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Nonlocal Cauchy Problem for Integrodifferential Equations of Sobolev Type in Banach Space

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Abstract In this paper, we prove the existence and uniqueness of mild and strong solutions of nonlinear integrodifferential equations of Sobolev type in Banach space. The results are obtained by using compact semigroups and the Schauder fixed point theorem.

Keywords Mild and Strong solutions; Nonlocal condition; Integrodifferential equations; Compact semigroup; Schauder fixed point theorem.

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1 Introduction

Byszewski [1] has established the existence and uniqueness of mild, strong and classical solutions of the following nonlocal Cauchy problem

$$\frac{du(t)}{dt} + Au(t) = f(t, u(t)), \quad t \in (0, a],$$

$$u(t_0) + g(t_1, t_2, \ldots, t_p, u(.)) = u_0$$

where A is the infinitesimal generator of a C_0-semigroup $T(t)$ on a Banach space X, $0 \le t_0 < t_1 < \cdots < t_p \le a$, $a > 0$, $u_0 \in X$ and $f : [0, a] \times X \to X$, $g : [0, a]^p \times X \to X$ are given functions. Subsequently, he has investigated the same problem for different types of evolution equations in Banach space [2–5]. Many papers have been written on nonlocal Cauchy problem for different classes of differential and integrodifferential equations [6–12]. Brill [13] and Showalter [14] established the existence of solutions of semilinear evolution equations of Sobolev type in Banach space. Such type of equations arises in various applications such as in the flow of fluid through fissured rocks [15], and thermodynamics [16]. Balachandran and Ravikumar [17] proved the existence of mild and strong solutions of nonlinear time varying delay integrodifferential equations of Sobolev type with nonlocal conditions in Banach spaces by using the theory of compact semigroup and Schaefer's fixed point theorem. Recently, Xu [18] has studied the existence of delay integrodifferential equations of Sobolev type with nonlocal conditions in Banach space

In this paper, we shall prove the existence of solutions for integrodifferential equations of Sobolev type with nonlocal conditions of the form

$$\begin{aligned}&(Bu(t))' + Au(t)\\ &\quad = f\left(t, u(t), \int_{t_0}^{t} k\left(t, s, u(s), \int_{t_0}^{s} h(s, \tau, u(\tau))d\tau\right)ds\right), \; t \in (t_0, t_0 + a] \qquad (1)\end{aligned}$$

$$u(t_0) + g(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) = u_0, \tag{2}$$

where $f : I \times X \times X \times X \to Y$, $k : I \times I \times X \times X \to X$, $h : I \times I \times X \to X$ and $g : I^p \times X^p \to Y$ are given functions, and $I = [t_0, t_0 + a]$.

2 Preliminaries

Definition 1 A continuous solution u of the integral equation

$$u(t) = B^{-1}T(t - t_0) Bu_0 - B^{-1}T(t - t_0) Bg(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) + \int_{t_0}^{t} B^{-1}T(t-s) f\left(s, u(s), \int_{t_0}^{s} k\left(s, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu)) d\mu\right) d\tau\right) ds, \ t \in I \tag{3}$$

is called a mild solution of problem (1) – (2) on I.

Definition 2 A function u is said to be a strong solution of problem (1) – (2) on I if u is differentiable almost everywhere on I $u^{'} \in L^1(I, X)$ and satisfying (1) – (2) almost everywhere on I.

In order to prove our main theorem we consider certain conditions on the operators A and B. Let X and Y be Banach spaces with norm $|.|$ and $\|.\|$ respectively. The operators $A : D(A) \subset X \to Y$ and $B : D(B) \subset X \to Y$ satisfy the following hypothesis:

(H_1) A and B are closed linear operators,

(H_2) $D(B) \subset D(A)$ and B is bijective,

(H_3) $B^{-1} : Y \to D(B)$ is continuous.

The hypothesis (H_1) and (H_2) and the closed graph theorem imply the boundedness of the linear operator $AB^{-1} : Y \to Y$. Further $-AB^{-1}$ generates a uniformly continuous semigroup $T(t)\, t \geq 0$, of bounded linear operators from Y into Y.

(H_4) For some $\lambda \in \rho\left(-AB^{-1}\right)$, the resolvent set of $-AB^{-1}$ the resolvent $R\left(\lambda, -AB^{-1}\right)$ is compact operator.

