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An Economic Production Inventory Problem with Reworking of Imperfect Items in Fuzzy Environment

Nirmal Kumar Mandal
Department of Mathematics, Silda Chandrasekhar College
Silda, Paschim Medinipur, West Bengal, India, Pin-721515
e-mail: mandal_n_k@yahoo.co.in

Abstract The classical inventory models are formulated with the assumptions that the items are produced with perfect quality. But in reality, product quality may not always be perfect. Due to machinery fault, unskilled labour fault etc., imperfect quality items may be produced. A proportion of the defective items are assumed to rework at a constant rate. Some of the perfect quality items may deteriorate perish or damage at the time of packaging or transportation and these defective items passes from the manufacturer or supplier to the customers. Customers then return these defective items to the suppliers. In this paper, we formulate a multi-objective imperfect quality inventory model with rework of defective items under the limited storage space restrictions in fuzzy environment. Cost parameters are assumed as fuzzy number with different types of left and right branches of membership functions. Problem is solved by modified geometric programming approach. A numerical example is provided to illustrate the proposed model.

Keywords inventory; nearest weighted interval; best approximationl interval; ranking fuzzy number; geometric programming.

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1 Introduction

Generally the inventory models are formulated by considering that only the perfect quality items are produced. But in reality, product quality is not always perfect and is usually a function of the production process. The process may deteriorate and produce defective or poor quality items. So, a proportion of the produced items can be found to be defective. Porteus [1] incorporated the effect of defective items into the inventory problem. Rosenblatt and Lee [2] studied the effect of substandard quality, due to deterioration process on lot sizing decisions. Cheng [3] proposed a classical inventory model with demand dependent unit production cost and imperfect production process. He formulated an inventory model with this idea and solved by Geometric Programming method. Salameh and Jaber [4] developed an inventory problem where all received items are not perfect quality and after 100% screening process imperfect quality items are withdrawn from the inventory and sold at a discounted price. Hayek and Salameh [5] formulated a finite production inventory model and studied the effect of imperfect quality items on it. Wee *et al.* [6] developed a single item inventory model for items with imperfect quality and shortage backordering. Krishnamoorthi and Panayappan [7] proposed an imperfect production inventory model with defect sales return.

In real life, it is not always possible to obtain the precise information about inventory parameters. This type of imprecise data is not always well represented by random variables selected from probability distribution. So decision making methods under uncertainty are needed. To deal with this uncertainty and imprecise data, the concept of fuzziness can be applied. The inventory cost parameters such as holding cost, set up cost, production cost, reworking cost are assumed to be flexible i.e. fuzzy in nature. These parameters can be represented by fuzzy numbers. An efficient method of ranking fuzzy numbers has a very important role to handle the fuzzy numbers in a fuzzy decision-making problem. Again, in real life situation, it is almost impossible to predict the total inventory cost precisely. These are also imprecise in nature. Decision maker may change these quantities within some limits as per demand of the situation. Hence, these quantities may be assumed uncertain in non-stochastic sense but fuzzy in nature. In this situation, the inventory problem along with constraints can be developed with the fuzzy set theory.

In 1965, Zadeh [8] first introduced the concept of fuzzy set theory. Later on, Bellman and Zadeh [9] used the fuzzy set theory to the decision-making problem. Tanaka et al. [10] introduced the objectives as fuzzy goals over the α-cut of a fuzzy constraint set and Zimmermann [11] gave the concept to solve multi-objective linear-programming problem using fuzzy programming technique. Fuzzy set theory now has made an entry into the inventory control systems. Sommer [12] applied the fuzzy concept to an inventory and production-scheduling problem. Park [13] examined the Economic Order Quantity (EOQ) formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Roy and Maiti [14] solved a single objective fuzzy EOQ model using Geometric Programming (GP) technique. De and Goswami [15] derived a replenishment policy for items with finite production rate and fuzzy deterioration rate represented by a triangular fuzzy number using extension principle. Jain [16] first proposed the method of ranking fuzzy numbers. Yager [17] proposed a procedure for ordering fuzzy subsets of the unit interval. A subjective approach for ranking fuzzy numbers was presented by Campos and Munoz [18]. In 1999, Dubois and Prade [19] proposed a unified view of ranking technique of fuzzy numbers. Wen and Quan [20] used best approximation interval to rank fuzzy numbers.

GP method, as introduced by Duffin et al. [21], is an effective method to solve a non-linear programming problem. It has certain advantages over the other optimization methods. The advantage is that this method converts a problem with highly non-linear and inequality constraints (primal problem) to an equivalent problem with linear and equality constraints (dual problem). It is easier to deal with the dual problem consisting linear and equality constraints than the primal problem with non-linear and inequality constraints. Kotchenberger [22] was first used GP method to solve the basic inventory problem. Warral and Hall [23] utilized this technique to solve a multi-item inventory problem with several constraints. This method is now widely used to solve the optimization problem in inventories. But to solve a non-linear programming problem by GP method, degree of difficulty (DD) plays a significant role. DD is defined as total number of terms in objective function and constraints − (total number of decision variables + 1). It will be difficult to solve the problem for higher values of DD. So, one always tries to reduce the DD to avoid such complexity. Ata et al. [24], Ata and Kotb [25]and Chen [26] developed some inventory problems and solved by GP method. Hariri and Ata [27] gave a new idea on GP to solve multi-item inventory problems. (Here, after, this new GP has called modified geometric programming (MGP)). Mandal et al. [28] used MGP technique to solve multi-item inventory problem. Liu [29] presented a profit maximization problem with interval coefficients and quantity discounts and solved by GP method. Leung [30] proposed an inventory problem with flexible and imperfect production process and used GP technique to obtain closed form optimal solution. Sadjadi et al. [31] proposed a new pricing and marketing planning problem where demand was a function of price and marketing expenditure with fuzzy environments and the resulted problem solved by GP method. Mandal [32] proposed an inventory model with ranking fuzzy number cost parameters and solved by modified GP method.

In this paper, a multi-objective economic order quantity problem with imperfect production without shortage is formulated along with total available storage space restriction. The model formulated with the assumptions that some of the defective items are reworkable and remaining scrap items are discarded. Some of the sold items are found to be defective by the customers and they returned those items to the manufacturer. Due to volatile nature of the market, the cost parameters are represented here by fuzzy numbers with different types of left and right branches of membership function. These parameters are first expressed as nearest weighted interval approximation and then expressed as ranking fuzzy numbers with best approximation interval. The objective goal cannot be predicted precisely in real life. The authority may allow the flexibility of these goals to some extent. In this context, the objective functions are considered here in fuzzy environment by giving some tolerance value. The problem has expressed in posynomial problem. MGP technique is used here to solve the problem. As a particular case, we also investigate the case when only perfect quality items are produced. The problems are illustrated by numerical examples.

