distribution u^* against ξ for Different Values of m in Swirling Jet

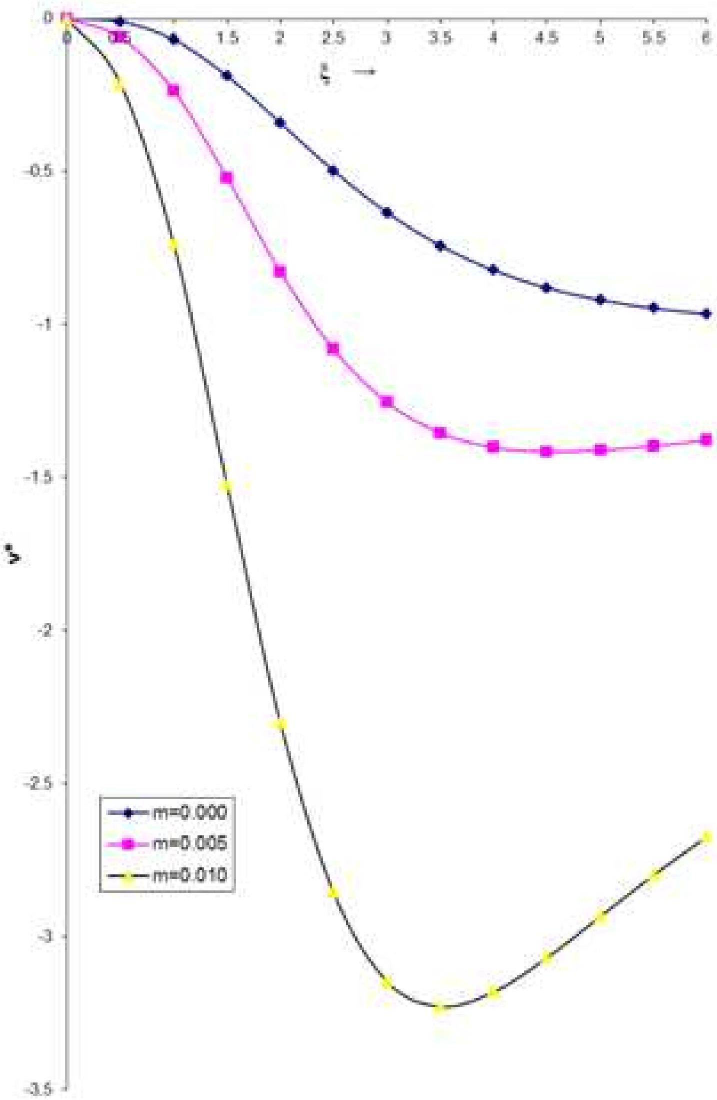

Figure 2: Tangential Velocity Distribution v^* against ξ for Different Values of m in Swirling Jet

