Effects of Non-Newtonian Parameter on Unsteady MHD Oscillatory Slip Flow of Viscoelastic Fluid in a Planer Channel

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Abstract An analysis has been carried out to obtain the effect of the non-Newtonian parameter on the problem of magnetohydrodynamic (MHD) mixed convective oscillatory flow of an electrically conducting optically thin viscoelastic fluid through a planer channel filled with saturated porous medium in the presence of a first-order chemical reaction. The effect of buoyancy, heat source, thermal radiation, and chemical reaction are taken into account embedded with slip boundary condition, varying temperature and concentration. The closed form analytical solutions are constructed for the problem. The effects of the magnetic field, porous parameter, solutal and thermal Grashof numbers on velocity profile for different values of the non-Newtonian parameter are illustrated with the help of figures. A table containing the numerical data for skin friction is also provided.

Keywords viscoelastic fluid; chemical reaction; MHD; oscillatory flow.

2010 Mathematics Subject Classification 76A10, 76W05.

1   Introduction

Combined heat and mass transfer problems under the influence of a magnetic field and chemical reaction arise in many transport processes both naturally and in many branches of science and engineering applications. They play an important role in many industries namely, in the chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces, cooling of nuclear reactors and magnetohydrodynamic (MHD) power generators.

The problems of convection flows arising in fluids as a result of interaction of the force of gravity and density difference caused by simultaneous diffusion of thermal energy and chemical species have been investigated by Raptis and Kafousias [1], Bejan and Khair [2]. Elbashbeshy [3] studied MHD heat and mass transfer problem along a vertical plate under the combined buoyancy effects of thermal and species diffusion. Gireesh Kumar *et al.* [4] investigated effects of chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink.

The role of thermal radiation is of major importance in engineering areas occurring at high temperatures and its effects cannot be neglected (Siegel and Howell [5], Modest [6]). The effect of radiation on MHD flow, heat and mass transfer become more important industrially. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes a very important for the design of the pertinent equipment. The interaction of radiation with mixed convection flows past a vertical plate was investigated by Hossain and Takhar [7]. Heat and mass transfer effects on moving plate in the presence of thermal radiation have been studied by Muthucumarswamy and Kumar Senthil [8] using Laplace technique. El-Hakiem [9] has studied the unsteady two-dimensional

hydromagnetic oscillatory free convection and thermal radiation flow of an electrically conducting viscous incompressible fluid through a highly porous medium with constant suction velocity. Pal and Talukdar [10] have studied the unsteady MHD convective heat and mass transfer in a vertical permeable plate with thermal radiation. Sivaraj and Kumar [11] have investigated the chemical reaction effects on unsteady MHD oscillatory slip flow in an optically thin fluid through a planer channel in the presence of a temperature-dependent heat source.

The purpose of the present work is to investigate the effects of non-Newtonian parameter on unsteady MHD oscillatory slip flow of viscoelastic fluid in a planer channel filled with saturated porous medium in presence of chemical reaction. One of the most popular models for non-Newtonian fluids is the model that is called the second-order fluid or fluid of second grade. It is reasonable to use the second-order fluid model to do numerical calculations. The effects of non-Newtonian parameter with the combinations of the other flow parameters have been studied thoroughly and presented graphically.

2   Formulation of the Problem

Consider the unsteady mixed convection, two dimensional slip flow of an electrically conducting, heat generating optically thin and chemically reacting oscillatory viscoelastic fluid flow in a planer channel filled with porous medium in the presence of thermal radiation with temperature and concentration variation. The $x-$axis is along the direction of flow, $y-$axis is normal to it. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of thermal and solutal buoyancy effects in the direction of $y-$axis. The constitutive equation for the incompressible second-order fluid is

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{1}$$

where S is the stress tensor, p is the hydrostatic pressure, I is the unit tensor, $A_n, n = 1, 2$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2, μ_3 are the material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively, where μ_1 and μ_3 are positive and μ_2 is negative (Coleman and Markovitz [12]). The equation (1) was derived by Coleman and Noll [13] from that of the simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

Applying Boussinesq incompressible fluid model, the governing equations of the motion are given by