2 Mathematical Formulation

A multi-objective inventory model is developed under the following notations. and assumptions

2.1 Notations

Parameters for i-th (i =1,2,. . . ,n) item are

D_i	demand per unit item
Q_i	lot size per unit item (decision variable) ($Q \equiv (Q_1, Q_2, ..., Q_n)^T$)
C_{0i}	production cost per unit item
C_{1i}	holding cost per unit item
C_{2i}	cost per unit item
C_{3i}	set up cost
I_i	rate of defective items from regular production
E_i	rate of defective items from end customers
x_i	proportion of defective items from regular production
y_i	proportion of defective items from customers
θ_i	proportion of defective items that cannot be reworked (scrap items)
t_{1i}	processing time
t_{2i}	rework time without scrap
t_{3i}	consumption time
W_i	storage space
$TC_i(Q_i)$	total average cost function
$SS(Q_i)$	function of total available storage area
TC_{0i}	goal of the objective function

p_{0i}	tolerance value for the goal TC_{0i}
W	total available storage space

2.2 Assumptions

(i) Rate of imperfect quality items from regular production (I_i) is equal to the production rate (P_i) times the percentage of defective items produced (x_i) i.e. $I_i = x_i P_i, \quad 0 \leq x_i \leq 1$.

(ii) Rate of defective items from the end customers (E_i) is equal to the some fraction (y_i) of the demand (D_i) of the items i.e. $E_i = y_i D_i, \quad 0 \leq y_i \leq 1$.

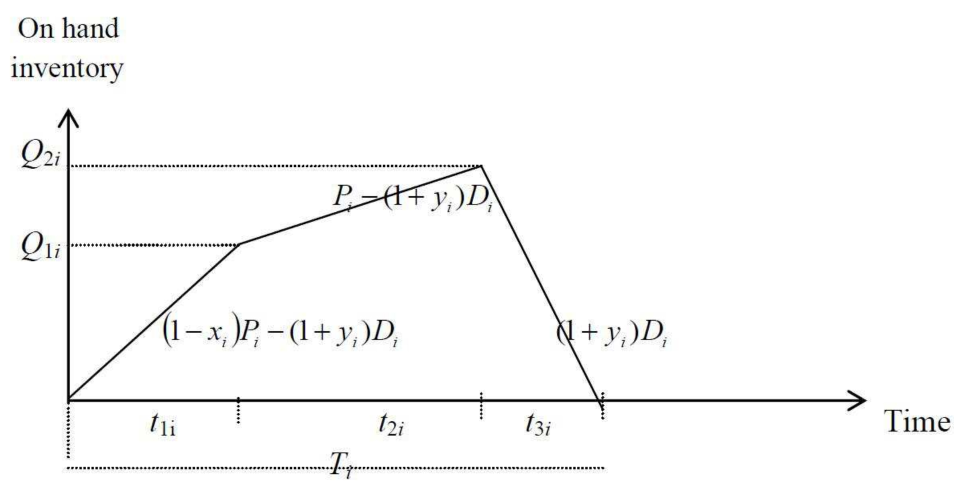

Figure 1: On-hand Inventory of i-th Item

The governing differential equation is

$$\begin{aligned}\frac{dq_i}{dt} &= (1-x_i)P_i - (1+y_i)D_i \text{ for } 0 \leqslant t \leqslant t_{1i},\\ &= P_i - (1+y_i)D_i \quad \text{for } t_{1i} \leqslant t \leqslant t_{1i}+t_{2i},\\ &= \ (1+y_i)D_i \text{ for } t_{1i}+t_{2i} \leqslant t \leqslant t_{1i}+t_{2i}+t_{3i}(=T_i).\end{aligned}$$

Imperfect quality items produced $= x_i Q_i, \ 0 \leqslant t \leqslant t$.
At $t = t_{1i}$, scrap items $= x_i \theta_i Q_i$.
Reworkable items $= x_i(1-\theta_i)Q_i$.
Total items produced during the production cycle is

$$Q_i = P_i t_{1i} \text{ i.e. } t_{1i} = \frac{Q_i}{P_i}.$$

Maximum level of on hand imperfect quality items $= I_i t_{1i} = P_i x_i t_{1i} = x_i Q_i$.

The time needed to consume all units Q_i at D_i is $T_i = \frac{Q_i}{D_i}$.

The inventory level of perfect quality items at time $t = t_{1i}$ is

$$Q_{1i} = \int_0^{t_{1i}} dq_i(t) = (P_i - D_i - I_i - E_i)\frac{Q_i}{P_i}.$$

The inventory level of reworkable items is $x_i(1-\theta_i)Q_i = P_i t_{2i}$, i.e.

$$t_{2i} = \frac{x_i(1-\theta_i)Q_i}{P_i}.$$

The inventory level of perfect quality items at time $t_{1i} + t_{2i}$ is

$$Q_{2i} = \int_0^{t_{1i}+t_{2i}} dq_i(t) = (P_i - D_i - I_i - E_i)\frac{Q_i}{P_i} + (P_i - D_i - E_i)\frac{x_i(1-\theta_i)Q_i}{P_i}.$$

Again $Q_{2i} = (D_i + E_i)t_{3i}$ *i.e.* $t_{3i} = \frac{Q_{2i}}{D_i+E_i}$.

Cycle time

$$\begin{aligned}
T_i &= t_{1i} + t_{2i} + t_{3i} \\
&= \frac{Q_i}{P_i} + \frac{x_i(1-\theta_i)Q_i}{P_i} + \frac{(P_i - D_i - I_i - E_i)Q_i}{(D_i + E_i)} + \frac{x_i(1-\theta_i)Q_i(P_i - D_i - E_i)}{P_i(D_i + E_i)}, \\
&= \frac{(1-\theta_i x_i)Q_i}{D_i + E_i}.
\end{aligned}$$

$$\begin{aligned}
\text{Holding cost } &= C_{1i}\int_0^{T_i} dq_i(t), \\
&= C_{1i}\int_0^{t_{1i}} (P_i - D_i - I_i - E_i)dt + C_{1i}\int_{t_{1i}}^{t_{1i}+t_{2i}} (P_i - D_i - E_i)dt \\
&\quad + C_{1i}\int_{t_{1i}+t_{2i}}^{t_{1i}+t_{2i}+t_{3i}} (D_i + E_i)dt, \\
&= \frac{C_{1i}Q_i}{2P_i(1-x_i\theta_i)}\left[P_i(1-x_i\theta_i)^2 - D_i(1+y_i)(1+x_i-2x_i\theta_i) + x^2(1-\theta^2)\right].
\end{aligned}$$

