

Portfolio Selection using Cointegration and Modern Portfolio Theory: An Application to the Colombo Stock Exchange

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Abstract Profitable stock market investments have never been easy due to the lack of predictability and higher risk of stock returns. Hence the necessity of portfolio selection has arisen in order to find the ideal portfolio which best suits the stock market behavior. This study aims at devising the ideal sector portfolio and identifying a better strategy for developing the ideal sector portfolio in the Colombo Stock Exchange (CSE). It covers a period when CSE was one of the best performing stock exchanges in the world; the post civil war period in Sri Lanka. In this study better portfolios were derived from two strategies: (1) Cointegration approach; and (2) Modern Portfolio Theory (including Capital Market Theory) where both are widely used to derive portfolios related to stock market trading. Then the performances of the better portfolios derived were compared by means of the Sharpe Ratio, Information Ratio, Return and Risk in order to determine the ideal portfolio as well as the better strategy. Final conclusion of the study states that “Market portfolio” obtained from the Modern Portfolio Theory performs better than “Best Cointegrated portfolio” obtained from the Cointegration approach in the considered period of time by dominating most of the comparison measures. Also the ideal portfolio consisted of eight sectors out of twenty sectors in CSE with varying weight percentages. Further the best portfolio selection method between the two strategies could have been obtained regardless the period if the methodology is implemented for several Stock Exchanges. However, limitations in accessing necessary data prevented this implementation in the study.

Keywords Sharpe Ratio; Information Ratio; Cointegration; Modern Portfolio Theory; Capital Market Theory; Colombo Stock Exchange.

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1 Introduction

Numerous ways could be listed for investing and trading money in the modern financial world. Bank accounts, Gold, Stocks, Real estate, Crude oil trading, Treasury bonds and other commodities could be given as some examples. However this study is focused only on stock market investments since its popularity among lot of investors in Sri Lanka as well as the rest of the world. A portfolio in a stock market can consist of different assets such as Stocks, Bonds, Treasury bills, Options, Futures and Swaps etc. As stock market investments rose to its top level, becoming the most anticipated investment opportunity among investors all over the world, necessity of maintaining a better portfolio occurred. That led investors and researchers to the concept of portfolio selection (optimization).

Portfolio selection could be done using numerous strategies. Modern Portfolio Theory (MPT), Genetic Algorithms, Neural Networks, Convex Quadratic Programming and Cointegration Technique can be given as some instances. Out of these strategies cointegration has emerged as a powerful tool in the recent past since its initial introduction in analyzing time series data [1]. Also cointegration models are now becoming common and popular in financial data analysis. On the other hand MPT is

the pioneering method used in portfolio selection [2]. Moreover most of the solid financial data analysis techniques such as Capital Asset Pricing Model and Arbitrage Pricing Theory are built on this MPT. Most of the underlying methods and techniques of the above two strategies are statistical and hence the results obtained would be more realistic since they incorporate the real world practicability. Hence consideration of the study is given only to Cointegration approach and MPT. In addition to the portfolio selection using the two strategies, their performances will be compared in different aspects.

Portfolio construction using cointegration as an application was first introduced by Alexander [3]. The concept of index tracking with the help of cointegration was brought forward with this. Index tracking strategy is an investment strategy which comes under passive portfolio approach where a portfolio of stocks is used to track a benchmark in the stock market. Gabriel [4] carried a study related to index tracking strategy and important part of that study is the use of an algorithm to determine the best set of cointegrated stocks. A similar study has been done on the Colombo Stock Exchange (CSE) [5]. That study has tracked the All Share Price Index (ASPI) from a portfolio of twenty sectors of CSE. In this study two portfolios were obtained by the unmanaged technique and the rebalancing technique.

On the other hand MPT is based on correlation analysis and it was introduced in the mid 90s [2, 6]. In MPT volatilities of returns of the assets were considered as risks and the best possible combination of assets is traced.

This study was based on CSE because from the beginning of 2009 there was a massive trend to invest in CSE. The end of the Civil War (which was prevailing in the country for 27 years), recovery from the world economic crisis, increased awareness of general public, easiness and development of technology can be given as the major reasons behind this trend. There are twenty sectors¹ which are identified in CSE. In total there are about 241 companies listed under CSE belonging to one of the above sectors. Consequently an investor can invest according to his/her risk aversion with foregoing listed companies. However in order to maximize the return, selection of these companies in which to invest is vital. Thus following objectives were set to achieve in this paper.