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu_1 \frac{\partial^2 u^*}{\partial y^{*2}} + \nu_2 \frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} - \frac{\sigma_e B_0^2 u^*}{\rho} - \frac{\nu_1 u^*}{K^*} + g\beta_T (T - T_1)$$
$$+ g\beta_c (C - C_1) \tag{2}$$

$$\frac{\partial T}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y^*} + \frac{Q(T - T_1)}{\rho C_p} \tag{3}$$

$$\frac{\partial C}{\partial t^*} = D\frac{\partial^2 C}{\partial y^{*2}} - K_R (C - C_1) \tag{4}$$

The boundary conditions of the problem are

$$u^* = \gamma^* \frac{\partial u^*}{\partial y^*}, \ \ u^* = \gamma^* \frac{\partial u^*}{\partial y^*}, \ T = T_1 + \delta_T^* \frac{\partial T}{\partial y^*}, \ C = C_1 + \delta_c^* \frac{\partial C}{\partial y^*} \ \ \text{at} \ \ y = 0 \tag{5}$$

$$u^* = 0\,, T = T_2 + \delta_T^* \frac{\partial T}{\partial y^*}\,, C = C_2 + \delta_c^* \frac{\partial C}{\partial y^*} \quad \text{at} \quad y = d \tag{6}$$

Here u^* is the velocity component in the x^*-direction, t^* is the dimensional time, ρ is the fluid density, $\nu_i = \frac{\mu_i}{\rho}$ $(i = 1, 2)$, g is the gravitational force, K^* is the dimensional porous permeability parameter, T is the dimensional fluid temperature, C is the dimensional fluid concentration, k is the thermal conductivity, σ_e is the electrical conductivity, β_T is the volumetric thermal expansion, β_c is the volumetric concentration expansion, C_p is the specific heat at constant pressure, Q is the dimensional heat source parameter, D is the mass diffusivity, K_R is the dimensional chemical reaction parameter, γ^* is the dimensional slip parameter, δ_T^* is the dimensional temperature variation parameter, δ_c^* is the dimensional concentration variation parameter, T_1 and T_2 are the wall temperatures, C_1 and C_2 are the wall concentrations.

The radiative heat flux is given by Cogley *et al.* ([14]) as

$$\frac{\partial q}{\partial y^*} = 4\,(T_1 - T)\,I' \tag{7}$$

where $I' = \int_0^\infty \kappa_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$, $\kappa_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Planck's function.

The following non-dimensional quantities are introduced

$$
\begin{aligned}
&x = \frac{x^*}{d}, \quad y = \frac{y^*}{d}, \quad p = \frac{p^* d}{\mu U_0}, \quad u = \frac{u^*}{U_0}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad \varphi = \frac{C - C_1}{C_2 - C_1}, \\
&t = \frac{U_0 t^*}{d}, \quad Re = \frac{U_0 d}{v_1}, \quad K = \frac{K^*}{d^2}, \quad M^2 = \frac{\sigma_e B_0^2 d^2}{\mu_1}, \quad Gr = \frac{g\beta_T \,(T_2 - T_1)\,d^2}{v_1 U_0}, \\
&F = \frac{4I' d^2}{k}, \quad Gc = \frac{g\beta_c \,(C_2 - C_1)\,d^2}{v_1 U_0}, \quad Pe = \frac{\rho C_P U_0 d}{k}, \quad \alpha_1 = \frac{Q d^2}{k}, \quad Sc = \frac{D}{U_0 d}, \\
&K_r = \frac{K_R d}{U_0}, \quad \gamma = \frac{\gamma^*}{d}, \quad \delta_T = \frac{\delta_T^*}{d}, \quad \delta_c = \frac{\delta_c^*}{d}.
\end{aligned} \tag{8}
$$

In view of the above dimensionless variables, the basic field equations (2) to (4) can be expressed in non-dimensional form as

$$Re\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{K} + M^2\right) u + \alpha \frac{\partial^3 u}{\partial y^2 \partial t} + Gr\theta + Gc\varphi \tag{9}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (F + \alpha_1)\,\theta \tag{10}$$

$$\frac{\partial \varphi}{\partial t} = Sc\frac{\partial^2 \varphi}{\partial y^2} - K_r \varphi \tag{11}$$

The corresponding boundary conditions are

$$
\begin{aligned}
&u = \gamma\frac{\partial u}{\partial y}, \quad \theta = \delta_T \frac{\partial \theta}{\partial y}, \quad \varphi = \delta_c \frac{\partial \varphi}{\partial y} && \text{at} \quad y = 0 \\
&u = 0, \quad \theta = 1 + \delta_T \frac{\partial \theta}{\partial y}, \quad \varphi = 1 + \delta_c \frac{\partial \varphi}{\partial y} && \text{at} \quad y = 1
\end{aligned} \tag{12}
$$

where $Re, M, Gr, Gc, Pe, Sc, \alpha = \frac{\mu_2 U_0}{\mu_1 d}$ are Reynolds number, Hartmann number, thermal Grashof number, solutal Grashof number, Peclet number, Schmidt number, and non-Newtonian parameter respectively.

