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Applications of Feket-Szegö Inequalities for Generalized Sakaguchi Type Functions to Fractional Derivative Operator

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Abstract In the present paper, we have given the applications of Feket-Szegö inequalities for generalized Sakaguchi type functions obtained recently by us. We have investigated Feket-Szegö inequalities of certain classes of functions defined through fractional derivatives. The applications of Feket-Szegö inequalities for subclass of functions defined by convolution with a normalized analytic functions are also given.

Keywords Sakaguchi functions; Subordination; Fekete-Szegö inequality; Convolution

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1 Introduction

Let A be the class of analytic functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad (z \in \Delta : z \in C : |z| < 1) \tag{1}$$

and S be the subclass of A consisting of univalent functions. For two functions $f, g \in A$, we say that the function $f(z)$ is subordinate to $g(z)$ in Δ and write $f \prec g$, or $f(z) \prec g(z)(z \in \Delta)$ if there exists an analytic function $w(z)$ with $w(0) = 0$ and $|w(z)| < 1 (z \in \Delta)$ such that $f(z) = g(w(z)), (z \in \Delta)$. In particular, if the function g is univalent in Δ, the above subordination is equivalent to $f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$. Recently, Frasin [1] introduced and studied a generalized Sakaguchi type class as $S(\alpha, s, t)$ and $T(\alpha, s, t)$. A function $f(z) \in A$ is said to be in the class $S(\alpha, s, t)$ if it satisfies

$$Re\left\{\frac{(s-t)zf'(z)}{f(sz) - f(tz)}\right\} > \alpha \tag{2}$$

for some $0 \le \alpha < 1, s, t \in C$ with $s \ne t, |t| \le 1$ and for all $z \in \Delta$. We also denote by the subclass $T(\alpha, s, t)$ the subclass of A consisting of all functions $f(z)$ such that $zf'(z) \in S(\alpha, s, t)$.

In this paper we define the following class $S^g(\phi, s, t)$ and $T^g(\phi, s, t)$ which are generalizations of the classes $S(\alpha, s, t)$ and $T(\alpha, s, t)$. For two analytic functions

$$f(z) = z + \sum_{n=0}^{\infty} a_n z^n \text{ and } g(z) = z + \sum_{n=0}^{\infty} g_n z^n,$$

their convolution or Hadamard product is defined to be the function

$$(f * g)(z) = z + \sum_{n=0}^{\infty} a_n g_n z^n.$$

For a fixed $g(z) \in A$, let $S^g(\phi, s, t)$ be the class of functions $f(z) \in A$ for which $(f * g)(z) \in S^*(\phi, s, t)$. We also denote by the subclass $T^g(\phi, s, t)$ the subclass of A consisting of all functions $g(z) \in A$ such that $z(f * g)'(z) \in S^g(\phi, s, t)$ for $f(z) \in A$.

Definition 1 Let $\phi(z) = 1 + B_1 z + B_2 z^2 + ...$ be univalent starlike function with respect to '1' which maps the unit disk Δ onto a region in the right half plane which is symmetric with respect to the real axis, and let $B_1 > 0$. The function $g \in A$ is in the class $S^g(\phi, s, t)$ for $f \in A$ if

$$\left\{\frac{(s-t)z(f*g)'(z)}{(f*g)(sz)-(f*g)(tz)}\right\} \prec \phi(z), \qquad s, t \in C, s \neq t, |t| \leq 1. \tag{3}$$

Again $T^g(\phi, s, t)$ denotes the subclass of A consisting functions $g(z) \in A$ such that $z((f * g)'(z)) \in S^g(\phi, s, t)$ for $f(z) \in A$.

Obviously $S^g(\phi, 1, t) \equiv S^g(\phi, t)$, which are the classes introduced and studied by Goyal and Goswami [2]. When

$$\phi(z) = \frac{(1 + Az)}{(1 + Bz)}, (-1 \leq B < A \leq 1)$$

in (3), we denote the subclasses $S^g(\phi, s, t)$and $T^g(\phi, s, t)$ by $S^g[A, B, s, t]$ and $T^g[A, B, s, t]$, respectively.