Total average cost $TC_i(Q_i)$ consists of Production cost, Set up cost, Holding cost, Reworking cost:

$$\begin{aligned}
TC_i(Q_i) &= \frac{1}{T_i}\left[C_{0i}Q_i + C_{3i} + C_{1i}\int_0^{T_i} dq_i(t) + C_{2i}x_i(1-\theta_i)Q_i\right] \\
&= \left[\frac{D_i + E_i}{1-\theta_i x_i}C_{0i} + \frac{D_i + E_i}{Q_i(1-\theta_i x_i)}C_{3i}\right. \\
&\quad + \frac{Q_i\left\{P_i(1-x_i\theta_i)^2 - D_i(1+y_i)(1+x_i-2x_i\theta_i) + x_i^2(1-\theta_i^2)\right\}}{2P_i(1-x_i\theta_i)}C_{1i} \\
&\quad \left. + \frac{x_i(1-\theta_i)(D_i+\omega_i)}{1-\theta_i x_i}C_{2i}\right] \text{ for } i = 1, 2, ..., n.
\end{aligned}$$

The maximum inventory level allowed is

$$\sum_{i=1}^{n} W_i Q_{2i} = \sum_{i=1}^{n} \left[(P_i - D_i - E_i)(1 + x_i(1-\theta_i)) - I_i\right] \frac{W_i Q_i}{P_i}.$$

The problem is to minimize the total average cost function i.e.

$$\begin{aligned} \text{Min } TC_i(Q_i) = {} & \frac{D_i(1+y_i)}{1-\theta_i x_i} C_{0i} + \frac{D_i(1+y_i)}{Q_i(1-\theta_i x_i)} C_{3i} \\ & + \frac{Q_i \left\{ P_i(1-x_i\theta_i)^2 - D_i(1+y_i)(1+x_i-2x_i\theta_i) + x_i^2(1-\theta_i^2) \right\}}{2P_i(1-x_i\theta_i)} C_{1i} \\ & + \frac{x_i D_i(1-\theta_i)(1+y_i)}{1-\theta_i x_i} C_{2i} \quad \text{for } i = 1, 2, 3, ..., n \end{aligned} \tag{1}$$

subject to

$$\sum_{i=1}^{n} \Big[(P_i - D_i(1+y_i))(1 + x_i(1-\theta_i)) - P_i x_i\Big] \frac{W_i Q_i}{P_i} \le W.$$

Fuzzy Model: Here we consider that the cost parameters are imprecise in nature i.e. expressed as fuzzy numbers

$$\begin{aligned} \text{Min } T\tilde{C}_i(Q_i) = {} & \frac{D_i(1+y_i)}{1-\theta_i x_i} \tilde{C}_{0i} + \frac{D_i(1+y_i)}{Q_i(1-\theta_i x_i)} \tilde{C}_{3i} \\ & + \frac{Q_i \left\{ P_i(1-x_i\theta_i)^2 - D_i(1+y_i)(1+x_i-2x_i\theta_i) + x_i^2(1-\theta_i^2) \right\}}{2P_i(1-x_i\theta_i)} \tilde{C}_{1i} \\ & + \frac{x_i D_i(1-\theta_i)(1+y_i)}{1-\theta_i x_i} \tilde{C}_{2i} \end{aligned} \tag{2}$$

subject to

$$SS(Q) = \sum_{i=1}^{n} \left[(P_i - D_i(1+y_i))(1 + x_i(1-\theta_i)) - P_i x_i\right] \frac{W_i Q_i}{P_i} \le W.$$

Special case 1: When only perfect quality items are produced, then $x_i = 0$.
Problem (1) is reduced to

$$\text{Min } TC(Q) = D_i(1+y_i)C_{0i} + \frac{D_i(1+y_i)}{Q_i} C_{3i} + \frac{Q_i\left\{P_i - D_i(1+y_i)\right\}}{2P_i} C_{1i} \; i = 1,2,...,n \tag{3}$$

subject to

$$\sum_{i=1}^{n} \left[(P_i - D_i(1+y_i))\right] \frac{W_i Q_i}{P_i} \leqslant W.$$

Special case 2: When only perfect quality items are produced, then $x_i = 0$ and customers are received perfect quality items, then $y_i = 0$.

Problem (1) reduced to

$$\text{Min } TC_i(Q_i) = D_i C_{0i} + \frac{D_i}{Q_i} C_{3i} + \frac{Q_i}{2}\left(1 - \frac{D_i}{P_i}\right) C_{1i} \quad i = 1,2,...,n \tag{4}$$

subject to

$$\sum_{i=1}^{n} \left[(1 - \frac{D_i}{P_i})\right] W_i Q_i \leqslant W.$$

3 Ranking Fuzzy Number of Cost Parameters with Best Approximation Interval

Fuzzy number: A real number $\tilde{A}$ described as a fuzzy subset on the real line $\Re$ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristics with $-\infty < a_1 \le a_2 \le a_3 < \infty$

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu^L_{\tilde{A}}(x) & \text{if} \quad a_1 \le x \le a_2\,, \\ 1 & \text{if} \quad x = a_2\,, \\ \mu^R_{\tilde{A}}(x) & \text{if} \quad a_2 \le x \le a_3\,, \\ 0 & \quad\quad \text{otherwise}\,. \end{cases}$$

where left branch membership function $\mu^L_{\tilde{A}}(x) : [a_1, a_2] \to [0,1]$ is continuous and strictly increasing; right branch membership function $\mu^R_{\tilde{A}}(x) : [a_2, a_3] \to [0,1]$ is continuous and strictly decreasing.

$\alpha-$level Set: The α-level set of a fuzzy number $\tilde{A}$ is defined as a crisp set $A(\alpha)$ which is a non-empty bounded closed interval contained in X and can be denoted by $A(\alpha) = [A_L(\alpha), A_R(\alpha)] \quad = [\inf\{x \in \Re : \mu_{\tilde{A}}(x) \ge \alpha\}, \sup\{x \in \Re : \mu_{\tilde{A}}(x) \ge \alpha\}]$, where $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval respectively, $\forall \alpha \in [0,1]$.