Theorem 1 *Let $T(t)$ be a C_0 semigroup. If $T(t)$ is compact for $t > t_0$ then $T(t)$ is uniformly continuous for $t > t_0$.*

We have the following characterization of a compact semigroup in terms of the resolvent operators $R(\lambda : A)$ of its generator A.

Theorem 2 *Let $T(t)$ be a C_0 semigroup and let A be its infinitesimal generator. $T(t)$ is a compact semigroup if and only if $T(t)$ is uniformly continuous for $t > 0$ and $R(\lambda : A)$ is compact for $\lambda \in \rho(A)$* [19].

From the above fact that $-AB^{-1}$ generates a compact semigroup $T(t)\, t \geq 0$, and so $\|T(t)\|$ is finite. We denote $M = \|T(t)\|$, $L = \|B\|$ and $L^* = \left\|B^{-1}\right\|$.

Further, we assume the following:

(H_5) $f : I \times X \times X \times X \to Y$ is continuous in t on I and there exist a constant $N > 0$ such that

$$\left\| f\left(t, u(t), \int_{t_0}^{t} k\left(t, s, u(s), \int_{t_0}^{s} h(s, \tau, u(\tau)) d\tau\right) ds\right) \right\| \le N$$

for $t \in I$ and $u \in X$,

(H_6) $k : I \times I \times X \times X \to X$ is continuous in t,

(H_7) $h : I \times I \times X \to X$ is continuous in t,

(H_8) $g : I^p \times X^p \to Y$ is continuous and there exists a constant $G > 0$ such that

$$G = \| g(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) \| .$$

3 Existence of a Mild Solution

Theorem 3 *Assume that $(H_1) - (H_8)$ hold, then the problem (1) - (2) has a mild solution on I.*

Proof Let $S = C([t_0, t_0 + a], Y)$ and

$$S_0 = \left\{ \begin{array}{r} u : u \in Y, u(t_0) + g(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) = u_0, \\ \|u\| \le r, t_0 \le t \le t_0 + a \end{array} \right\},$$

where $r := L^* ML \|u_0\| + L^* MLG + L^* MNa$. Clearly, S_0 is a bounded closed convex subset of S. We define a mapping $F : S \to S_0$ by

$$(Fu)(t) = B^{-1} T(t - t_0) Bu_0 - B^{-1} T(t - t_0) Bg(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p))$$

$$+ \int_{t_0}^{t} B^{-1} T(t - s) f\left(s, u(s), \int_{t_0}^{s} k\left(s, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu)) d\mu\right) d\tau\right) ds, \ t \in I.$$

Since

$$\begin{aligned} &\|(Fu)(t)\| \\ &\le \left\| B^{-1} T(t - t_0) Bu_0 \right\| + \left\| B^{-1} T(t - t_0) Bg(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) \right\| \\ &+ \int_{t_0}^{t} \left\| B^{-1} T(t - s) \right\| \left[\left\| f\left(s, u(s), \int_{t_0}^{s} k\left(s, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu)) d\mu\right) d\tau\right) \right\| ds \right] \\ &\le L^* ML \|u_0\| + L^* MLG + L^* MNa = r, \end{aligned}$$

then F maps S_0 into S_0. Further, the continuity of F from S_0 to S_0 follows that fk and h are continuous on $[t_0, t_0 + a] \times X^3$, $[t_0, t_0 + a]^2 \times X^2$ and $[t_0, t_0 + a]^2 \times X$ respectively. Moreover, F maps S_0 into a precompact subset of S_0. We prove that, the set $S_0(t) :=$

$\{(Fu)(t) : u \in S_0\}$ is precompact in X for every fixed $0 \le t \le a$. Obviously for $t = t_0$ the set $S_0(t_0) = \{u_0 - g\}$ is precompact. Let $t > t_0$ be fixed. Define

$$\begin{aligned}
&(F_\epsilon u)(t) = B^{-1}T(t-t_0)Bu_0 - B^{-1}T(t-t_0)Bg(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) \\
&+ \int_{t_0}^{t-\epsilon} B^{-1}T(t-s) f\left(s, u(s), \int_{t_0}^{s} k\left(s, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu))\, d\mu\right) d\tau\right) ds \\
&= B^{-1}T(t-t_0)Bu_0 - B^{-1}T(t-t_0)Bg(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p)) \\
&+ T(\epsilon)\int_{t_0}^{t-\epsilon} B^{-1}T(t-\epsilon-s) f\left(s, u(s), \int_{t_0}^{s} k\left(s, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu))\, d\mu\right) d\tau\right) ds \qquad (4)
\end{aligned}$$

for $t_0 < \epsilon < t$.