In the present paper, we obtain applications of Fekete-Szegö inequality for the functions in the subclass $S^g(\alpha, s, t)$. To prove our main results, we need the following lemmas:

Lemma 1 [3] If $p(z) = 1 + c_1 z + c_2 z^2 + ...$ is a function with positive real part in Δ, then for any complex number μ

$$|c_2 - \mu c_1^2| \leq 2max\{1, |2\mu - 1|\}$$

and the result is sharp for the functions given by

$$p(z) = \frac{1+z}{1-z}, \qquad p(z) = \frac{1+z^2}{1-z^2}.$$

Lemma 2 [4] If $p(z) = 1 + c_1 z + c_2 z^2 + ...$ is an analytic function with positive real part in Δ, then for a real number ν

$$|c_2 - \nu c_1^2| \leq \begin{cases} -4\nu + 2, & \nu \leq 0, \\ 2, & 0 \leq \nu \leq 1, \\ 4\nu - 2 & \nu \geq 1. \end{cases}$$

When $\nu < 0$ or $\nu > 1$, the equality holds if and only if $p(z)$ is $(1 + z)/(1 - z)$ or one of its rotations. If $0 < \nu < 1$, then the equality holds if and only if $p(z)$ is $(1 + z^2)/(1 - z^2)$ or one of its rotations. If $\nu = 0$, the equality holds if and only if

$$p(z) = \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{1}{2}\lambda\right)\frac{1-z}{1+z}, \quad (0 \leq \lambda \leq 1),$$

or one of its rotations. If $\nu = 1$, the equality holds if and only if $p(z)$ is the reciprocal of one of its functions such that the equality holds in the case of $\nu = 0$. Also the above upper bound is sharp, and it can be improved as follows when $0 < \nu < 1$:

$$|c_2 - \nu c_1^2| + \nu |c_1|^2 \le 2 \qquad (0 < \nu \le 1/2)$$

and

$$|c_2 - \nu c_1^2| + (1-\nu)|c_1|^2 \le 2 \qquad (1/2 < \nu < 1)$$

Lemma 3 [5] If the function $f(z)$ given by (1) belongs to $S(\phi, s, t)$, then

$$|a_3 - \mu a_2^2| \le \frac{1}{|3 - s^2 - st - t^2|} max \left\{ B_1, \left| B_2 + B_1^2 \frac{(s+t)}{(2-s-t)} - \mu B_1^2 \frac{(3 - s^2 - st - t^2)}{(2-s-t)} \right| \right\}$$

provided $s + t \ne 2, s \ne t, |t| \le 1$. The result is sharp.

Corollary 1 [5] If the function $f(z)$ given by (1) belongs to $S(\phi, s, t)$, for real parameters s and t such that $s + t \ne 2$ and $s \ne t, |t| \le 1$, then

$$|a_3 - \mu a_2^2| \le \frac{1}{|3 - s^2 - st - t^2|} \begin{cases} \left| B_2 + B_1^2 \left(\frac{s+t}{2-s-t} \right) - \mu B_1^2 \left(\frac{3 - s^2 - st - t^2}{(2-s-t)^2} \right) \right|, & \mu \le \sigma_1, \\ B_1, \qquad \sigma_1 \le \mu \le \sigma_2, \\ \left| \mu B_1^2 \left(\frac{3 - s^2 - st - t^2}{(2-s-t)^2} \right) - B_1^2 \left(\frac{s+t}{2-s-t} \right) - B_2 \right|, & \mu \ge \sigma_2, \end{cases}$$

where

$$\sigma_1 = \frac{(2-s-t)^2}{B_1(3 - s^2 - st - t^2)} \left[-1 + \frac{B_2}{B_1} + B_1 \left(\frac{s+t}{2-s-t} \right) \right],$$

$$\sigma_2 = \frac{(2-s-t)^2}{B_1(3 - s^2 - st - t^2)} \left[1 + \frac{B_2}{B_1} + B_1 \left(\frac{s+t}{2-s-t} \right) \right].$$

The result is sharp.

Definition 2 Let $f(z)$ be analytic in a simply connected region of the z-plane containing the origin. The fractional derivative of $f(z)$ of order λ is defined by

$$_0D_z^\lambda f(z) = \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz} \int_0^z (z-\xi)^{-\lambda} f(\xi) d\xi \qquad (0 \le \lambda < 1), \tag{4}$$

where the multiplicity of $(z-\xi)^{-\lambda}$ is removed by requiring that $\log(z-\xi)$ is real for $(z-\xi) > 0$

Using definition 2, Owa and Srivastava(see[6], [7]; see also [8], [9]) introduced a fractional derivative operator $\Omega^\lambda : A \to A$ defined by

$$(\Omega^\lambda f)(z) = \Gamma(2-\lambda) z^\lambda \; _0D_z^\lambda f(z), \quad (\lambda \ne 2, 3, 4 ...).$$

