MATEMATIKA, 2014, Volume 30, Number 1, 79–88

Computing Irreducible Representation of Finite Metacyclic Groups of Order 16 Using Burnside Method

[1]Nizar Majeed Samin, [2]Nor Haniza Sarmin and [3]Hamisan Rahmat
[1]Iraq Kurdistan Ministry of Education
[2,3]Department of Mathematical Sciences, Faculty of Science,
Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor.
e-mail: [1]nizar3944@yahoo.com, [2]nhs@utm.my, [3]hamisan@utm.my

Abstract Irreducible representation is the nucleus of a character table and is of great importance in chemistry. This paper focuses on finite metacyclic groups and their irreducible representation. This study aims to find out the irreducible representation of finite metacyclic groups of class two and finite metacyclic group of class at least three of negative type that can have order 16 by using Burnside method.

Keywords Irreducible representation, finite metacyclic groups, Burnside method.

2010 Mathematics Subject Classification 20G05, 20C99

1 Introduction

This paper focuses on metacyclic groups. The study of metacyclic groups has been done in [1–3]. In this paper, the irreducible representation of finite metacyclic groups is determined. The study of irreducible representation has been done for many groups including symmetric group by Murnaghan [4], finite classical groups by Lusztig [5] and finite metacyclic groups with faithful irreducible representations by Sim [6]. However, it has not been done for metacyclic groups. In this paper we use Burnside method to obtain the irreducible representations of finite metacyclic groups. Burnside method has been used to obtain irreducible representations of groups of order 8 by Sarmin and Fong [7].

There are fourteen types of finite metacyclic groups. In this paper we choose the types that have order 16. They are of type 1, type 7, type 8 and type 9. The first one is of class two while the rest are of negative type of class at least three.

2 Preliminaries

In 2005, Beuerle [8] classified the non-abelian metacyclic p-groups. The presentation of Type 1 metacyclic group of nilpotency class two is as below:

$$G \approx \left\langle a, b \;\middle|\; a^{p^\alpha} = b^{p^\beta} = 1, [a,b] = a^{p^{\alpha-\gamma}} \right\rangle \text{ where } \alpha, \beta, \gamma \in \mathbb{N}, \alpha \geqslant 2\gamma, \text{ and } \beta \geqslant \gamma \geqslant 1.$$

Our focus is on those groups that have order 16. When $\alpha = \beta = 2$ and $\gamma = 1$ with $p = 2$ then by using the formula $|G| = p^{\alpha+\beta}$ [1], the order of this group is

$$|G| = p^{\alpha+\beta} = 2^{2+2} = 2^4 = 16.$$

Thus the presentation becomes $G \approx \left\langle a, b \mid a^4 = b^4 = 1, [a,b] = a^2 \right\rangle$.

Then the elements are

$$\{1, a, a^2, a^3, b, ab, a^2b, a^3b, b^2, ab^2, a^2b^2, a^3b^2, b^3, ab^3, a^2b^3, a^3b^3\}.$$

There are ten classes in this group as listed in Table 1.

Table 1: Classes in Finite Metacyclic Group of Class Two of Order 16

C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
1	a, a^3	a^2	b, a^2b	ab, a^3b	b^2	ab^2, a^3b^2	a^2, b^2	b^3, a^2b^3	ab^3, a^3b^3

According to Beuerle [8], the presentation for type 7, 8 and 9 of metacyclic groups of negative type of nilpotency class at least three are as below:

$$G \approx \left\langle a, b \;\middle|\; a^{2^\alpha} = 1, b^2 = a^{2^{\alpha-1}}, [a, b] = a^{-2} \right\rangle \text{ where } \alpha \in \mathbb{N}, \alpha \geqslant 3,$$
$$G \approx \left\langle a, b \;\middle|\; a^{2^\alpha} = 1, b^2 = 1, [b, a] = a^{-2} \right\rangle \text{ where } \alpha \in \mathbb{N}, \alpha \geqslant 3,$$
$$G \approx \left\langle a, b \;\middle|\; a^{2^\alpha} = 1, b^2 = 1, [b, a] = a^{2^{\alpha-1}-2} \right\rangle \text{ where } \alpha \in \mathbb{N}, \alpha \geqslant 3.$$