- (a) Construct portfolios of sectors in CSE using daily sector indices from two different strategies: (1) Cointegration approach, and (2) Modern Portfolio Theory (Markowitz's theory) with Capital Market Theory
- (b) Compare and contrast the portfolios obtained via two strategies to determine, the ideal portfolio that should be possessed in CSE and the most suitable portfolio selection strategy for the considered period of time

This paper constructs a sector portfolio rather than a security portfolio. Yet it should be mentioned that the same methodology can be extended to construct a security portfolio.

Two types of portfolio selection strategies were used in this study. Also the ideal portfolio was found among several better portfolios which help to determine the best strategy for portfolio selection during the considered period in Sri Lankan context. In this regard present study can be considered as a unique study and the ideal portfolio obtained would be the best out of the best from two strategies. No similar study to this has been found in Sri Lankan and global context and hence this would be an important research for Sri Lankan and Asian investors in making their investment decisions.

2 Materials and Methods

Methodology will be discussed under the two strategies along with the brief explanation of the methods that will be used to compare the portfolios in this section. Data used in the study consist of daily closing sector indices and the benchmark index, ASPI. The data set comprised of data from 02nd January 2009 to 31st March 2011. Data from beginning of 2009 to end of 2011 was used as the in-sample data. Data from 03rd January 2011 to 31st March 2011 was used as the back-test sample.

¹ Twenty sectors are listed in www.cse.lk

2.1 Portfolio Selection using Cointegration

If a linear combination of non stationary time series is stationary, then those time series are termed as cointegrated [7]. Cointegration approach discussed here determines the Best Cointegrated sector portfolio which tracks the benchmark index (i.e. index tracking strategy). Portfolio construction and back testing under this strategy would be done as Alexander and Dimitriu [8] emphasized and the iterative procedure that Gabriel [4] used will be adapted in selecting the ideal portfolio. In addition, the inclusion of non negativity constraints on the cointegration model would be adapted from Fernandopulle [5] since CSE does not allow short selling.

As the first step each sector index was transformed to its natural logarithm in order to scale down all the series. This transformation was done because the comparison would be easier between each series and it does not affect the portfolio construction procedure [8]. Then if a series seemed to have a stochastic trend it was employed Augmented Dickey-Fuller (ADF) test [9] and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test [10] else Phillips-Perron (PP) test [11] for formally determining the stationarity. Initial sector series which make up the benchmark index should be of order 1 including the benchmark series (i.e. $I(1)$). Hence non stationary sectors which are of $I(1)$ were considered to the portfolio construction and others were dropped. Then the following iterative procedure was employed for sector selection.

Step 1: The Engle-Granger cointegration equation was estimated using all n sectors of order 1 as explanatory variables and ASPI as the dependent variable². Then an ADF test was performed on the residuals of the regression. Let the corresponding ADF test statistic value from the unit root test³ be ADF_0 .

Step 2: Then the most cointegrated portfolio was constructed by eliminating the i^{th} sector from the above regression successively, for every $i = 1, 2, \dots, n$. Hence there were n cointegration equations each containing $n-1$ sectors. Then for every i^{th} equation, residuals were generated and subjected to ADF tests. Thus there were n ADF test statistics and they were named $ADF_{0,1}, ADF_{0,2}, \dots, ADF_{0,n}$. From these values minimum one (most negative one) was chosen. If that value is $ADF_{0,j}$ where, $1 \leq j \leq n$ then according to minimum ADF test statistic criterion the variable j was completely eliminated from the portfolio. Then it was obtained the most cointegrated portfolio at 1st iteration.

Step 3: Then the second iteration was started. Now the portfolio will be consisted of $n-1$ sectors. The above process was repeated again by successively eliminating i^{th} sector series from the cointegration regression for every $i = 1, 2, \dots, n-1$. Then there were $n-1$ equations and their residuals were subjected to ADF tests. Obtained ADF test statistics were named as $ADF_{1,1}, ADF_{1,2}, \dots, ADF_{1,(n-1)}$. Again the most negative ADF value was chosen. If it is $ADF_{1,k}$, where $1 \leq k \leq n-1$, then the k^{th} sector was completely eliminated from the portfolio as done in *Step 2*. This process was repeated until the portfolio consists of predetermined m number of sectors.