The class $S^{\lambda}(\phi,s,t)$ consists of the functions $f(z)\in A$ for which $\Omega^{\lambda}f(z)\in S(\phi,s,t)$. The class $S^{\lambda}(\phi,s,t)$ is a special case of the class $S^{g}(\phi,s,t)$ when

$$g(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)}z^n \quad (z\in\Delta).$$

Now applying lemma 3 for the function $(f*g)(z)=z+g_2a_2z^2+g_3a_3z^3+\ldots$, we get following theorem after an obvious change of the parameter μ.

2 Main Results

Theorem 1 *Let $g(z)=z+\sum\limits_{n=2}^{\infty}g_nz^n$ $(g_n>0)$. If $f(z)$ is given by (1) belongs to $S^{g}(\phi,s,t)$, then*

$$|a_3-\mu a_2^2|\le\frac{1}{g_3|3-s^2-t^2-st|}max\left\{B_1,\left|\frac{B_1^2(s+t)}{2-s-t}+B_2-\frac{\mu g_3B_1^2(3-s^2-t^2-st)}{g_2^2(2-s-t)^2}\right|\right\}$$

for real parameters s and t such that $s+t\neq 2$ and $s\neq t, |t|\le 1$

Proof Let $f*g=L\in S(\phi,s,t)$. Then there exists a *Schwarz* function $w(z)\in A$ such that

$$\left\{\frac{(s-t)zL'(z)}{L(sz)-L(tz)}\right\}=\phi(w(z)),\quad (z\in\Delta,s\neq t). \tag{5}$$

If $p_1(z)$ is analytic and has positive real part in Δ and $p_1(0)=1$, then

$$p_1(z)=\frac{1+w(z)}{1-w(z)}=1+c_1z+c_2z^2+\cdots.\quad (z\in\Delta). \tag{6}$$

From (6) we obtain

$$w(z)=\frac{c_1}{2}z+\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)z^2+\cdots \tag{7}$$

Let

$$p(z)=\frac{(s-t)zL'(z)}{L(sz)-L(tz)}=1+b_1z+b_2z^2+\cdots\quad (z\in\Delta) \tag{8}$$

$$\frac{(s-t)z\left[1+2g_2a_2z+3g_3a_3z^2+\ldots\right]}{(s-t)z+g_2a_2(s^2-t^2)z^2+g_3a_3(s^3-t^3)z^3}=1+b_1z+b_2z^2+\cdots.$$

which gives

$$b_1=(2-s-t)g_2a_2\quad \textit{and}\qquad b_2=(s+t)(s+t-2)g_2^2a_2^2+(3-s^2-st-t^2)g_3a_3. \tag{9}$$

Since $\phi(z)$ is univalent and $p\prec\phi$, therefore using (7), we obtain:

$$p(z)=\phi(w(z))=1+\frac{B_1c_1}{2}+\left\{\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)B_1+\frac{1}{4}c_1^2B_2\right\}z^2+\cdots\quad (z\in\Delta). \tag{10}$$

Now from (8),(9) and (10), we have

$$(2-s-t)g_2a_2 = \frac{B_1c_1}{2}, \quad \frac{1}{2}\left(c_2 - \frac{c_1^2}{2}\right)B_1 + \frac{1}{4}c_1^2B_2 = (s+t)(s+t+2)g_2^2a_2^2 + (3-s^2-st-t^2)g_3a_3.$$

Therefore we have

$$a_3 - \mu a_2^2 = \frac{B_1}{2g_3(3-s^2-st-t^2)}\left\{c_2 - \nu c_1^2\right\}, \quad (s+t \neq 2, s \neq t), \tag{11}$$

where

$$\nu = \frac{1}{2}\left\{1 - \frac{B_2}{B_1} - \left(\frac{s+t}{2-s-t}\right)B_1 + \left(\frac{\mu g_3 B_1(3-s^2-st-t^2)}{g_2^2(2-s-t^2)}\right)\right\}, \quad (s+t \neq 2, s \neq t).$$

Our result now follows by an application of Lemma 1. □

The result is sharp for the function defined by

$$\left\{\frac{(s-t)zL'(z)}{L(sz)-L(tz)}\right\} = \phi(z), \qquad s \neq t, \tag{12}$$

and

$$\left\{\frac{(s-t)zL'(z)}{L(sz)-L(tz)}\right\} = \phi(z^2), \qquad s \neq t. \tag{13}$$