Interval Number: An interval number A is defined by an ordered pair of real numbers as follows $A = [a_L, a_R] = \{x : a_L \le x \le a_R, x \in R\}$, where a_L and a_R are the left and right bounds of interval A respectively.

Here we want to approximate a fuzzy number by a crisp model. Suppose $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers with α-cuts are $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$ and $B(\alpha) = [B_L(\alpha), B_R(\alpha)]$ respectively. The distance $d(A(\alpha), B(\alpha))$ between $A(\alpha)$ and $B(\alpha)$ is given by Wen and Quan [20],

$$\begin{aligned} d^2(A(\alpha), B(\alpha)) &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\left\{\frac{A_L(\alpha) + A_R(\alpha)}{2} + x\,(A_R(\alpha) - A_L(\alpha))\right\}\right. \\ &\quad \left. - \left\{\frac{B_L(\alpha) + B_R(\alpha)}{2} + x\,(B_R(\alpha) - B_L(\alpha))\right\}\right]^2 dx, \\ &= \left(\frac{A_L(\alpha) + A_R(\alpha)}{2} - \frac{B_L(\alpha) + B_R(\alpha)}{2}\right)^2 \\ &\quad + \frac{1}{12}\left[(A_R(\alpha) - A_L(\alpha)) - (B_R(\alpha) - B_L(\alpha))\right]^2 . \end{aligned}$$

The distance between fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are defined by $D(\tilde{A}, \tilde{B})$ where

$$D^2(\tilde{A},\tilde{B}) = \frac{\int_0^1 d^2\left(A(\alpha),B(\alpha)\right)f(\alpha)d\alpha}{\int_0^1 f(\alpha)d\alpha}.$$

The weight function $f(\alpha) > 0$ is a continuous function defined on [0,1], $\forall\alpha \in (0,1]$.

Nearest Interval Approximation:

A weighting function $\psi = (\psi_L, \psi_R) : ([0,1],[0,1]) \rightarrow (R,R)$ such that the functions ψ_L, ψ_R are non-negative, monotone increasing and satisfies the normalization conditions

$$\int_0^1 \psi_L(\alpha)d\alpha = \int_0^1 \psi_R(\alpha)d\alpha = 1.$$

Let $\tilde{A}$ be a fuzzy number with $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$ and $\psi(\alpha) = (\psi_L(\alpha), \psi_R(\alpha))$ be a weighted function. Then, the nearest interval approximation is

$$A_\psi = \left[\int_0^1 \psi_L(\alpha)A_L(\alpha)d\alpha, \int_0^1 \psi_R(\alpha)A_R(\alpha)d\beta\right].$$

Best Approximation using Nearest Interval Approximation of Fuzzy Cost Parameters:

The cost parameters $\tilde{C}_{ji}$ (j= 0,1,2,3 and i= 1,2,...,n) are represented by fuzzy numbers. The α-level interval of $\tilde{C}_{ji}$ is $\tilde{C}_{ji}(\alpha) = [C_{jiL}(\alpha), C_{jiR}(\alpha)]$, $\forall\alpha \in (0,1]$. The nearest weighted interval approximation to fuzzy cost parameters $\tilde{C}_{ji}$ is

$$\tilde{C}_{ji\psi}(\alpha) = [\psi_L(\alpha)C_{jiL}(\alpha), \psi_R(\alpha)C_{jiR}(\alpha)].$$

Since each interval is also a fuzzy number with constant α-cuts, we can find a best approximation interval $C_D(\tilde{C}_{ji}) = [C_{jiL}, C_{jiR}]$ which is nearest to $\tilde{C}_{ji}$ with respect to metric D. Now, we have to minimize $g_\psi\left(C_{jiL}, C_{jiR}\right) = D^2(\tilde{C}_{ji}, C_D(\tilde{C}_{ji}))$, i.e.

$$\begin{aligned} g_\psi\left(C_{jiL}, C_{jiR}\right) = \int_0^1 \Bigg\{ & \left[\frac{1}{2}\left(\psi_L(\alpha)C_{jiL}(\alpha) + \psi_R(\alpha)C_{jiR}(\alpha)\right) - \frac{1}{2}\left(C_{jiL} + C_{jiR}\right)\right]^2 \\ & + \frac{1}{12}\left[\left(\psi_R(\alpha)C_{jiR}(\alpha) - \psi_L(\alpha)C_{jiL}(\alpha)\right) \ -\left(C_{jiR} - C_{jiL}\right)^2\right]\Bigg\} f(\alpha)d\alpha / \int_0^1 f(\alpha)d\alpha \end{aligned}$$

with respect to C_{jiL} and C_{jiR}.

To solve the problem, we find partial derivatives for $g_\psi\left(C_{jiL}, C_{jiR}\right)$ with respect to C_{jiL} and C_{jiR};

$$\begin{aligned} \frac{\partial g_\psi\left(C_{jiL}, C_{jiR}\right)}{\partial C_{jiL}} = & -\frac{1}{3}\int_0^1 [2\psi_L(\alpha)C_{jiL}(\alpha) + \psi_R(\alpha)C_{jiR}(\alpha)]\, f(\alpha)d\alpha / \int_0^1 f(\alpha)d\alpha \\ & + \frac{1}{3}(2C_{jiL} + C_{jiR}) \end{aligned}$$

and

$$\frac{\partial g_\psi(C_{jiL}, C_{jiR})}{\partial C_{jiR}} = -\frac{1}{3}\int_0^1 [\psi_L(\alpha)C_{jiL}(\alpha) + 2\psi_R(\alpha)C_{jiR}(\alpha)]\, f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha + \frac{1}{3}(C_{jiL} + 2C_{jiR}).$$

Solving

$$\frac{\partial g_\psi\left(C_{jiL}, C_{jiR}\right)}{\partial C_{jiL}} = 0, \text{ and } \frac{\partial g_\psi\left(C_{jiL}, C_{jiR}\right)}{\partial C_{jiR}} = 0.$$

We have

$$C_{jiL} = \int_0^1 \psi_L(\alpha)C_{jiL}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha$$

and

$$C_{jiR} = \int_0^1 \psi_R(\alpha)C_{jiR}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha.$$

Therefore, the best approximation interval fuzzy number $\tilde{C}_{ji}$ with respect to distance D is

$$C_D(\tilde{C}_{ji}) = \left[\int_0^1 \psi_L(\alpha)C_{jiL}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha, \int_0^1 \psi_R(\alpha)C_{jiR}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha\right].$$

Note: If $f(\alpha) = 1$ and $\psi_L(\alpha) = 1 = \psi_R(\alpha)$, $\forall \alpha \in (0, 1]$, the best approximation interval

$$C_D(\tilde{C}_{ji}) = \left[\int_0^1 C_{jiL}(\alpha)d\alpha, \int_0^1 C_{jiR}(\alpha)d\alpha\right]$$

which was defined by Campose and Munoz [18].