The compactness of the semigroup $T(t)$ for every $t > 0$ and (4) imply that for every ϵ, $t < \epsilon < t_0$ the set

$$S_\epsilon(t) = \{(F_\epsilon u)(t) : u \in S_0\}$$

is precompact in X. Now, for any $u \in S_0$ we have

$$\begin{aligned}
&\|(Fu)(t) - (F_\epsilon u)(t)\| \\
&\le \int_{t-\epsilon}^{t} \left\|B^{-1}\right\| \|T(t-s)\| \left\| f\left(s, u(s), \int_{t_0}^{s} k\left(s, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu))\, d\mu\right) d\tau\right)\right\| ds \\
&\le L^* M N \epsilon. \qquad (5)
\end{aligned}$$

From (5) it follows that the set $S_0(t)$ is precompact. Now we show that $F(S_0) = \tilde{S} = \{Fu : u \in S_0\}$ is an equicontinuous.

For $t_0 < t < s$ we have

$$\begin{aligned}
&\|(Fu)(t) - (Fu)(s)\| \le \|B^{-1}(T(t-t_0) - T(s-t_0))Bu_0\| \\
&+ \|B^{-1}(T(t-t_0) - T(s-t_0))Bg(t_1, t_2, \ldots, t_p, u(t_1), u(t_2), \ldots, u(t_p))\| \\
&+ \int_{t_0}^{t} \|B^{-1}\| \|T(t-\xi) - T(s-\xi)\| \left\| f(\xi, u(\xi), \int_{t_0}^{\xi} k(\xi, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu)) d\mu) d\tau) \right\| d\xi \\
&+ \int_{t}^{s} \|B^{-1}\| \|T(s-\xi)\| \left\| f(\xi, u(\xi), \int_{t_0}^{\xi} k(\xi, \tau, u(\tau), \int_{t_0}^{\tau} h(\tau, \mu, u(\mu)) d\mu) d\tau) \right\| d\xi \\
&\le (L^* L \|u_0\| + L^* L G) \|T(t-t_0) - T(s-t_0)\| \\
&\qquad + L^* N \int_{t_0}^{t} \|T(t-\xi) - T(s-\xi)\| d\xi + L^* M N |s-t|. \qquad (6)
\end{aligned}$$

Since $T(t)$ is compact, Theorem 1 implies that $T(t)$ is continuous in the uniform operator topology for $t > 0$. Therefore the righthand side of (6) tends to zero as $s \to t$ tends to zero. Thus $\tilde{S}$ is equicontinuous. Also, $\tilde{S}$ is bounded. It follows from Arzela-Ascoli theorem (see Dieudonne [2]), that $\tilde{S}$ is precompact. Hence by the Schauder fixed point theorem, F has a fixed point in S_0 and any fixed point of F is a mild solution of (1) – (2) on I such that $u(t) \in X$ for $t \in I$.□

4 Existence of a Strong Solution

Theorem 4 *Assume that*

(i) *Conditions* $(H_1)-(H_8)$ *hold,*

(ii) Y *is reflexive Banach space with norm* $\|.\|$,

(iii) $f: I\times X\times X\times X\to Y$ *is continuous in* t *on* I *and there exist constants* $N>0$ *and* $N_1>0$ *such that*

$$\left\|f\Big(t,u\Big(t\Big),\int_{t_0}^{t}k\Big(t,s,u\Big(s\Big),\int_{t_0}^{s}h\Big(s,\tau,u\Big(\tau\Big)\Big)d\tau\Big)ds\Big)\right\|\le N$$

and $\|f(t,u_1,u_2,u_3)-f(s,v_1,v_2,v_3)\|$
$\le N_1\left[\|t-s\|+\|u_1-v_1\|+\|u_2-v_2\|+\|u_3-v_3\|\right]$ *for* $ts\in I,\ u_iv_i\in X,$