Our interest would be for those that have order 16. When $\alpha = 3$, then by using this formula $|G| = 2^{\alpha+1}$ [1], the order of these groups is

$$|G| = p^{\alpha+1} = 2^{3+1} = 2^4 = 16.$$

The presentation becomes

$$G \approx \langle a, b \mid a^8 = 1, b^2 = a^4, [b, a] = a^{-2} \rangle,$$
$$G \approx \langle a, b \mid a^8 = 1, b^2 = 1, [b, a] = a^{-2} \rangle,$$
$$G \approx \langle a, b \mid a^8 = 1, b^2 = 1, [b, a] = a^2 \rangle.$$

Then the elements of type 7 are

$$\{1, a, a^2, a^3, b, ab, a^2b, a^3b, b^2, ab^2, a^2b^2, a^3b^2, b^3, ab^3, a^2b^3, a^3b^3\},$$

and the elements type 8 and 9 are

$$\{1, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}.$$

Then there are seven classes in these groups listed in Table 2,3 and 4.

Table 2: Classes in Finite Metacyclic Group of Class at Least Three Negative Type of Order 16 for Type 7

C_0	C_1	C_2	C_3	C_4	C_5	C_6
1	a, a^3b^3	a^2, a^2b^2	a^3, ab^2	b, b^3, a^2b, a^2b^3	b^2	ab, a^3b, b^3, a^3b^3

Table 3: Classes in Finite Metacyclic Group of Class at Least Three Negative Type of Order 16 for Type 8

C_0	C_1	C_2	C_3	C_4	C_5	C_6
1	a, a^7	a^2, a^6	a^3, a^5	a^4	b, a^2b, a^4b, a^6b	ab, a^3b, a^5b, a^7b

Table 4: Classes in Finite Metacyclic Group of Class at Least Three Negative Type of Order 16 for Type 9

C_0	C_1	C_2	C_3	C_4	C_5	C_6
1	a, a^3	a^2, a^6	a^4	a^5, a^7	b, a^2b, a^4b, a^6b	ab, a^3b, a^5b, a^7b

Next, the irreducible representation of these types of finite metacyclic groups of order 16 are found using the Burnside method.

3 Burnside Method [2]

There are three formulas in this method. The first step in getting the irreducible representation is by obtaining the class multiplication coefficients

$$C_iC_j = \sum_s C_{ij,s}C_s \tag{1}$$

and the $C_{ij,s}$ are the class multiplication coefficients.

The second step is to obtain the characters of the irreducible representations in terms of d_k using the result given by Burnside (1911)

$$h_ih_j\chi_i^k\chi_j^k = d_k\sum_{s=1}^{\gamma} C_{ij,s}h_s\chi_s^k \tag{2}$$

where h_i is the order of the class C_i, χ_i^k is the character of elements in class C_i in the irreducible representation labelled by k, d_k is the dimension of the irreducible representation, $C_{ij,s}$ is the class multiplication coefficient and $r$is the number of classes in the group.

The last step of getting the irreducible representations of a group is to obtain the numerical values for d_k using the following equation:

$$\sum_i h_i \chi_i^j \chi_i^k = N\delta_{jk} \tag{3}$$

where $N$is the order of the group, δ_{jk} is the Kronecker Delta symbol, $r$is the number of classes in the group and χ_i^j is the character of elements in class C_i in the irreducible representation labelled by j. For χ_i^k it is necessary to take the complex conjugate of χ_i^j whenever imaginary or complex numbers are involved.

3.1 Irreducible Representations of Finite Metacyclic Groups of Class Two of Order 16

We use Equation (1) to obtain the class multiplication coefficients. For example, multiplying C_1 with C_6 will give elements in the Table 5 below.