Ranking Fuzzy Numbers of Cost Parameters with Best Approximation Interval

The best approximation interval of $\tilde{C}_{ji}$ is $[C_{jiL}, C_{jiR}]$. The ranking fuzzy number of the best approximation interval $[C_{jiL}, C_{jiR}]$ is defined as a convex combination of lower and upper boundary of the best approximation interval. Let $\lambda \in [0, 1]$ is a pre-assigned parameter, called degree of optimism. Therefore, the ranking fuzzy number of $\tilde{C}_{ji}$ is defined by $R_{\lambda f\psi}(\tilde{C}_{ji}) = \lambda C_{jiR} + (1 - \lambda)C_{jiL}$. A large value of $\lambda \in [0, 1]$ specifies the higher degree of optimism. When $\lambda = 0$, $R_{0f\psi}(\tilde{C}_{ji}) = C_{jiL}$ expresses that the decision maker's viewpoint is completely pessimistic. When $\lambda = 1$, $R_{1f\psi}(\tilde{C}_{ji}) = C_{jiR}$ expresses that the decision maker's attitude is completely optimistic. When $\lambda = \frac{1}{2}$,

$$R_{\frac{1}{2}f\psi}(\tilde{C}_{ji}) = \tfrac{1}{2}[C_{jiR} + C_{jiL}]$$

reflects moderately optimistic or neutral attitude of the decision maker. To find the ranking fuzzy numbers of $\tilde{C}_{ji}$, i=1,2,...,n, j=0,1,2,3, firstly, transform these fuzzy numbers into best approximation interval numbers, $C_D(\tilde{C}_{ji}) = [C_{jiL}, C_{jiR}]$ by means of the best approximation operator C_D. Then, by using the convex combination of the boundaries of

$C_D(\tilde{C}_{ji}) = [C_{jiL}, C_{jiR}]$, we change these interval numbers into real values. Ranking fuzzy numbers of $\tilde{C}_{ji}$ is as follows:

$$R_{\lambda f\psi}(\tilde{C}_{ji}) = \int_0^1 \left[\lambda\psi_R(\alpha)C_{jiR}(\alpha) + (1-\lambda)\psi_L(\alpha)C_{jiL}(\alpha)\right] f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha. \qquad (5)$$

Taking $f(\alpha) = \alpha,$ and $\psi_{\mathrm{L}}(\alpha) = 2\alpha,\ \psi_R(\alpha) = 3\alpha^2, \forall\alpha \in (0,1]$, then

$$R_{\lambda f\psi}(\tilde{C}_{ji}) = 4\int_0^1 \alpha^2 \left[\lambda C_{jiR}(\alpha) + (1-\lambda)C_{jiL}(\alpha)\right] d\alpha.$$

If $\tilde{C}_{ji} = (C_{ji1}, C_{ji2}, C_{ji3})$ is a Linear Fuzzy Number (LFN), then

$$C_{jiL}(\alpha) = C_{ji1} + \alpha(C_{ji2} - C_{ji1}) \text{ and } C_{jiR}(\alpha) = C_{ji3} - \alpha(C_{ji3} - C_{ji2}).$$

The lower limit of the interval is

$$C_{jiL} = \int_0^1 \psi_L(\alpha)C_{jiL}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha = \frac{1}{3}\left(C_{ji1} + 3C_{ji2}\right)$$

and the upper limit of the interval is

$$C_{jiR} = \int_0^1 \psi_R(\alpha)C_{jiR}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha = \frac{3}{10}\left(4C_{ji2} + C_{ji3}\right).$$

Corresponding ranking fuzzy number is

$$R_{\lambda f\psi}(\tilde{C}_{ji}) = \frac{1}{3}\left[(1-\lambda)c_{ji1} + 3c_{ji2} + \lambda c_{ji3}\right].$$

If $\tilde{C}_{ji} = (C_{ji1}, C_{ji2}, C_{ji3})$ is a Parabolic Fuzzy Number (PFN), then

$$C_{jiL}(\alpha) = C_{ji2} - (C_{ji2} - C_{ji1})\sqrt{1-\alpha} \text{ and } C_{jiR}(\alpha) = C_{ji2} + (C_{ji3} - C_{ji2})\sqrt{1-\alpha}.$$

The lower limit of the interval is

$$C_{jiL} = \int_0^1 \psi_L(\alpha)C_{jiL}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha = \frac{4}{105}\left(16C_{ji2} + 19C_{ji1}\right)$$

and the upper limit of the interval is

$$C_{jiR} = \int_0^1 \psi_R(\alpha)C_{jiR}(\alpha)f(\alpha)d\alpha \Big/ \int_0^1 f(\alpha)d\alpha = \frac{1}{210}\left(187C_{ji2} + 128C_{ji3}\right).$$

Corresponding ranking fuzzy number is

$$R_{\lambda f\psi}(\tilde{C}_{ji}) = \frac{76(1-\lambda)}{105}C_{ji1} + \frac{128+59\lambda}{210}C_{ji2} + \frac{64\lambda}{105}C_{ji3}.$$

If $\tilde{C}_{ji} = (C_{ji1}, C_{ji2}, C_{ji3})$ is Exponential Fuzzy Number (EFN), then

$$C_{jiL}(\alpha) = C_{ji1} - \frac{(C_{ji2} - C_{ji1})}{\delta_1}\log\left(1 - \frac{\alpha}{\nu_1}\right)$$

and

$$C_{jiR}(\alpha) = C_{ji3} + \frac{(C_{ji3} - C_{ji2})}{\delta_2} \log\left(1 - \frac{\alpha}{\nu_2}\right).$$

The lower limit of the interval is

$$\begin{aligned}
C_{jiL} &= \int_0^1 \psi_L(\alpha) C_{jiL}(\alpha) f(\alpha) d\alpha \Big/ \int_0^1 f(\alpha) d\alpha, \\
&= \frac{4}{3} C_{ji1} + \frac{4\nu_1^3 (C_{ji2} - C_{ji1})}{18\delta_1} \left[18 \left\{ \log\left(1 - \frac{1}{\nu_1}\right) - 1 \right\} \left(1 - \frac{1}{\nu_1}\right) \right. \\
&\left. - 9 \left\{ 2\log\left(1 - \frac{1}{\nu_1}\right) - 1 \right\} \left(1 - \frac{1}{\nu_1}\right)^2 + 2 \left\{ 3\log\left(1 - \frac{1}{\nu_1}\right) - 1 \right\} \left(1 - \frac{1}{\nu_1}\right)^3 + 11 \right].
\end{aligned}$$