(iv) $k: I\times I\times X\times X\to X$ *is continuous in* t *and there exist constants* $N_2>0$ *and* $N_3>0$ *such that* $\|k(t,s,u,v)\|\le N_2,\ \|k(t,\sigma,u,v)-k(s,\sigma,u,v)\|\le N_3|t-s|$ *for* $t,\sigma,s\in I,\ u\in X,$

(v) $g(t_1,t_2,\dots,t_p,u(t_1),u(t_2),\dots,u(t_p))\in D\left(-AB^{-1}\right),\ u_0\in D\left(-AB^{-1}\right).$

Then u *is a strong solution of problem (1) – (2) on* I.

Proof Since all the assumptions of Theorem 3 are satisfied, then the problem (1) – (2) has a mild solution belonging to $C(I,X)$ Now we shall show that u is a strong solution of problem (1) – (2) on I. For any $t\in I$ we have

$$\begin{aligned}
&u\Big(t+h\Big)-u\Big(t\Big)=B^{-1}\Big[T\Big(t+h-t_0\Big)-T\Big(t-t_0\Big)\Big]Bu_0\\
&-B^{-1}\Big[T\Big(t+h-t_0\Big)-T\Big(t-t_0\Big)\Big]Bg\Big(t_1,t_2,\dots,t_p,u\Big(t_1\Big),u\Big(t_2\Big),\dots,u\Big(t_p\Big)\Big)\\
&+\int_{t_0}^{t_0+h}B^{-1}T\Big(t+h-s\Big)f\Big(s,u\Big(s\Big),\int_{t_0}^{s}k\Big(s,\tau,u\Big(\tau\Big),\int_{t_0}^{\tau}h\Big(\tau,\mu,u\Big(\mu\Big)\Big)d\mu\Big)d\tau\Big)ds\\
&+\int_{t_0+h}^{t+h}B^{-1}T\Big(t+h-s\Big)f\Big(s,u\Big(s\Big),\int_{t_0}^{s}k\Big(s,\tau,u\Big(\tau\Big),\int_{t_0}^{\tau}h\Big(\tau,\mu,u\Big(\mu\Big)\Big)d\mu\Big)d\tau\Big)ds\\
&-\int_{t_0}^{t}B^{-1}T\Big(t-s\Big)f\Big(s,u\Big(s\Big),\int_{t_0}^{s}k\Big(s,\tau,u\Big(\tau\Big),\int_{t_0}^{\tau}h\Big(\tau,\mu,u\Big(\mu\Big)\Big)d\mu\Big)d\tau\Big)ds\\
&=B^{-1}T\Big(t-t_0\Big)\Big[T\Big(h\Big)-I\Big]Bu_0\\
&-B^{-1}T\Big(t-t_0\Big)\Big[T\Big(h\Big)-I\Big]Bg\Big(t_1,t_2,\dots,t_p,u\Big(t_1\Big),u\Big(t_2\Big),\dots,u\Big(t_p\Big)\Big)\\
&+\int_{t_0}^{t_0+h}B^{-1}T\Big(t+h-s\Big)f\Big(s,u\Big(s\Big),\int_{t_0}^{s}k\Big(s,\tau,u\Big(\tau\Big),\int_{t_0}^{\tau}h\Big(\tau,\mu,u\Big(\mu\Big)\Big)d\mu\Big)d\tau\Big)ds\\
&+\int_{t_0}^{t}B^{-1}T\Big(t-s\Big)\Big[f\Big(s+h,u\Big(s+h\Big),\int_{t_0}^{s+h}k\Big(s+h,\tau,u\Big(\tau\Big),\int_{t_0}^{\tau}h\Big(\tau,\mu,u\Big(\mu\Big)\Big)d\mu\Big)d\tau\Big)\\
&-f\Big(s,u\Big(s\Big),\int_{t_0}^{s}k\Big(s,\tau,u\Big(\tau\Big),\int_{t_0}^{\tau}h\Big(\tau,\mu,u\Big(\mu\Big)\Big)d\mu\Big)d\tau\Big)\Big]ds.
\end{aligned}$$