Table 5: Multiplying C_1 with C_6

	ab^2	a^3b2
a	a^2b^2	b^2
a^3	b^2	a^2b^2

Then $C_1 \cdot C_6 = 2C_5 + 2C_7$.
Therefore,

$$\begin{aligned} C_1 \cdot C_6 &= c_{16,0}C_0 + c_{16,1}C_1 + c_{16,2}C_2 + c_{16,3}C_3 + c_{16,4}C_4 + c_{16,5}C_5 + c_{16,6}C_6 + c_{16,7}C_7 \\ &\quad + c_{16,8}C_8 + c_{16,9}C_9 \\ 2C_5 + 2C_7 &= c_{16,0}C_0 + c_{16,1}C_1 + c_{16,2}C_2 + c_{16,3}C_3 + c_{16,4}C_4 + c_{16,5}C_5 + c_{16,6}C_6 + c_{16,7}C_7 \\ &\quad + c_{16,8}C_8 + c_{16,9}C_9 \end{aligned}$$

Thus

$$c_{16,2} = 2 \text{ and } c_{16,7} = 2.$$

Applying equation (1) for all cases gives us

$c_{00,0} = 1$	$c_{11,0} = 2$	$c_{22,0} = 1$	$c_{33,5} = 2$	$c_{44,5} = 2$	$c_{55,0} = 1$	$c_{66,0} = 2$
$c_{01,1} = 1$	$c_{11,2} = 2$	$c_{23,3} = 1$	$c_{33,7} = 2$	$c_{44,7} = 2$	$c_{56,1} = 1$	$c_{66,2} = 2$
$c_{02,2} = 1$	$c_{12,1} = 1$	$c_{25,5} = 1$	$c_{34,6} = 2$	$c_{45,9} = 1$	$c_{57,2} = 1$	$c_{67,1} = 1$
$c_{03,3} = 1$	$c_{13,4} = 2$	$c_{25,7} = 1$	$c_{35,8} = 1$	$c_{46,8} = 2$	$c_{58,3} = 1$	$c_{68,4} = 2$
$c_{04,4} = 1$	$c_{14,3} = 2$	$c_{25,7} = 1$	$c_{36,9} = 2$	$c_{47,9} = 1$	$c_{59,4} = 1$	$c_{69,3} = 2$
$c_{05,5} = 1$	$c_{15,4} = 1$	$c_{26,6} = 1$	$c_{37,8} = 1$	$c_{48,1} = 2$		
$c_{06,6} = 1$	$c_{16,5} = 2$	$c_{27,5} = 1$	$c_{38,0} = 2$	$c_{49,0} = 2$		
$c_{07,7} = 1$	$c_{16,7} = 2$	$c_{28,8} = 1$	$c_{38,2} = 2$	$c_{49,3} = 2$		
$c_{08,8} = 1$	$c_{17,6} = 1$	$c_{29,9} = 1$	$c_{39,1} = 2$			
$c_{09,9} = 1$	$c_{18,9} = 2$					
	$c_{19,8} = 2$					

$c_{77,0} = 1$ $\quad c_{88,5} = 2$ $\quad c_{99,5} = 2$
$c_{78,3} = 1$ $\quad c_{88,7} = 2$ $\quad c_{99,7} = 2$
$c_{79,4} = 1$ $\quad c_{89,6} = 2$

Next, using Equation (2) for example i=j=0

$$\begin{aligned}
h_0h_0\chi_0^k\chi_0^k &= d_k\sum_{s=0}^{9} C_{00,s}h_s\chi_s^k \\
&= d_k(C_{00,0}h_0\chi_0^k + C_{00,1}h_1\chi_1^k + C_{00,2}h_2\chi_2^k + C_{00,3}h_3\chi_3^k + C_{00,4}h_4\chi_4^k \\
&+ C_{00,5}h_5\chi_5^k + C_{00,6}h_6\chi_6^k + C_{00,7}h_7\chi_7^k + C_{00,8}h_8\chi_8^k + C_{00,9}h_9\chi_9^k \\
&= d_k((1)h_0\chi_0^k + (0)h_1\chi_1^k + (0)h_2\chi_2^k + (0)h_3\chi_3^k + (0)h_4\chi_4^k + (0)h_5\chi_5^k \\
&+ (0)h_6\chi_6^k + (0)h_7\chi_7^k + (0)h_8\chi_8^k + (0)h_9\chi_9^k \\
&= d_kh_0\chi_0^k.
\end{aligned}$$