3   Solution of the Problem

In order to solve the equations (9)-(11) for purely oscillatory flow, let

$$-\frac{\partial p}{\partial x} = \lambda e^{i\omega t}, \ u(y,t) = u_0(y)e^{i\omega t}, \ \theta(y,t) = \theta_0(y)e^{i\omega t}, \ \varphi(y,t) = \varphi_0(y)e^{i\omega t}, \quad (13)$$

where λ is a constant and ω is the frequency of oscillation.
Substituting the expressions in equation (13) into equations (9) to (11), the following equations are obtained

$$(1 + i\alpha\omega)u_0'' - \left(i\omega R_0 + \frac{1}{K} + M^2\right)u_0 + (A + Gr\theta_0 + Gc\varphi_0) = 0 \quad (14)$$

$$\theta_0'' + (F + \alpha_1 - i\omega Pe)\theta_0 = 0 \quad (15)$$

$$\varphi_0'' - \left(\frac{K_r + i\omega}{Sc}\right)\varphi_0 = 0 \quad (16)$$

with boundary conditions

$$\begin{aligned} u_0 = \gamma u_0', \quad \theta_0 = \delta_T \theta_0', \quad \varphi_0 = \delta_c \varphi_0' && \text{at} \quad y = 0, \\ u_0 = 0, \quad \theta_0 = 1 + \delta_T \theta_0', \quad \varphi_0 = 1 + \delta_c \varphi_0' && \text{at} \quad y = 1 \quad (17) \end{aligned}$$

where the primes denote differentiation with respect to y.

Equations (14) to (16) are solved and the expressions for the velocity, the temperature and concentration profiles are obtained as follows

$$u_0 = A_{17}e^{A_{10}y}\{\cos(A_{11}y) + \sin(A_{11}y)\} + A_{18}e^{A_{12}y}\{\cos(A_{13}y) + \sin(A_{13}y)\} + A_{27}y \quad (18)$$
$$\theta_0 = A_{23}\cos(A_{22}y) + A_{24}\sin(A_{22}y) \quad (19)$$
$$\varphi_0 = A_{25}e^{A_{23}y} + A_{26}e^{-A_{23}y} \quad (20)$$

Here the constants are not given due to sake of brevity.