If we take parameters s and t to be real numbers then by using Lemma 2 we obtain following result:

Corollary 2 If the function $f(z)$ given by (1) belongs to $S^g(\phi, s, t)$, for real parameters s and t such that $s+t \neq 2$ and $s \neq t, |t| \leq 1$, then

$|a_3 - \mu a_2^2|$

$$\leq \frac{1}{g_3|3-s^2-st-t^2|}\begin{cases} \left|B_2 + B_1^2\left(\frac{s+t}{2-s-t}\right) - \mu B_1^2\left(\frac{g_3(3-s^2-st-t^2)}{g_2^2(2-s-t)^2}\right)\right|, & \mu \leq \eta_1, \\ B_1, \qquad \eta_1 \leq \mu \leq \eta_2, \\ \left|\mu B_1^2\left(\frac{g_3(3-s^2-st-t^2)}{g_2^2(2-s-t)^2}\right) - B_1^2\left(\frac{s+t}{2-s-t}\right) - B_2\right|, & \mu \geq \eta_2, \end{cases}$$

where

$$\eta_1 = \frac{g_2^2(2-s-t)^2}{g_3B_1(3-s^2-st-t^2)}\left[-1 + \frac{B_2}{B_1} + B_1\left(\frac{s+t}{2-s-t}\right)\right],$$

$$\eta_2 = \frac{g_2^2(2-s-t)^2}{g_3B_1(3-s^2-st-t^2)}\left[1 + \frac{B_2}{B_1} + B_1\left(\frac{s+t}{2-s-t}\right)\right].$$

The result is sharp.

Example 1 Let $-1 \leq B < A \leq 1$. If $f(z)$ given by (1) belongs to $S^g[A, B, s, t]$, for real parameters s and t such that $s+t \neq 2$ and $s \neq t, |t| \leq 1$, then

$$|a_3-\mu a_2^2|$$

$$\leq \frac{(A-B)}{2g_3|3-s^2-st-t^2|}\begin{cases}\left|-B+\dfrac{(A-B)(s+t)}{(2-s-t)}-\dfrac{\mu(A-B)g_3(3-s^2-st-t^2)}{g_2^2(2-s-t)^2}\right|, & \mu\leq\sigma_1,\\ 1, & \sigma_1\leq\mu\leq\sigma_2,\\ \left|B-\dfrac{(A-B)(s+t)}{(2-s-t)}+\dfrac{\mu(A-B)g_3(3-s^2-st-t^2)}{g_2^2(2-s-t)^2}\right|, & \mu\geq\sigma_2,\end{cases}$$

where

$$\sigma_1=\frac{g_2^2(2-s-t)^2}{g_3(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B+1}{A-B}\right)\right],$$

$$\sigma_2=\frac{g_2^2(2-s-t)^2}{g_3(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B-1}{A-B}\right)\right].$$

If $\sigma_1\leq\mu\leq\sigma_2$, in view of Lemma 2, Corollary 2 can be improved.

Theorem 2 *If the function $f(z)$ given by* (1) *belongs to $T^g(\phi,s,t)$, then*

$$|a_3-\mu a_2^2|\leq\frac{1}{3g_3|3-s^2-st-t^2|}max\left\{B_1,\left|B_2+\frac{B_1^2(s+t)}{(2-s-t)}-\frac{3\mu g_3B_1^2(3-s^2-st-t^2)}{4g_2^2(2-s-t)^2}\right|\right\}$$

provided $s+t\neq2$ and $s\neq t,|t|\leq1$. The result is sharp.

If we take parameter s and t to be real numbers, then we have following result.

Corollary 3 If the function $f(z)$ given by (1) belongs to $T^g(\phi,s,t)$, for real parameters s and t such that $s+t\neq2$ and $s\neq t,|t|\leq1$, then

$$|a_3-\mu a_2^2|$$

$$\leq\frac{1}{3g_3|3-s^2-st-t^2|}\begin{cases}\left|B_2+\dfrac{B_1^2(s+t)}{(2-s-t)}-\dfrac{3\mu B_1^2g_3(3-s^2-st-t^2)}{4g_2^2(2-s-t)^2}\right|, & \mu\leq\sigma_1^*,\\ B_1, & \sigma_1^*\leq\mu\leq\sigma_2^*\\ \left|-B_2-\dfrac{B_1^2(s+t)}{(2-s-t)}+\dfrac{3\mu g_3B_1^2(3-s^2-st-t^2)}{4g_2^2(2-s-t)^2}\right|, & \mu\geq\sigma_2^*,\end{cases}$$

where

$$\sigma_1^*=\frac{4g_2^2(2-s-t)^2}{3g_3B_1(3-s^2-st-t^2)}\left[-1+B_1\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B_2}{B_1}\right)\right],$$

$$\sigma_2^*=\frac{4g_2^2(2-s-t)^2}{3g_3B_1(3-s^2-st-t^2)}\left[1+B_1\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B_2}{B_1}\right)\right].$$

The result is sharp.