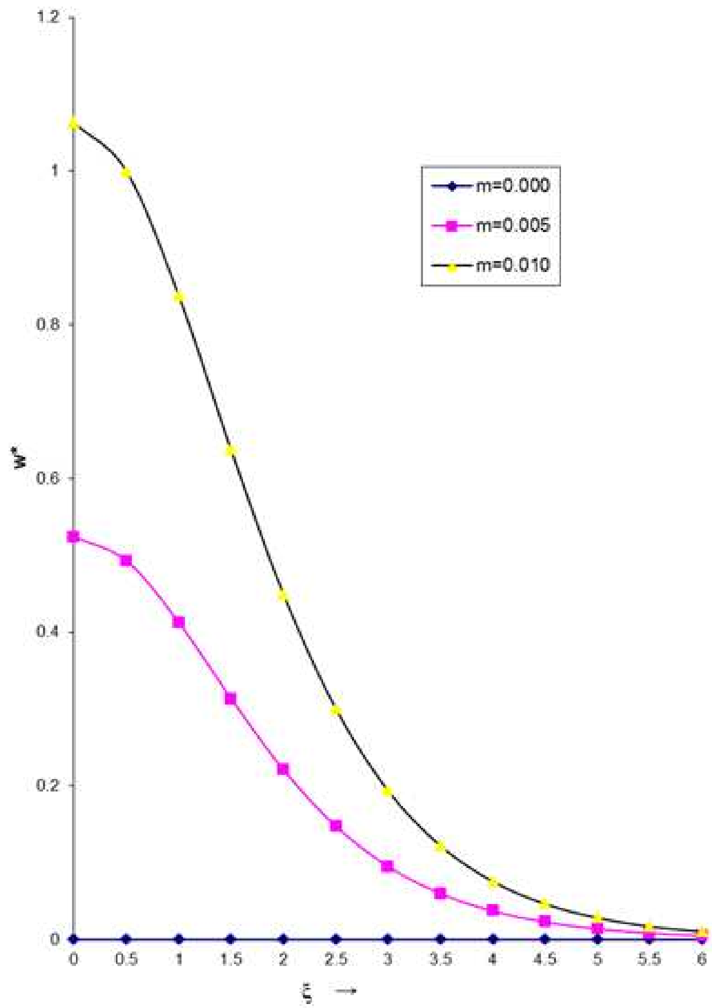

Figure 3: Axial Velocity Distribution w^* against ξ for Different Values of m in Swirling Jet