4   Results and Discussion

The purpose of this study is to bring out the effects of non-Newtonian parameter on the governing flow with the combination of the other flow parameters. The non-Newtonian effect is exhibited through the non-dimensional parameter α. The corresponding results for Newtonian fluid are obtained by setting $\alpha = 0$ and these results coincide with the results obtained by Sivaraj and Kumar [11]. The real parts of the results are considered throughout for numerical validation. Figures 1-5 depict the velocity profile u against y for various sets of values of the magnetic field parameter(M), porous permeability parameter(K), solutal Grashof number (Gc), thermal Grashof number (Gr) with fixed values of $t = 1$, $\lambda = 1$, $\omega = 0.5$, $Re = 1$, $F = 2$, $\alpha_1 = 3$, $Pe = 4$, $K_r = 2$, $Sc = 1$, $\gamma = 0.1$, $\delta_T = 0.002$, $\delta_c = 0.002$.
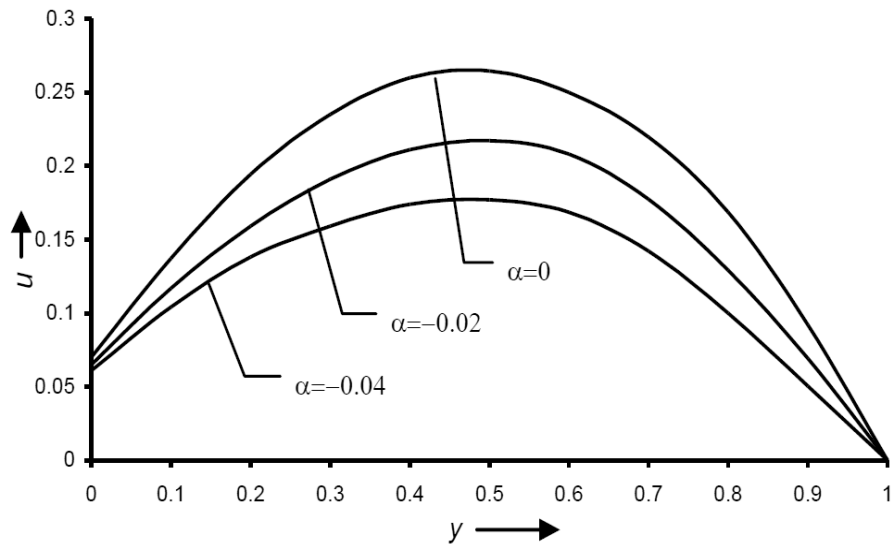
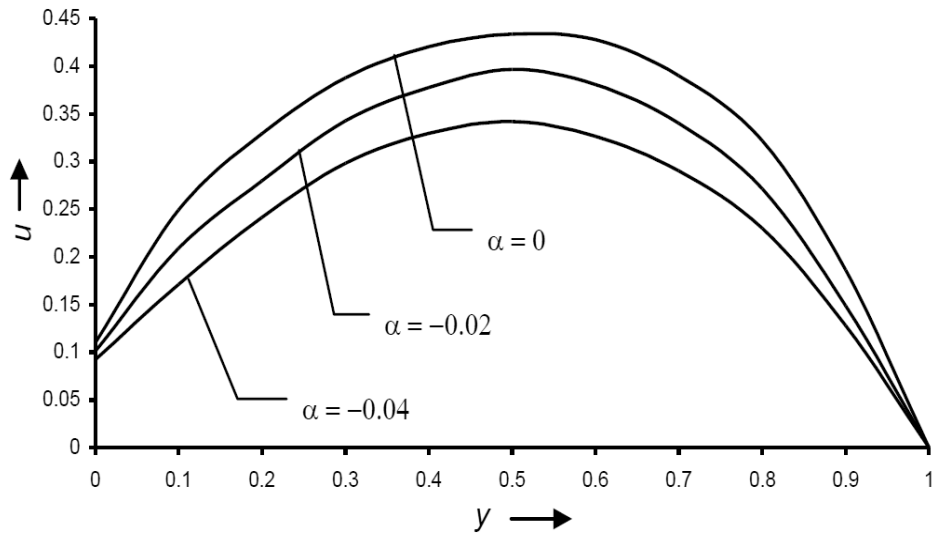Figure 1: Variation of u Against y when $M = 2, K = 2, Gc = 1, Gr = 2.$