Thus $\chi_0^k = d_k$.
Similarly $\chi_2^k = \pm d_k, \chi_5^k = \pm d_k, \chi_7^k = \pm d_k$.
Using $c_{25,7} = 1$ we get $\chi_2^k \cdot \chi_5^k = d_k\chi_7^k$.
Using $c_{11,0} = 2$ and $c_{11,2} = 2$ we obtain

$$\chi_1^k = \pm d_k, \text{if} \chi_2^k = d_k, \text{ and } \chi_1^k = 0, \text{if} \chi_2^k = -d_k.$$

Similarly for χ_6^k.
Using $c_{33,5} = 2$ and $c_{33,7} = 2$ then

$$\chi_5^k = \pm d_k, \text{if} \chi_5^k = \chi_7^k \text{ and } \chi_5^k = 0, \text{if} \chi_5^k \neq \chi_7^k.$$

Similarly for χ_4^k, χ_8^k and χ_9^k.
Then the characters of the irreducible representations in terms of d_k is given in Table 6.

Table 6: The Characters of the Irreducible Representations in Terms of d_k

C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
d_k	d_k	d_k	d_k	d_k	d_k	d_k	d_k	d_k	d_k
d_k	d_k	d_k	$-d_k$	$-d_k$	d_k	d_k	d_k	$-d_k$	$-d_k$
d_k	$-d_k$	d_k	d_k	$-d_k$	d_k	$-d_k$	d_k	d_k	$-d_k$
d_k	$-d_k$	d_k	$-d_k$	d_k	d_k	$-d_k$	d_k	$-d_k$	d_k
d_k	0	$-d_k$	0	0	d_k	0	$-d_k$	0	0
d_k	d_k	d_k	d_k	d_k	$-d_k$	$-d_k$	$-d_k$	$-d_k$	$-d_k$
d_k	d_k	d_k	$-d_k$	$-d_k$	$-d_k$	$-d_k$	$-d_k$	d_k	d_k
d_k	$-d_k$	d_k	d_k	$-d_k$	$-d_k$	d_k	$-d_k$	$-d_k$	d_k
d_k	$-d_k$	d_k	$-d_k$	d_k	$-d_k$	d_k	$-d_k$	d_k	$-d_k$
d_k	0	$-d_k$	0	0	$-d_k$	0	d_k	0	0

Finally, using Equation (3) to obtain d_k

$$\begin{aligned}\sum_i h_i\left(\chi_i^k\right)^2 &= h_0\chi_0^k\chi_0^k + h_1\chi_1^k\chi_1^k + h_2\chi_2^k\chi_2^k + h_3\chi_3^k\chi_3^k + h_4\chi_4^k\chi_4^k h_5\chi_5^k\chi_5^k + h_6\chi_6^k\chi_6^k \\ &\quad + h_7\chi_7^k\chi_7^k + h_8\chi_8^k\chi_8^k + h_9\chi_9^k\chi_9^k \\ &= \chi_0^k\chi_0^k + 2\chi_1^k\chi_1^k + \chi_2^k\chi_2^k + 2\chi_3^k\chi_3^k + 2\chi_4^k\chi_4^k + \chi_5^k\chi_5^k + 2\chi_6^k\chi_6^k + \chi_7^k\chi_7^k \\ &\quad + 2\chi_8^k\chi_8^k + 2\chi_9^k\chi_9^k \\ &= 16.\end{aligned}$$

For example, using the characters of the third irreducible representation,

$$\begin{aligned}\sum_i h_i\left(\chi_i^k\right)^2 &= d_kd_k + 2\left(-d_k\right)\left(-d_k\right) + d_kd_k + 2d_kd_k + 2\left(-d_k\right)\left(-d_k\right) + d_kd_k \\ &\quad + 2\left(-d_k\right)\left(-d_k\right) \quad + d_kd_k + 2d_kd_k + 2\left(-d_k\right)\left(-d_k\right) \\ &= 16d_k^2 \\ &= 16.\end{aligned}$$

Thus, $d_k = 1$.
As for the fifth irreducible representation,

$$\begin{aligned}\sum_i h_i\left(\chi_i^k\right)^2 &= d_kd_k + 2\left(0\right)\left(0\right) + \left(-d_k\right)\left(-d_k\right) + 2(0)(0) + 2\left(0\right)\left(0\right) + d_kd_k \\ &\quad + 2\left(0\right)\left(0\right) + \left(-d_k\right)\left(-d_k\right) + 2(0)(0) + 2\left(0\right)\left(0\right) \\ &= 4d_k^2 \\ &= 16.\end{aligned}$$

Thus, $d_k = 2$.