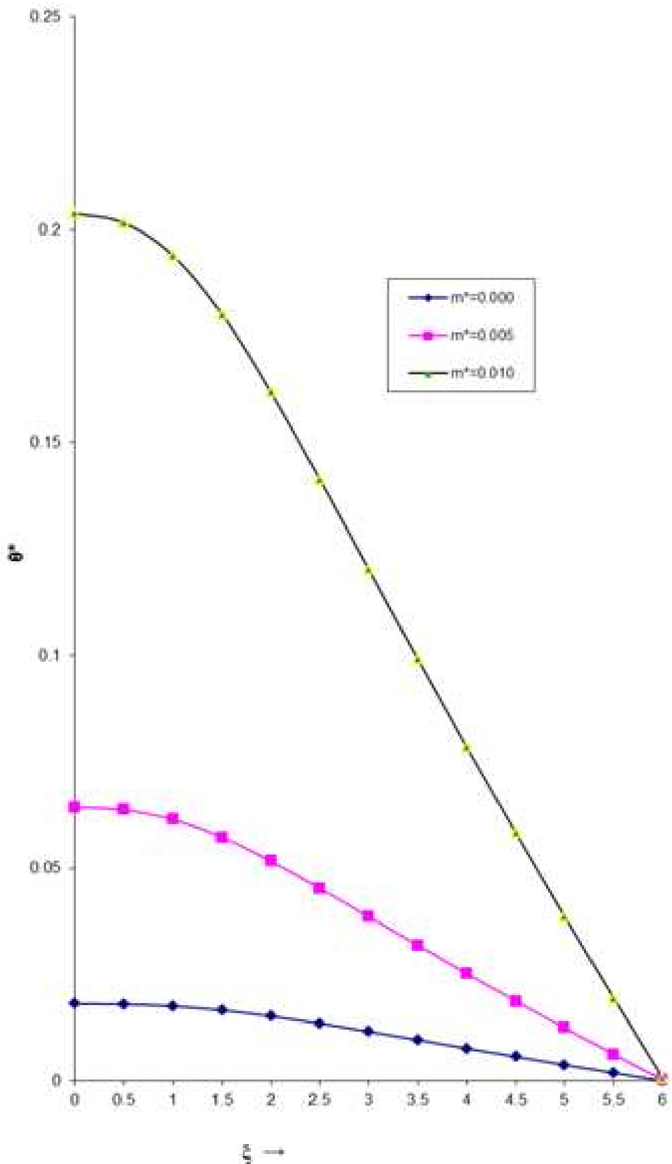

Figure 4: Temperature Distribution in Swirling Jet for $Pr = 0.044$

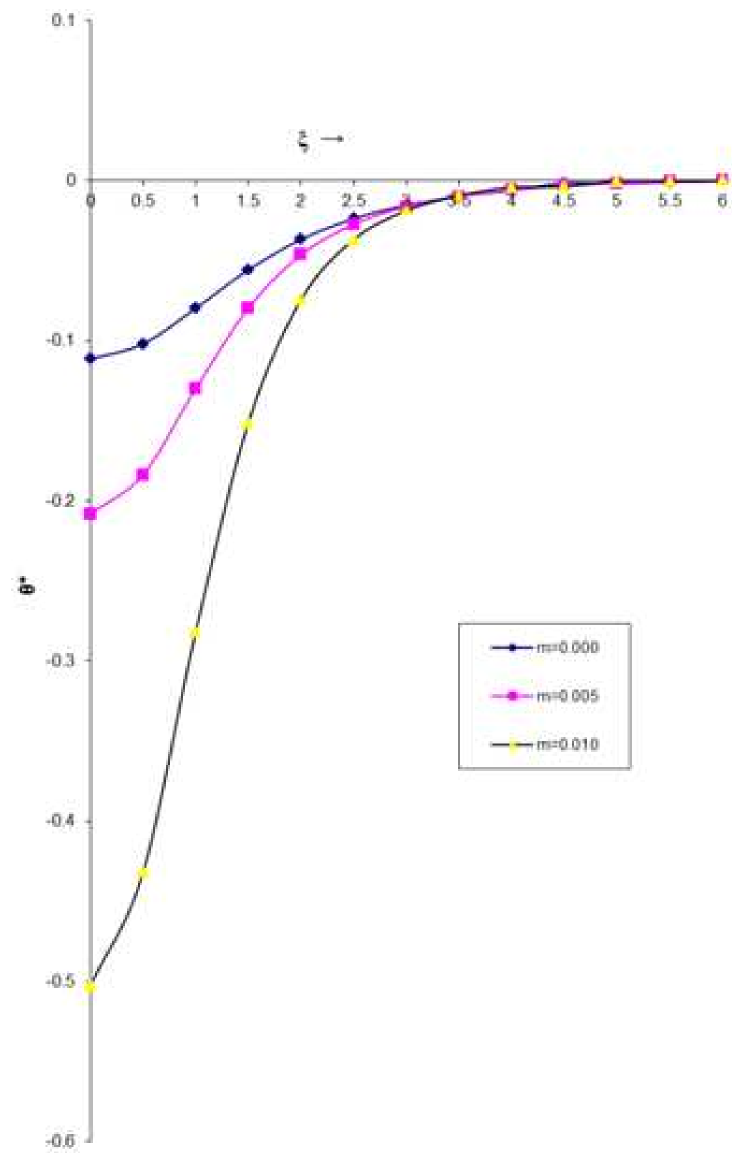

Figure 5: Temperature Distribution in Swirling Jet for $Pr = 0.72$

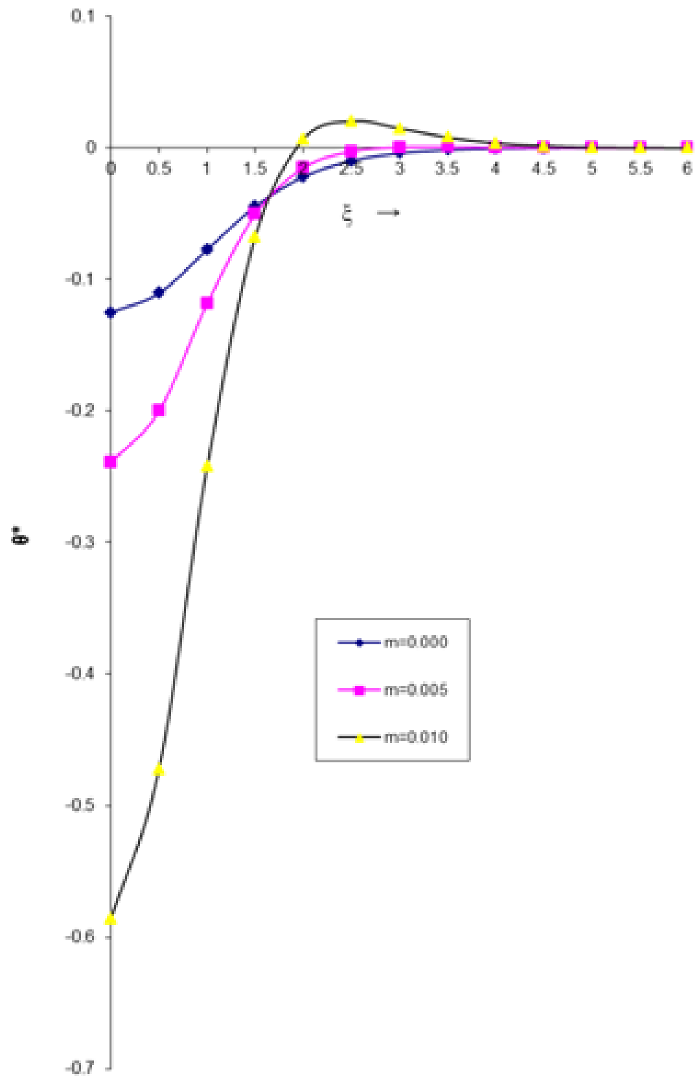