The value m can be varied according to researcher's choice [8]. This was applied because of its popularity and the practicability. Normally it takes integer values 5, 10 and 15. Purpose of different m values is that, it can be compared and examined several portfolios and their performances on ASPI with respect to different number of sectors. It would make the portfolio selection more efficient.

Once the portfolio with m sectors was obtained it was subjected to non-negativity constraint by enforcing negative regression coefficients to be zero and then again re-estimating cointegration equation by remaining sectors [7].

The equation with the overall minimum ADF value was considered as the best cointegration equation and its p sectors comprised the Best Cointegrated portfolio. This value p can be 5, 10, 15 or even a number between them. Finally all the portfolios were normalized in order to make them interpretable as weights of the portfolio. This was done by dividing each sector coefficient by the summation of all sector coefficients in a particular cointegration equation.

² ASPI was set to be the dependent variable throughout the process

³ This is a common name for stationary tests

2.2 Portfolio Selection using Modern Portfolio Theory and Capital Market Theory

In this strategy, two portfolios namely Minimum Risk and Market portfolios were found. Minimum Risk portfolio is the one that has minimum standard deviation of returns. Market portfolio is the one which incorporates risk free rate and it was found by using both MPT and Capital Market Theory (CMT). In this strategy back testing was done similar to the Cointegration approach.

First step in constructing the market portfolio was to obtain the rate of return series for each and every sector⁴. Then the highly influential extreme outliers were removed from the rate of return series and they were estimated by taking the average of adjacent two points of a particular outlier, in order to secure the continuity of the series. Values lie beyond the range, $(Q_{1,i} - 3 * IQR_i, Q_{3,i} + 3 * IQR_i)$ can be considered as extreme outliers of the i^{th} sector. Here $Q_{1,i}$ and $Q_{3,i}$ are the 1st and 3rd quartile of the i^{th} sector's rate of return and IQR_i is the inter quartile range of the i^{th} sector's rate of return.

Then mean values (expected return of the sectors), variances of returns and co-variances between returns were calculated for each and every sector using in-sample. From these results, variance co-variance matrix was also obtained for the sector return series. Next step was the determination of efficient frontier [2, 6]. Efficient frontier is the curve in risk-return plane which gives the set of portfolios which have the minimum risk for a given level of return. The determination of the efficient frontier was achieved from an optimization process. This optimization can be formulated as below.

$$\left. \begin{array}{l} \text{Min}(\omega' \Omega \omega) = \sigma_{port} \\ \text{Subject to constraints,} \end{array} \right\} \begin{array}{l} \omega' \mu = E(R_{port}) \\ \omega' \tau = 1 \\ \omega \geq 0 \end{array} \quad (1)$$

where, ω is a $n \times 1$ vector of sector weights, Ω is the $n \times n$ covariance matrix, σ_{port} is the portfolio standard deviation, μ is the $n \times 1$ vector of sector returns, $E(R_{port})$ is the expected portfolio return, τ is a $n \times 1$ vector of ones, n is the number of sectors.

Also,

$$E(R_{port}) = \sum_{i=1}^n w_i R_i \quad (2)$$

$$\sigma_{port} = \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1, j \neq i}^n w_i w_j \text{cov}_{i,j}} \quad (3)$$

where, R_i is the expected return of i^{th} sector, w_i is the weight of the i^{th} sector in the portfolio, σ_i^2 is the variance of the i^{th} sector, $\text{cov}_{i,j}$ is the covariance between i^{th} and j^{th} sectors.

By specifying different values for $E(R_{port})$ different ω vectors were found which are considered as the weight combinations such that σ_{port} is minimum. Additionally the weight combination of Minimum Risk portfolio was found without using $\omega' \mu = E(R_{port})$ constraint. The use of $\omega \geq 0$ is to restrict the short selling similar to the Cointegration approach. By plotting specified $E(R_{port})$ against σ_{port} efficient frontier was obtained. Portfolio at the turning point of the curve is the Minimum Risk portfolio. Then the Market portfolio was obtained by drawing the Capital Market Line (CML) which

⁴ Rate of return is same as the return. In this paper both terms were used interchangeably

becomes a tangent with the efficient frontier. The intercept of the CML was the Risk Free Rate (RFR). This tangency portfolio is known as the Market portfolio [12].