Therefore, $d_k = 2$ for the fifth and tenth irreducible representations and for the others $d_k = 1$.

Thus the character irreducible representations table can be completed in Table 7.

Table 7: Character Irreducible Representations of Finite Metacyclic Group of Order 16

	C_0	$2C_1$	C_2	$2C_3$	$2C_4$	C_5	$2C_6$	C_7	$2C_8$	$2C_9$
Γ_0	1	1	1	1	1	1	1	1	1	1
Γ_1	1	1	1	−1	−1	1	1	1	−1	−1
Γ_2	1	−1	1	1	−1	1	−1	1	1	−1
Γ_3	1	−1	1	−1	1	1	−1	1	−1	1
Γ_4	2	0	−2	0	0	2	0	−2	0	0
Γ_5	1	1	1	1	1	−1	−1	−1	−1	−1
Γ_6	1	1	1	−1	−1	−1	−1	−1	1	1
Γ_7	1	−1	1	1	−1	−1	1	−1	−1	1
Γ_8	1	−1	1	−1	1	−1	1	−1	1	−1
Γ_9	2	0	−2	0	0	−2	0	2	0	0

3.2 Irreducible Representations of Finite Metacyclic Groups of Class At Least Three of Order 16 of Type 7

We use Equation (1) to obtain the class multiplication coefficients for all cases. Then we have

$c_{00,0} = 1$	$c_{11,0} = 2$	$c_{22,0} = 2$	$c_{33,0} = 2$	$c_{44,0} = 4$	$c_{55,0} = 1$	$c_{66,0} = 4$
$c_{01,1} = 1$	$c_{11,3} = 1$	$c_{22,5} = 2$	$c_{33,2} = 1$	$c_{44,2} = 4$	$c_{56,6} = 1$	$c_{66,2} = 4$
$c_{02,2} = 1$	$c_{12,1} = 1$	$c_{23,1} = 1$	$c_{34,6} = 2$	$c_{44,5} = 4$		$c_{66,5} = 4$
$c_{03,3} = 1$	$c_{12,3} = 1$	$c_{23,3} = 1$	$c_{35,1} = 1$	$c_{45,4} = 1$		
$c_{04,4} = 1$	$c_{13,2} = 1$	$c_{24,6} = 2$	$c_{36,4} = 2$	$c_{46,1} = 4$		
$c_{05,5} = 1$	$c_{13,5} = 1$	$c_{25,2} = 1$		$c_{46,3} = 4$		
$c_{06,6} = 1$	$c_{14,6} = 2$	$c_{26,6} = 2$				
	$c_{15,3} = 1$					
	$c_{16,4} = 2$					

Next, using Equation (2) for example $i = j = 0$,

$$\begin{aligned}
h_0 h_0 \chi_0^k \chi_0^k &= d_k \sum_{s=0}^{9} C_{00,s} h_s \chi_s^k \\
&= d_k (C_{00,0} h_0 \chi_0^k + C_{00,1} h_1 \chi_1^k + C_{00,2} h_2 \chi_2^k + C_{00,3} h_3 \chi_3^k + C_{00,4} h_4 \chi_4^k + C_{00,5} h_5 \chi_5^k \\
&\quad + C_{00,6} h_6 \chi_6^k \\
&= d_k ((1) h_0 \chi_0^k + (0) h_1 \chi_1^k + (0) h_2 \chi_2^k + (0) h_3 \chi_3^k + (0) h_4 \chi_4^k + (0) h_5 \chi_5^k + (0) h_6 \chi_6^k \\
&= d_k h_0 \chi_0^k.
\end{aligned}$$

Thus, $\chi_0^k = d_k$.
Similarly,

$$\begin{aligned}
&\chi_5^k = \pm d_k, \\
&\chi_2^k = \pm d_k, \text{ if } \chi_5^k = d_k, \\
&\chi_2^k = 0, \text{ if } \chi_5^k = -d_k.
\end{aligned}$$