Figure 2: Variation of u Against y when $M = 0, K = 2, Gc = 1, Gr = 2.$
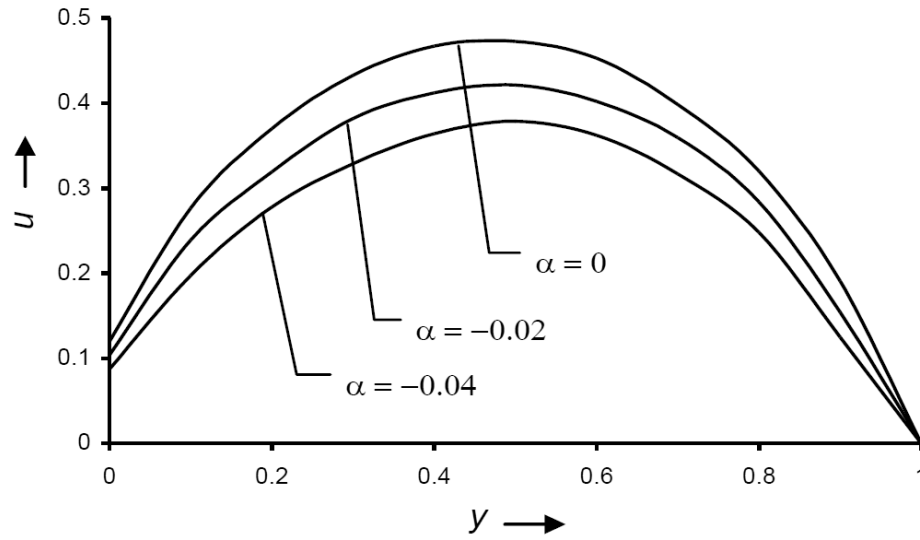
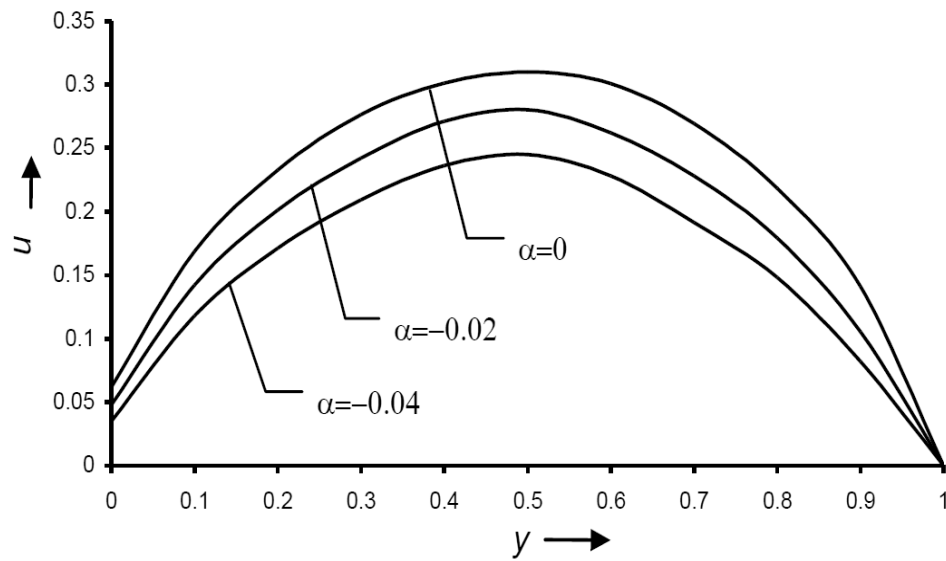Figure 3: Variation of u Against y when $M = 2, K = 3, Gc = 1, Gr = 2$.

Figure 4: Variation of u Against y when $M = 2, K = 2, Gc = 2, Gr = 2$.
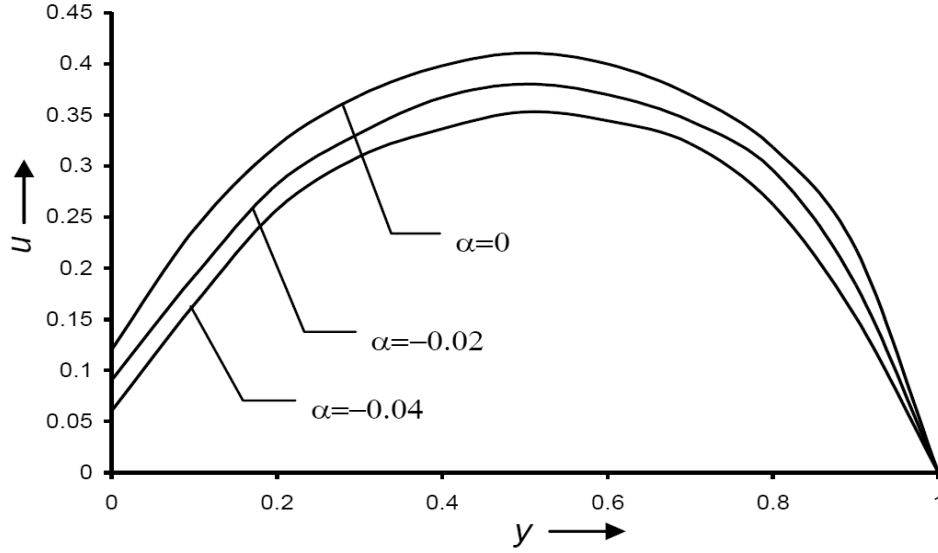
Figure 5: Variation of u Against y when $M = 2, K = 2, Gc = 1, Gr = 4$.