2.3 Comparison

Comparison among the obtained portfolios was done using several measurements. Namely: Return of the portfolio, Risk of the portfolio (standard deviation), Excess returns and their volatility, Correlation with the benchmark index, Sharpe Ratio and Information Ratio.

3 Results and Discussion

Only the important and necessary results are given in this section. Also the results from the Cointegration approach and MPT are given separately. All series were found to be non stationary and of order 1 except the “Construction and Engineering” sector series. All non stationary series of order 1 were considered for Cointegration approach. From the preliminary analysis for MPT, it was found that every rate of return series is consisted of possible extreme outliers. Thus they were eliminated and estimated using the method explained in the previous section.

3.1 Cointegration Results

Under this strategy portfolios which are consisted of 15, 10 and 5 sectors were constructed. Once the non negativity constraint was imposed the same were consisted of 13, 9 and 5 sectors respectively. Then the Best Cointegrated portfolio was found by considering the equation with overall minimum ADF and it was consisted of 11 sectors when the non negativity constraint was imposed.

3.2 Modern Portfolio Theory Results

Under the MPT expected rate of returns and variances of expected returns were calculated at the first place. Then as mentioned in the previous section, optimization was carried out with the respective constraints. Then the efficient frontier was drawn on the risk-return plane and corresponding Minimum Risk portfolio was obtained from the turning point of the efficient frontier. RFR considered here was the government Treasury bond rate in 2009 which was 21%. By incorporating this RFR, CML was drawn and hence the Market portfolio was obtained. Derivation of the two portfolios is given in Figure 1.

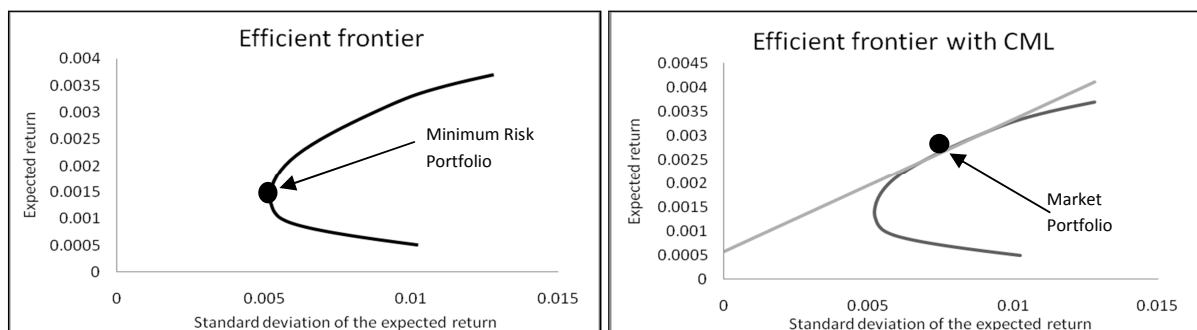


Figure 1 (a) Minimum Risk portfolio and (b) Market portfolio

3.3 Weights

Percentage weights of the six constructed portfolios are given in Table 1.

Table 1 Percentage weights of sectors in portfolios

Sector Name	13 Sector Portfolio	9 Sector Portfolio	5 Sector Portfolio	Best Cointegrated Portfolio	Minimum Risk Portfolio	Market Portfolio
Banks, Finance and Insurance	-	-	-	-	-	19.22%
Beverage, Food and Tobacco	21.29%	22.10%	-	22.21%	10.57%	26.53%
Chemicals and Pharmaceuticals	7.08%	7.33%	-	6.97%	-	1.38%
Construction and Engineering	-	-	-	-	3.51%	6.39%
Diversified Holdings	15.24%	16.55%	-	14.99%	-	15.32%
Footwear and Textile	1.09%	-	-	-	1.09%	-
Healthcare	-	-	-	-	7.19%	-
Hotels and Travels	10.93%	12.78%	42.90%	10.76%	-	-
Investment Trusts	4.83%	4.28%	2.57%	4.72%	-	-
Information Technology	-	-	-	-	0.07%	-
Land and Property	0.43%	-	-	1.09%	-	-
Manufacturing	0.49%	-	-	-	-	-
Motors	0.09%	1.97%	5.80%	1.00%	9.56%	7.03%
Oil Palms	-	-	-	-	3.89%	-
Plantations	4.98%	6.00%	-	5.23%	-	-
Power and Energy	-	-	-	-	-	-
Services	-	-	-	-	3.50%	-
Stores and Supplies	5.97%	6.38%	11.01%	5.91%	58.37%	18.01%
Telecommunications	24.92%	22.60%	37.72%	25.16%	2.25%	-
Trading	1.89%	-	-	1.96%	-	6.11%

3.4 Comparison

Comparison was done with respect to given measurements in Materials and Methods section. Summary of the comparison results is given in Table 2.