Upper limit of the interval is

$$\begin{aligned}
C_{jiR} &= \int_0^1 \psi_R(\alpha) C_{jiR}(\alpha) f(\alpha) d\alpha \Big/ \int_0^1 f(\alpha) d\alpha, \\
&= \frac{3}{2} C_{ji3} - \frac{\nu_2^4 (C_{ji3} - C_{ji2})}{8\delta_2} \left[48 \left\{ \log\left(1 - \frac{1}{\nu_2}\right) - 1 \right\} \left(1 - \frac{1}{\nu_2}\right) \right. \\
&- 36 \left\{ 2\log\left(1 - \frac{1}{\nu_2}\right) - 1 \right\} \left(1 - \frac{1}{\nu_2}\right)^2 + 16 \left\{ 3\log\left(1 - \frac{1}{\nu_2}\right) - 1 \right\} \left(1 - \frac{1}{\nu_2}\right)^3 \\
&\left. - 3 \left\{ 4\log\left(1 - \frac{1}{\nu_2}\right) - 1 \right\} \left(1 - \frac{1}{\nu_2}\right)^4 + 31 \right].
\end{aligned}$$

Corresponding ranking fuzzy number is

$$R_{\lambda f \psi}(\tilde{C}_{ji}) = \lambda C_{jiR} + (1 - \lambda) C_{jiL}.$$

4 Geometric Programming Technique to Solve Fuzzy Inventory Problem

The triangular shaped fuzzy numbers $\tilde{C}_{ji}$ are represented by $\tilde{C}_{ji} = (C_{ji1}, C_{ji2}, C_{ji3})$ for $j =$ 0,1,2,3 and i= 1,2. . . .n. Then, the objective functions are represented by

$$T\tilde{C}_i = (TC_{i1}, TC_{i2}, TC_{i3}), \quad i = 1, 2, ..., n$$

where

$$\begin{aligned}
R_{\lambda f \psi}\left(T\tilde{C}_i(Q)\right) &= \frac{D_i(1 + y_i)}{1 - \theta_i x_i} R_{\lambda f \psi}(\tilde{C}_{0i}) + \frac{D_i(1 + y_i)}{Q_i(1 - \theta_i x_i)} R_{\lambda f \psi}(\tilde{C}_{3i}) \\
&+ \frac{Q_i \left\{ P_i(1 - x_i\theta_i)^2 - D_i(1 + y_i)(1 + x_i - 2x_i\theta_i) + x_i^2(1 - \theta_i^2) \right\}}{2P_i(1 - x_i\theta_i)} R_{\lambda f \psi}(\tilde{C}_{1i}) \\
&+ \frac{x_i D_i(1 - \theta_i)(1 + y_i)}{1 - \theta_i x_i} R_{\lambda f \psi}(\tilde{C}_{2i})
\end{aligned}$$

subject to

$$SS(Q) = \sum_{i=1}^{n} \left[(P_i - D_i(1+y_i))(1+x_i(1-\theta_i)) - P_i x_i\right] \frac{W_i Q_i}{P_i} \leq W.$$

According to Werners [33], the objective functions should be fuzzy in nature. So, for given $\lambda \in [0,1]$, (2) is equivalent to the following fuzzy goal programming problem

$$\begin{array}{l} \text{Find}\, Q \\ \quad R_{\lambda f \psi}\left(T\tilde{C}_i\,(Q_i)\right) \tilde{\leq} TC_{0i}, \quad \text{for}\, i = 1, 2, ..., n \\ \quad SS(Q) \leq W \end{array} \tag{6}$$

In this formulation, it is assumed that the manufacturer has a target of expenditure TC_{0i} for i-th item. As before it may happen that in course of business, he or she may be compelled to augment some more capital to spend more say, p_{0i} for i-item to take some business advantages, if such a situation occurs. Here, we assume that the objective goals are imprecise having a minimum targets TC_{01}, ,TC_{0n} with positive tolerances $p_{01} \ldots,$,p_{0n} for $\lambda \in [0,1]$.

In fuzzy set theory, the imprecise objectives are defined by their membership functions, which also may be linear and or non-linear. Membership functions for *i-th* objective is

$$\mu_i\left(R_{\lambda f\psi}\left(T\tilde{C}_i\right)\right) = \begin{cases} 0, & R_{\lambda f\psi}\left(T\tilde{C}_i\right) > TC_{0i} + p_{0i}, \\ 1 - \frac{R_{\lambda f\psi}\left(T\tilde{C}_i\right) - TC_{0i}}{p_{0i}}, & TC_{0i} \leq R_{\lambda f\psi}\left(T\tilde{C}_i\right) \leq TC_{0i} + p_{0i}, \\ 1, & R_{\lambda f\psi}\left(T\tilde{C}_i\right) < TC_{0i}. \end{cases}$$

for $i = 1, 2, ..., n$,

Following Bellman and Zadeh [9], max-min operator or convex combination operator the fuzzy goal programming problem (6) may be reduced to a crisp Primal Geometric Programming (PGP) problem. To reduce the DD, here convex combination operator is used. So, the problem (6) can be formulated as

$$\text{Max} \quad V = \sum_{i=1}^{n} \omega_i \mu_i \left(R_{\lambda f\psi}\left(T\tilde{C}_i\right)\right) \tag{7}$$

subject to $SS(Q) \leq W$ where

$$\mu_i\left(R_{\lambda f\psi}\left(T\tilde{C}_i\right)\right) = 1 - \frac{R_{\lambda f\psi}\left(T\tilde{C}_i\right) - TC_{0i}}{p_{0i}}, \quad \text{for } i = 1, 2, \ldots, n$$

and $\mu_i\left(R_{\lambda f\psi}\left(T\tilde{C}_i\right)\right) \in [0, 1]$.

Here ω_i may be taken as positive normalized preference values (i.e. weights) of objective functions i.e. $\sum_{i=1}^{n} \omega_i = 1$.