According to our assumption we observe that

$$
\begin{aligned}
&\|u(t+h)-u(t)\| \\
&\quad \le L^*MLh\left\|B^{-1}Au_0\right\| + L^*MLh\left\|B^{-1}Ag\left(t_1,t_2,\ldots,t_p,u\left(t_1\right),u\left(t_2\right),\ldots,u\left(t_p\right)\right)\right\| \\
&\qquad + L^*MNh + L^*M\int_{t_0}^{t} N_1\Big[h + \|u(s+h)-u(s)\| \\
&\qquad + \int_{t_0}^{s}\Big\|k\Big(s+h,\tau,u(\tau),\int_{t_0}^{\tau}h(\tau,\mu,u(\mu))d\mu\Big) - k\Big(s,\tau,u(\tau),\int_{t_0}^{\tau}h(\tau,\mu,u(\mu))d\mu\Big)\Big\|d\tau \\
&\qquad + \int_{s}^{s+h}\Big\|k\Big(s+h,\tau,u(\tau),\int_{t_0}^{\tau}h(\tau,\mu,u(\mu))d\mu\Big)\Big\|d\tau\Big]ds \\
&\quad \le L^*MLh\left\|B^{-1}Au_0\right\| + L^*MLh\left\|B^{-1}Ag\left(t_1,t_2,\ldots,t_p,u\left(t_1\right),u\left(t_2\right),\ldots,u\left(t_p\right)\right)\right\| \\
&\qquad + L^*MNh + L^*MN_1\int_{t_0}^{t}\left[h + \left\|u\left(s+h\right)-u\left(s\right)\right\| + N_2ha + N_3h\right]ds \\
&\quad \le Ch + L^*MN_1\int_{t_0}^{t}\left\|u\left(s+h\right)-u\left(s\right)\right\|ds,
\end{aligned}
$$

where

$$
\begin{aligned}
C = {} & L^*ML\left\|B^{-1}Au_0\right\| + L^*ML\left\|B^{-1}Ag\left(t_1,t_2,\ldots,t_p,u\left(t_1\right),u\left(t_2\right),\ldots,u\left(t_p\right)\right)\right\| \\
& + L^*MN + L^*MN_1a + L^*MN_1N_2a^2 + L^*MN_1N_3a
\end{aligned}
$$

which is independent of h and $t \in I$. Thanks to Gronwall's inequality, we obtain

$$\left\|u\left(t+h\right)-u\left(t\right)\right\| \le Che^{L^*MN_1a} \text{ for } t \in I.$$

Therefore, u is Lipschitz continuous on I. The Lipschitz continuity of u on I combined with (iii) and (iv) of Theorem 4 implies

$$t \to f\left(t,u\left(t\right),\int_{t_0}^{t}k\left(t,s,u\left(s\right),\int_{t_0}^{s}h\left(s,\tau,u\left(\tau\right)\right)d\tau\right)ds\right)$$

is Lipschitz continuous on I. By Pazy [19, Corollary 4.2.11], we observe that the equation

$$
\begin{aligned}
&\left(Bv\left(t\right)\right)' + Av\left(t\right) = f\left(t,u\left(t\right),\int_{t_0}^{t}k\left(t,s,u\left(s\right),\int_{t_0}^{s}h\left(s,\tau,u\left(\tau\right)\right)d\tau\right)ds\right), \ t \in \left(t_0,t_0+a\right] \\
&v\left(t_0\right) = u_0 - g\left(t_1,t_2,\ldots,t_p,u\left(t_1\right),u\left(t_2\right),\ldots,u\left(t_p\right)\right)
\end{aligned}
$$

has a unique strong solution v satisfying the equation

$$
\begin{aligned}
v\left(t\right) = {} & B^{-1}T\left(t-t_0\right)Bu_0 - B^{-1}T\left(t-t_0\right)Bg\left(t_1,t_2,\ldots,t_p,u\left(t_1\right),u\left(t_2\right),\ldots,u\left(t_p\right)\right) \\
& + \int_{t_0}^{t}B^{-1}T(t-s)f\Big(s,u(s),\int_{t_0}^{s}k\Big(s,\tau,u(\tau),\int_{t_0}^{\tau}h(\tau,\mu,u(\mu))d\mu\Big)d\tau\Big)ds, \ \ t \in I \\
= {} & u\left(t\right).
\end{aligned}
$$

Consequently, $u\left(t\right)$ is the strong solution of initial value problem (1) – (2) on I. This completes the proof of Theorem 4. □

5 Conclusion

Thus we got the existence and uniqueness of mild and strong solutions of nonlinear integrodifferential equations of Sobolev type in Banach space by using compact semigroup and Schauder fixed point theorem. Our results are generalization of the results of Balachandran and Park [7, 9].