Figure 6: Temperature Distribution in Swirling Jet for $Pr = 1.000$

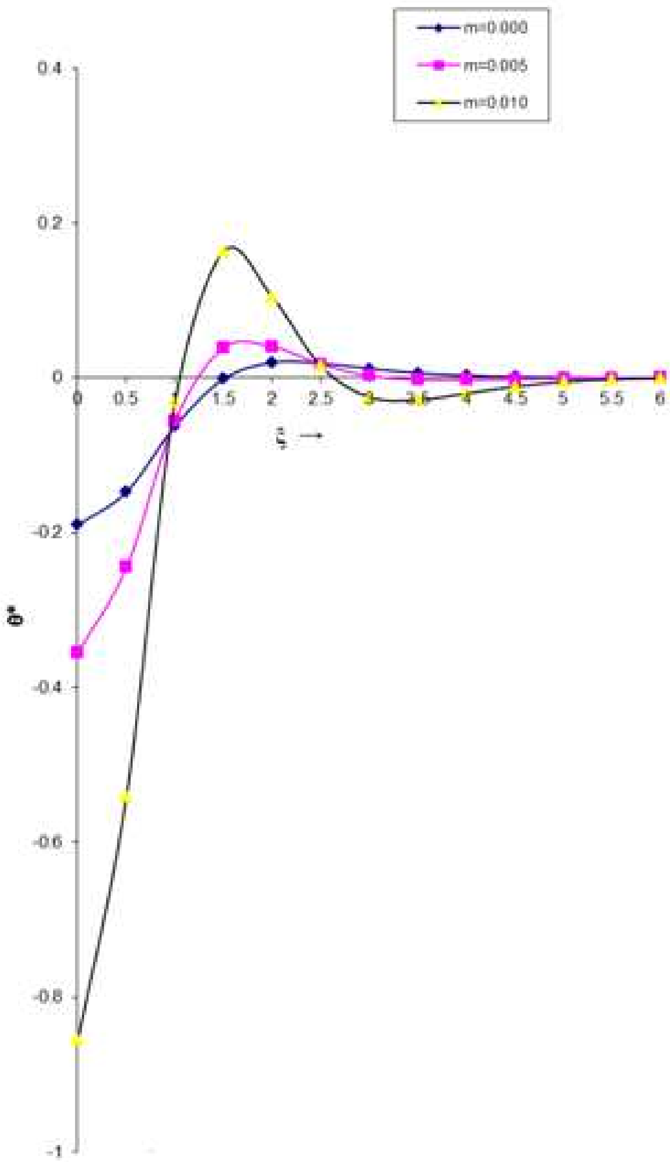

Figure 7: Temperature Distribution in Swirling Jet for $Pr = 2.000$

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