Problem (7) may be written as

$$\text{Max} \quad V = \sum_{i=1}^{n} \left(\omega_i + \frac{TC_{0i}}{p_{0i}}\right) - U(Q) \tag{8}$$

subject to the same constraint of (7), where

$$U(Q) = \sum_{i=1}^{n} \frac{\omega_i R_{\lambda f \psi}(T\tilde{C}_i)}{p_{0i}}$$

Problem (8) can be written as possynomial geometric programming problem as

$$\text{Min } \; U_0(Q) = \sum_{i=1}^{n} \left(\frac{A_i}{Q_i} + B_i Q_i \right) \tag{9}$$

subject to $\sum_{i=1}^{n} W_{Si} Q_i \leq W, Q > 0$, where

$$U(Q) = \sum_{i=1}^{n} \left[\frac{\omega_i}{p_{0i}} \left\{ \frac{D_i(1+y_i)}{1-\theta_i x_i} R_{\lambda f \psi}(\tilde{C}_{0i}) + \frac{x_i D_i (1-\theta_i)(1+y_i)}{1-\theta_i x_i} R_{\lambda f \psi}(\tilde{C}_{2i}) \right\} \right] + U_0(Q),$$

$$A_i = \frac{\omega_i D_i (1+y_i)}{p_{0i}(1-\theta_i x_i)} R_{\lambda f \psi}(\tilde{C}_{3i}),$$

$$B_i = \frac{\omega_i}{2 p_{0i} P_i (1-\theta_i x_i)} \left\{ P_i (1-\theta_i x_i)^2 - D_i(1+y_i)(1+x_i-2x_i\theta_i) + x_i^2(1-\theta_i^2) \right\} R_{\lambda f \psi}(\tilde{C}_{3i}),$$

$$W_{Si} = [(P_i - D_i(1+y_i))(1+x_i(1-\theta_i)) - P_i x_i] \frac{W_i}{P_i}.$$

Problem (9) is an unconstrained possynomial geometric programming problem with $(2n-1)$ degree of difficulty. For large value of n, it will be very cumbersome to solve the problem by GP method.But in MGP method, the DD reduces to 1. The corresponding dual problem of (9) is

$$\text{Max } dw = \prod_{i=1}^{n} \left(\frac{A_i}{w_{1i}} \right)^{w_{1i}} \left(\frac{B_i}{w_{2i}} \right)^{w_{2i}} \left(\frac{W_{Si}}{w_{3i}} \right)^{w_{3i}} \left(\sum_{i=1}^{n} w_{3i} \right)^{\sum_{i=1}^{n} w_{3i}} \tag{10}$$

subject to the normality and orthogonality conditions

$$\begin{aligned} & w_{1i} + w_{2i} = 1, \\ & -w_{1i} + w_{2i} + w_{3i} = 0, \\ & 0 < w_{1i}, w_{2i} < 1. \end{aligned}$$

Solving the equality constraints, we get,$w_{2i} = 1 - w_{1i}$ and $w_{3i} = 2w_{1i} - 1$. Putting these values in the dual function (10) and then differentiating $log(dw)$ with respect to w_{1i} we get

$$A_i W_{Si}^2 (1-w_{1i}) \left(\sum_{i=1}^{n} (2w_{1i}-1) \right)^2 - B_i w_{1i} (2w_{1i}-1)^2 = 0$$

for $i = 1,2,...,n$ where the optimality criteria is $0.5 < w_{1i} < 1$. The relation gives the optimal value w_{1i}^* and hence other optimal values are w_{2i}^* and w_{3i}^*. The optimal value of the decision variable will be found from the relation

$$\frac{A_i}{w_{1i}^* Q_i^*} = \frac{B_i Q_i^*}{w_{2i}^*},$$

which gives

$$Q_i^* = \sqrt{\frac{A_i w_{2i}^*}{B_i w_{1i}^*}}$$

and hence the optimal values of the objective functions are $TC_i^*(i = 1,2,...,n)$.

5 Numerical Example

A manufacturing company produces three types of machines A, B, C in lots. The company has a warehouse whose total floor area is $W = 100\ m^2$. The production rates of three machines are 570, 880 and 700 units per months respectively. From the past records, it is found that the demand of the items are 160, 170 and 200 units per months respectively. Rate of defective items from regular productions are 2%, 1% and 3% respectively. Rate of defective items from the end customers are 1%, 2% and 1% respectively. Rate of defective items that cannot be reworked are 3%, 2% and 1% respectively. The holding cost of the machine A is near about \$5.5 but never less than \$5.2 and never above than \$5.6 i.e. $\tilde{c}_{11} \equiv \$\,(5.2,\ 5.5,\ 5.6)$. Similarly, holding cost of machine B is $\tilde{c}_{12} \equiv \$\ (6.1,\ 6.4,\ 6.7)$and for C is $\tilde{c}_{13} \equiv \$\ (5.5,\ 5.8,\ 6.6)$. The production costs, set up costs and reworking costs of three machines are $\tilde{c}_{01} \equiv \$\ (1.1,\ 1.2,\ 1.3)$, $\tilde{c}_{02} \equiv \$\ (1.1,\ 1.3,\ 1.4)$, $\tilde{c}_{03} \equiv \$\ (1.2,\ 1.4,\ 1.5)$ and $\tilde{c}_{31} \equiv \$\ (120,\ 130,\ 140)$, $\tilde{c}_{32} \equiv \$\ (100,\ 120,\ 130)$, $\tilde{c}_{33} \equiv \$\ (110,\ 140,\ 170)$ and $\tilde{c}_{21} \equiv \$\ (1.8,\ 2,\ 2.2)$, $\tilde{c}_{22} \equiv \$\ (2,\ 2.2,\ 2.4)$, $\tilde{c}_{23} \equiv \$\ (21,\ 2.4,\ 2.6)$ respectively. The space required for three types of machines are 6.2 m^2, 5.5 m^2 and 5.8 m^2 respectively. The authority decides to spend \$800 to produce machine A, \$850 to produce machine B and \$1200 to produce machine C and allows a tolerance value of \$250 for each machines.

From the past experiences, it is found that the membership functions of different cost parameters are not same. They may be linear, parabolic or exponential type membership functions for left and right branches of different cost parameters. Input values of different types of membership functions are given in Table 1 below. Here, left branch of the cost parameter $\tilde{C}_{01}$ is taken as exponential type membership function whereas right branch the same parameter is taken as linear type membership function. Similarly, membership functions of other cost parameters are expressed in Table 1, where L, P, E stands for linear, parabolic, exponential membership functions respectively and Lt and Rt stands for left and right branches of fuzzy cost parameters.