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References

[1] Byszewski, L. Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem, *J. Math. Anal. Appl.* 1991. 162: 494-505.

[2] Byszewski, L., and Akca, H. Existence of solutions of a semilinear functional differential evolution nonlocal problem, *Nonlinear Analysis, Theory, Methods and Applications.* 1998. 34: 65-712.

[3] Byszewski, L. On weak solution of functional-differential abstract nonlocal Cauchy problems, *Annales Polonici Mathematici.* 1997. 2: 163-170.

[4] Byszewski, L., On a mild solution of a semilinear functional-differential evolution nonlocal problems, *Journal of Applied Mathematics and Stochastic Analysis.* 1997. 10: 265-271.

[5] Byszewski, L. Existence, uniqueness and asymptotic stability of weak and strong solutions of functional-differential abstract nonlocal Cauchy problems, *Dynamics System and Application.* 1996. 5: 595-606.

[6] Kumar, Kamalendra and Kumar, Rakesh. Existence of mild solution of a delay integrodifferential equation with nonlocal condition, *Egyptian-Chinese Journal of Computational and Applied Mathematics.* (accepted).

[7] Balachandran, K., and Park, J. Y. Sobolev type integrodifferential equation with nonlocal condition in Banach spaces, *Taiwanese Journal of Mathematics*. 2003. 7(1): 155-163.

[8] Kanakaraj, M. and Balachandran, K. Existence of solutions of Sobolev-type semilinear mixed integrodifferential inclusions in Banach spaces, *Journal of Applied Mathematics and Stochastic Analysis*. 2003. 16(2): 163-170.

[9] Balachandran, K., and Park, J. Y., Nonlocal Cauchy problem for Sobolev type functional integrodifferential equation, *Bull. Korean Math. Soc.* 2002. 39: 561-569.

[10] Balachandran, K., and Chandrasekaran, M., Existence of solutions of a delay differential equation with nonlocal condition, *Indian J. Pure Appl. Math.*. 1996. 27(5): 443-449.

[11] Lin, Y., and Liu, J. H. Semilinear integrodifferential equations with nonlocal Cauchy problem, *Nonlinear, Analysis Theory, Methods and Applications.* 1996. 26: 1023-1033.

[12] Kumar K., Kumar R. and Shukla R.K. Existence of solutions of nonlinear delayed integrodifferential equation with nonlocal condition, in the *Proceedings "Advance in Applied Mathematics and Computational Physics"* (Eds. Rakesh Kumar et.al.), World Education Publishers, Delhi, India. 2012. 227-235.

[13] Brill, H. A semilinear Sobolev evolution equation in Banach space, *J. Differential Equations.* 1997. 24: 412-425.

[14] Showalter, R. E. Existence and representation theorem for a semilinear Sobolev equation in Banach space, *SIAM J. Math. Anal.* 1972. 3: 527-543.

[15] Barenblat, G., Zheltor, J., and Kochiva, I. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. *J. Appl. Math. Mech.* 1960. 24: 1286-1303.

[16] Chen, P. J. and Curtin, M. E. On a theory of heat conduction involving two temperatures. *Z. Angew. Math. Phys.* 1968. 19: 614-627.

[17] Balachandran, K., and Ravikumar, R., Nonlocal Cauchy problem for time varying delay integrodifferential equations of Sobolev type in Banach spaces, *Tamkang Journal of mathematics.* 2006. 27(3): 193-205.

[18] Xu, Xiaoping. Existence of delay integrodifferential equations of Sobolev type with nonlocal conditions, *International Journal of Nonlinear Science* 2012. 12(3): 263-269.

[19] Pazy, A. *Semigroup of Linear Operators and Applications to Partial Differential Equations.* New York: Springer Verlag. 1983.

[20] Dieudonne, J. *Foundation of Modern Analysis.* New York: Academic Press. 1960.