Table 2 Comparison results

Portfolio	Average annualized return	Portfolio risk	Annualized volatility of daily return	Average annualized daily excess return	Annualized volatility of daily excess return	Correlation between ASPI and portfolio returns	Correlation between ASPI and portfolio excess returns	Sharpe ratio	Information ratio
13 sector	143.83%	0.0245	38.96%	92.30%	29.26%	0.7017	0.3166	3.1527	3.6917
9 sector	143.90%	0.0238	37.79%	92.38%	28.47%	0.6915	0.2832	3.2515	3.8071
5 sector	166.24%	0.0261	41.48%	114.71%	34.57%	0.5686	0.1595	3.5012	4.0074
Best Cointegrated	142.32%	0.0242	38.40%	90.79%	28.73%	0.7033	0.3111	3.1587	3.7054

Minimum risk	226.17%	0.0297	47.13%	174.64%	41.32%	0.4939	0.126	4.3525	4.798
Market	151.67%	0.0211	33.54%	100.14%	29.54%	0.4779	-0.0691	3.8948	4.5208

When the summary results are observed in Table 2, it can be seen that constructed 13 sector and 5 sector portfolios have not performed well by any means even if they have constructed from the well known Cointegration approach. However it can be justified; when a portfolio is over diversified or less diversified its performance may get worse. Hence it can be believed that over diversification and less diversification might be the reasons for this.

There was a suspicion in selecting the Market portfolio as the ideal one over the Minimum Risk portfolio. Even though it seemed that Minimum Risk portfolio is the ideal portfolio by the domination of majority of considered aspects (return, Sharpe ratio and Information ratio) in the comparison, surprisingly it possessed the highest risk among the constructed portfolio. Minimum risk portfolio should never possess the highest risk since it is the choice of more risk averse investor. However, if the Minimum Risk portfolio was selected as the ideal one, this scenario would be a contradicted. Hence the Market portfolio was selected as the ideal portfolio which performs well in risk and seconds only to Minimum Risk portfolio in terms of returns, Sharpe ratio and Information ratio.

4 Conclusion

The Best Cointegrated portfolio outperformed every other portfolio in tracking behaviors while becoming the best tracking portfolio. On the other hand Market portfolio became the safest (less risky) and the best portfolio when the entire concentration is on return and risk. Also it should be noted that there is a risk in investing in the Best Cointegrated portfolio since it tracks the ASPI over the time and hence there is a possibility to generate negative returns (losses) when ASPI performs poorly⁵. However there is no such shortcoming in the Market portfolio. Furthermore, the considered market not being a 100% efficient market and volatility issues may cause unwanted results in the Best Cointegrated portfolio to the investor.

Additionally constructed portfolios perform much better than ASPI index in terms of return, Sharpe ratio and Information ratio. This could be observed when the estimated prices of portfolios are considered. Hence, any of the constructed portfolios would satisfy the investors' desires in terms of profitability with respect to their risk aversion. Finally it can be concluded that Modern Portfolio Theory with Capital Market Theory suits more than the Cointegration approach in portfolio selection at Colombo Stock Exchange for the considered period of time.

Market portfolio was chosen as the ideal portfolio and its eight sectors were 'Banks Finance and Insurance', 'Beverage Food and Tobacco', 'Chemicals and Pharmaceuticals', 'Construction and Engineering', 'Diversified Holdings', 'Motors', 'Stores and Supplies' and 'Trading'. Percentage capital allocation should be done as 19.22%, 26.53%, 1.38%, 6.39%, 15.32%, 7.03%, 18.01% and 6.11% in above sectors respectively.

This study could not be extended to foreign stock markets, due to the unavailability of data. Future research direction will be a determination of the best portfolio optimization strategy by applying both to several global stock markets.

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⁵ In this regard rebalancing strategy could be used

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