Table 1: Left and Right Branches of Fuzzy Cost Parameters

Br	$\tilde{C}_{01}$	$\tilde{C}_{02}$	$\tilde{C}_{03}$	$\tilde{C}_{11}$	$\tilde{C}_{12}$	C_{13}	$\tilde{C}_{21}$	$\tilde{C}_{22}$	$\tilde{C}_{23}$	$\tilde{C}_{31}$	$\tilde{C}_{32}$	$\tilde{C}_{33}$
Lt	E	L	P	E	P	E	P	L	P	E	P	E
Rt	L	P	L	E	P	P	L	L	E	P	P	L

The parameter (ν_1, δ_1) is chosen for the left branch of exponential type membership function $\tilde{C}_{01}$,$\tilde{C}_{11}$,$\tilde{C}_{13}$, $\tilde{C}_{31}$ and $\tilde{C}_{33}$ whereas the parameters (ν_2, δ_2) is chosen for the right branch of exponential type membership function $\tilde{C}_{11}$ and $\tilde{C}_{23}$. Input values of the parameters (ν_1, δ_1) and (ν_2, δ_2) are given in Table 2.

Table 2: Values of (ν_1, δ_1) and (ν_2, δ_2) for the Membership Functions of $\tilde{C}_{01}$,$\tilde{C}_{11}$,$\tilde{C}_{13}$, $\tilde{C}_{23}$, $\tilde{C}_{31}$ and $\tilde{C}_{33}$

Br	$\tilde{C}_{01}$	$\tilde{C}_{11}$	$\tilde{C}_{13}$	$\tilde{C}_{23}$	$\tilde{C}_{31}$	$\tilde{C}_{33}$
Left (ν_1, δ_1)	(1.2, 2.4)	(1.2, 4.5)	(1.5,4.5)	-	(1.3,4.7)	(1.7,4.3)
Right(ν_2, δ_2)	-	(1.3, 2.5)	-	(1.6,4.6)	-	-

Optimal values of the objective functions are given in Table 3 below for the degree of optimism $\lambda = 0.7$ which reflects the higher degree of optimism. Here, optimal values of objective functions and decision variables are found for different preference values to objective functions. It is seen from the Table 3 that optimal value of the objective function TC_1 is minimum (bold numeral) when more preference is given to that objective function. Similarly TC_2^* and TC_3^* is minimum (bold numeral) when more preference values are given to the objective function TC_2 and TC_3 respectively. It is also found from the Table 3 that maximum inventory can be achieved from that objective function which has more preference value. When more preference value is given to 1^{st} objective function then the optimal value Q_1^* reflects more inventory size than the others. Similarly when more preference values are given to 2^{nd} and 3^{rd} objective functions than the others then optimal value Q_2^* and Q_3^* reflects more inventory size than the others respectively.

Table 3: The Optimal Values (for $\lambda = 0.7$)

Preference values $(\sigma_1, \sigma_2, \sigma_3)$	Q_1^*	Q_2^*	Q_3^*	$TC_1^*(\$)$	$TC_2^*(\$)$	$TC_3^*(\$)$
Equal Preference values (1/3,1/3,1/3)	95.03837	79.99869	140.0891	889.5551	934.4553	1375.394
More preference to the 1^{st} objective function (0.5,0.3,0.2)	99.43225	80.34273	135.2541	**887.2317**	934.1975	1379.312
More preference to the 2^{nd} objective function (0.2,0.7,0.1)	96.33066	86.02499	131.9133	888.7356	**931.6344**	1382.845
More preference to the 3^{rd} objective function (0.3,0.2,0.5)	94.55871	76.23727	144.8617	889.8896	938.1351	**1372.787**

Optimal values of the decision variables and objective functions for special case I (when only perfect quality items are produced) and special case II (when perfect quality items are produced and customers also received perfect quality items) are shown in Table 4 below.

It is seen from Table 4 that optimal values of the objective functions for special case II are minimum than the special case I.

Table 4: Optimal Solutions for Special Cases (for $\lambda = 0.7$ and Equal Preference Values)

	Q_1^*	Q_2^*	Q_3^*	$TC_1^*(\$)$	$TC_2^*(\$)$	$TC_3^*(\$)$
Special Case – I	94.18679	79.62115	138.5716	883.5897	930.2416	1362.675
Special Case - II	93.94308	78.92917	138.2436	878.6096	919.3442	1355.603

6 Sensitivity analysis

Here, the nature of changes of optimal values of decision variables and objective functions are investigated for the corresponding changes of degree of optimism λ. It is seen from Table 5, when the degree of optimism λ increases, then the optimal values Q_1^*, Q_2^*, TC_1^*, TC_2^* are gradually decreases. But the optimal values Q_3^* and TC_3^* increases when the degree of optimism λ increases.

Table 5: Effect of Change in λ When Preference Values are Equal

λ	Q_1^*	Q_2^*	Q_3^*	$TC_1^*(\$)$	$TC_2^*(\$)$	$TC_3^*(\$)$
0	105.4398	90.84590	117.2074	923.0230	1065.615	1208.056
0.1	103.7456	89.06796	120.9470	917.8126	1046.556	1235.806
0.2	101.9708	87.38799	124.6566	912.8040	1027.649	1261.991
0.3	100.3301	85.84456	128.0749	907.9613	1008.843	1286.831
0.4	98.86998	84.29796	131.3142	903.2275	990.1509	1310.471
0.5	97.46184	82.88948	134.3435	898.6172	971.4912	1333.062
0.6	96.29420	81.36379	137.2633	894.0060	952.9869	1354.677
0.7	95.03837	79.99869	140.0891	889.5551	934.4553	1375.394
0.8	93.93331	78.60679	142.7931	885.1063	916.0060	1395.312
0.9	92.85580	77.28962	145.3840	880.7272	897.5442	1414.501
1	91.85971	76.02833	147.8289	876.3623	879.0636	1433.058

7 Conclusion

In this paper, a multi-objective imperfect production inventory problem with reworking of defective items along-with space constraint is formulated. The cost components are considered here as triangular shaped fuzzy numbers with linear, parabolic and exponential types of left and right branch membership functions. These fuzzy numbers are then defined by ranking fuzzy numbers with respect to the best approximation interval number. The objective goals are not precise. The authority allows some flexibility to attain his target. The company can achieve its target varying the level of optimistic value λ from 0 and 1. The model is illustrated with a practical example (manufacturing company). Hence, MGP method is used here to solve the problem. The model can be easily extended to generic inventory problems with other constraints. The method presented here is quite general and can be applied to the real life inventory problems faced by the practitioners in industry or in other areas. This method may be applied to several type of fuzzy model in engineering optimization (like structural optimization).

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