Using $c_{11,0} = 2$ and $c_{11,2} = 1$, we obtain

$$\begin{aligned}
&\chi_1^k = \pm \frac{1}{\sqrt{2}} d_k, \text{ if } \chi_2^k = 0, \\
&\chi_1^k = \pm d_k, \text{ if } \chi_2^k = d_k, \\
&\chi_1^k = 0, \text{ if } \chi_2^k = -d_k.
\end{aligned}$$

Similar as χ_3^k,

$$\begin{aligned}
&\chi_4^k = \pm d_k, \text{ if } x_5^k = d_k, \chi_2^k = 0, \\
&\chi_4^k = 0, \text{ if } x_5^k = d_k, \chi_2^k = 0, \\
&\chi_4^k = 0, \text{ if } x_5^k = -d_k, \chi_2^k = 0.
\end{aligned}$$

and the others can be similarly shown.

Finally, using Equation (3) to obtain d_k,

$$\begin{aligned}\sum_i h_i\left(\chi_i^k\right)^2 &= h_0\chi_0^k\chi_0^k + h_1\chi_1^k\chi_1^k + h_2\chi_2^k\chi_2^k + h_3\chi_3^k\chi_3^k + h_4\chi_4^k\chi_4^k + h_5\chi_5^k\chi_5^k + h_6\chi_6^k\chi_6^k \\ &= \chi_0^k\chi_0^k + 2\chi_1^k\chi_1^k + \chi_2^k\chi_2^k + 2\chi_3^k\chi_3^k + 2\chi_4^k\chi_4^k + \chi_5^k\chi_5^k + 2\chi_6^k\chi_6^k \\ &= 16.\end{aligned}$$

Then the complete character irreducible representations is shown in Table 8.

Table 8: Character Irreducible Representations of Finite Metacyclic Group of Order 16 of Type 7

	C_0	$2C_1$	$2C_2$	$2C_3$	$4C_4$	C_5	$4C_6$
Γ_0	1	1	1	1	1	1	1
Γ_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	0	-2	0
Γ_2	2	0	-2	0	0	2	0
Γ_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	0	-2	0
Γ_4	1	1	1	1	-1	1	-1
Γ_5	1	-1	1	-1	1	1	-1
Γ_6	1	-1	1	-1	-1	1	1

3.3 Irreducible Representations of Finite Metacyclic Groups of Class At Least Three of Order 16 of Type 8

Using Equation (1), we obtain the class multiplication coefficients for all cases as given below:

$c_{00,0}=1$	$c_{11,0}=2$	$c_{22,0}=2$	$c_{33,0}=2$	$c_{44,0}=1$	$c_{55,0}=4$	$c_{66,0}=4$
$c_{01,1}=1$	$c_{11,3}=1$	$c_{22,4}=2$	$c_{33,2}=1$	$c_{45,5}=1$	$c_{55,2}=4$	$c_{66,2}=4$
$c_{02,2}=1$	$c_{12,1}=1$	$c_{23,2}=1$	$c_{34,1}=1$	$c_{46,6}=1$	$c_{55,4}=4$	$c_{66,4}=4$
$c_{03,3}=1$	$c_{12,3}=1$	$c_{23,3}=1$	$c_{35,6}=2$		$c_{56,1}=4$	
$c_{04,4}=1$	$c_{13,4}=2$	$c_{24,3}=1$	$c_{36,5}=2$		$c_{56,3}=4$	
$c_{05,5}=1$	$c_{13,2}=1$	$c_{25,5}=2$				
$c_{06,6}=1$	$c_{14,3}=1$					
	$c_{15,6}=2$					
	$c_{16,5}=2$					

Next, using Equation (2) for example $i=j=0$,

$$\begin{aligned}h_0h_0\chi_0^k\chi_0^k &= d_k\sum_{s=0}^{9} C_{00,s}h_s\chi_s^k \\ &= d_k(C_{00,0}h_0\chi_0^k + C_{00,1}h_1\chi_1^k + C_{00,2}h_2\chi_2^k + C_{00,3}h_3\chi_3^k + C_{00,4}h_4\chi_4^k \\ &\quad + C_{00,5}h_5\chi_5^k + C_{00,6}h_6\chi_6^k \\ &= d_k((1)h_0\chi_0^k + (0)h_1\chi_1^k + (0)h_2\chi_2^k + (0)h_3\chi_3^k + (0)h_4\chi_4^k + (0)h_5\chi_5^k + (0)h_6\chi_6^k \\ &= d_kh_0\chi_0^k.\end{aligned}$$