From Figures 1-5, it is seen that the velocity decrease with the increasing values of the non-Newtonian parameter $|\alpha|$, $(\alpha = 0, -0.02, -0.04)$ in comparison with Newtonian fluid $(\alpha = 0)$. It is observed that velocity increase with the increasing values of the permeability parameter (Figures 1 and 3), solutal Grashof number (Figures 1 and 4), and thermal Grashof number (Figures 1 and 5), whereas the velocity decrease in the presence of magnetic field (Figures 1 and 2) for both Newtonian and non-Newtonian cases.

The shear tress at any point in the fluid can be characterized by

$$\tau = \frac{\tau^*}{\frac{\mu_1 U_0}{d}} = -\left(\frac{\partial u}{\partial y} + \alpha \frac{\partial^2 u}{\partial y \partial t}\right) \tag{21}$$

The skin-friction (τ) at the walls $y = 0$ and $y = 1$ are given by

$$\tau = [\tau]_{y=0} \quad \text{and} \quad \tau = [\tau]_{y=1}. \tag{22}$$

The computed values of the skin friction are tabulated in Table 1. It is clear from the table that the absolute value of skin friction increases with the increasing values of the non-Newtonian parameter $|\alpha|$ $(\alpha = 0, -0.02, -0.04)$ in comparison with Newtonian fluid. From Table 1, it is observed that the growing magnetic field parameter M(cases I and II) increases the absolute value of the skin friction at both the walls $y = 0$ and $y = 1$ due to the action of Lorentz force in the flow field. It is further observed that both the thermal Grashof number (cases I and III) and Schimdt number (I and IV) enhance the skin friction at the walls $y = 0$ and $y = 1$ whereas the Peclect number decrease the absolute value of the skin friction for both Newtonian and non-Newtonian cases.

Table 1: Values of Skin Friction τ with Fixed Values of $t = 1$, $\lambda = 1$, $\omega = 0.5$, $Re = 1$, $F = 2$, $\alpha_1 = 3$, $Gc = 1$, $K_r = 2$, $\gamma = 0.1$, $K = 2$, $\delta_T = 0.002$, $\delta_c = 0.002$.

cases	M	Gr	Sc	Pe	α	τ_0	τ_1
					0	-0.7076	1.3343
I	2	2	1	4	-0.02	-0.7098	1.3349
					-0.04	-0.7120	1.3368
					0	-1.4462	2.2782
II	5	2	1	4	-0.02	-1.4482	2.2812
					-0.04	-1.4504	2.2846
					0	-1.2968	2.1246
III	2	4	1	4	-0.02	-1.2976	2.1264
					-0.04	-1.2992	2.1289
					0	-0.7264	1.3564
IV	2	2	2	4	-0.02	-0.7295	1.3572
					-0.04	-0.7306	1.3598
					0	-0.7214	1.3546
V	2	2	1	2	-0.02	-0.7246	1.3588
					-0.04	-0.7269	1.3602

The co-efficient of the rate of heat transfer and the rate of mass transfer at any point in the fluid can be characterized by

$$Nu = \left[\frac{Nu^*d}{k\,(T_1 - T_0)}\right] = -\frac{\partial\theta}{\partial y} \tag{23}$$

$$Sh = \left(\frac{Sh^*d}{C_1 - C_0}\right) = -\frac{\partial\varphi}{\partial y} \tag{24}$$

From (23) and (24), it has been observed that the Nusselt number and Sherwood number are not significantly affected by the non-Newtonian parameter.

5 Conclusion

An analysis is presented for the problem of unsteady two-dimensional MHD oscillatory slip flow of a viscoelastic fluid in a planer channel with variable temperature and concentration.

The effects of non-Newtonian parameter for different values of the parameters involved in the flow problem are studied in detail. The second-order fluid model for viscoelastic fluid is assumed. It was found that the velocity decrease with the increasing values of the non-Newtonian parameter in comparison with the Newtonian fluid. Further, it was found that the velocity increase with the increasing values of the permeability parameter, solutal Grashof number, and thermal Grashof number, whereas the velocity decrease in the presence of magnetic field for both Newtonian and non-Newtonian cases. Also, the absolute value of the skin friction co-efficient increase with the increasing values of the non-Newtonian parameter ($|\alpha|$). The magnetic field parameter, thermal Grashof number, Schmidt numbers are found to enhance the skin friction while Peclet number decrease the skin friction for both Newtonian and non-Newtonian cases.

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