Thus $\chi_0^k = d_k$.
Similarly, we obtain

$$\chi_4^k = \pm d_k,$$
$$\chi_2^k = \pm d_k, if \chi_4^k = d_k,$$
$$\chi_2^k = 0, if \chi_4^k = -d_k.$$

and the others can similarly be shown.
Finally, using Equation (3) to obtain d_k:

$$\begin{aligned}\sum_i h_i \left(\chi_i^k\right)^2 &= h_0\chi_0^k\chi_0^k + h_1\chi_1^k\chi_1^k + h_2\chi_2^k\chi_2^k + h_3\chi_3^k\chi_3^k + h_4\chi_4^k\chi_4^k + h_5\chi_5^k\chi_5^k + h_6\chi_6^k\chi_6^k \\ &= \chi_0^k\chi_0^k + 2\chi_1^k\chi_1^k + \chi_2^k\chi_2^k + 2\chi_3^k\chi_3^k + 2\chi_4^k\chi_4^k + \chi_5^k\chi_5^k + 2\chi_6^k\chi_6^k \\ &= 16.\end{aligned}$$

Thus the character irreducible representations are listed in Table 9.

Table 9: Character Irreducible Representations of Finite Metacyclic Group of Order 16 of Type 8

	C_0	$2C_1$	$2C_2$	$2C_3$	C_4	$4C_5$	$4C_6$
Γ_0	1	1	1	1	1	1	1
Γ_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0
Γ_2	2	0	-2	0	2	0	0
Γ_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0
Γ_4	1	1	1	1	1	-1	-1
Γ_5	1	-1	1	-1	1	1	-1
Γ_6	1	-1	1	-1	1	-1	1

3.4 Irreducible Representations of Finite Metacyclic Groups of Class At Least Three of Order 16 of Type 9

We use the same steps as above and obtain the complete character irreducible representations as given in Table 10.

4 CONCLUSION

Burnside method can be applied to any type of groups without having to consider the structure of the group. Because of that, we used this method to obtain the irreducible representation for some types of finite metacyclic groups.

Acknowledgements

The authors would like to thank Universiti Teknologi Malaysia, Johor Bahru, Malaysia for the financial funding through Research University Grant (RUG) with Vote No. 07J43.

Table 10: Character Irreducible Representations of Finite Metacyclic Group of Order 16 of Type 9

	C_0	$2C_1$	$2C_2$	$2C_3$	$4C_4$	C_5	$4C_6$
Γ_0	1	1	1	1	1	1	1
Γ_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	0	-2	0
Γ_2	2	0	-2	0	0	2	0
Γ_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	0	-2	0
Γ_4	1	1	1	1	-1	1	-1
Γ_5	1	-1	1	-1	1	1	-1
Γ_6	1	-1	1	-1	-1	1	1

References

[1] Brandl, R. and Verardi, L. Metacyclic p-groups and their conjugacy classes of subgroups. *Glasgow Mathematical Journal*. 1993. 35(3): 339–344.

[2] Hempel, C. E. Metacyclic groups. *Bulletin of the Australian Mathematical Society*. 2009. 61(3): 523.

[3] Sim, H. S. Metacyclic groups of odd order. *Proceedings of the London Mathematical Society*. 1994. 3(1): 47–71.

[4] Murnaghan, F. D. Irreducible representations of the symmetric group. *Proceedings of the National Academy of Sciences of the United States of America*. 1955. 41(12): 1096–103.

[5] Lusztig, G. Irreducible representations of finite classical groups. *Inventiones Mathematicae*. 1977. 43(2): 125–175.

[6] Sim, H. S. Finite metacyclic groups with faithful irreducible representations. *Bulletin-Korean Mathematical Society*. 2003. 40(2): 177–182.

[7] Sarmin, N. H. and Fong, W. Irreducible representations of groups of order 8. *Matematika*. 2006. 22(1): 1–16.

[8] Beuerle, J. R. An elementary classification of finite metacyclic p-groups of class at least three. In *Algebra Colloquium*. World Scientific. 2005. 12(4): 553–562.