Example 2 Let $-1\leq B<A\leq1$. If $f(z)$ given by (1) belongs to $T^g[A,B,s,t]$, for real parameters s and $t$such that $s+t\neq2,s\neq t,|t|\leq1$, then

$$|a_3-\mu a_2^2|$$
$$\leq \frac{A-B}{3g_3|3-s^2-st-t^2|}\begin{cases}\left|-B+\dfrac{(A-B)(s+t)}{(2-s-t)}-\dfrac{3\mu g_3(A-B)(3-s^2-st-t^2)}{4g_2^2(2-s-t)^2}\right|, & \mu\leq\sigma_1^{**},\\ 1, \qquad \sigma_1^{**}\leq\mu\leq\sigma_2^{**} \\ \left|B-\dfrac{(A-B)(s+t)}{(2-s-t)}+\dfrac{3\mu g_3(A-B)(3-s^2-st-t^2)}{4g_2^2(2-s-t)^2}, \ \mu\geq\sigma_2^{**}\right|,\end{cases}$$

where
$$\sigma_1^{**}=\frac{4g_2^2(2-s-t)^2}{3g_3(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B+1}{A-B}\right)\right],$$
$$\sigma_2^{**}=\frac{4g_2^2(2-s-t)^2}{3g_3(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B-1}{A-B}\right)\right].$$

Since
$$\Omega^\lambda f(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)}z^n,$$
we have
$$g_2:=\frac{\Gamma(3)\Gamma(2-\lambda)}{\Gamma(3-\lambda)}=\frac{2}{2-\lambda}, \tag{14}$$
and
$$g_3:=\frac{\Gamma(4)\Gamma(2-\lambda)}{\Gamma(4-\lambda)}=\frac{6}{(2-\lambda)(3-\lambda)}. \tag{15}$$

For g_2, g_3 given by (14) and (15) respectively, Theorem 1 reduces to the following.

Theorem 3 *Let $\lambda<2$. If $f(z)$ given by* (1) *belongs to $S^\lambda(\phi,s,t)$ for $s+t\neq 2, s\neq t, |t|\leq 1$, then*
$$|a_3-\mu a_2^2|$$
$$\leq\frac{(2-\lambda)(3-\lambda)}{6|3-s^2-t^2-st|}max\left\{B_1,\left|\frac{B_1^2(s+t)}{2-s-t}+B_2-\frac{3}{2}\frac{\mu(2-\lambda)B_1^2(3-s^2-t^2-st)}{(3-\lambda)(2-s-t)^2}\right|\right\}$$

Corollary 4 If the function $f(z)$ given by (1) belongs to $S^\lambda(\phi,s,t)$, for real parameters s and t such that $s+t\neq 2$ and $s\neq t, |t|\leq 1$, then
$$|a_3-\mu a_2^2|$$
$$\leq\frac{(2-\lambda)(3-\lambda)}{6|3-s^2-st-t^2|}\begin{cases}\left|B_2+B_1^2\left(\dfrac{s+t}{2-s-t}\right)-\dfrac{3}{2}\mu B_1^2\left(\dfrac{(3-\lambda)(3-s^2-st-t^2)}{(2-\lambda)(2-s-t)^2}\right)\right|, & \mu\leq\eta_1^*,\\ B_1, \qquad \eta_1^*\leq\mu\leq\eta_2^*, \\ \left|\dfrac{3}{2}\mu B_1^2\left(\dfrac{(3-\lambda)(3-s^2-st-t^2)}{(2-\lambda)(2-s-t)^2}\right)-B_1^2\left(\dfrac{s+t}{2-s-t}\right)-B_2\right|, & \mu\geq\eta_2^*\end{cases}$$

where
$$\eta_1^*=\frac{2}{3}\frac{(3-\lambda)(2-s-t)^2}{(2-\lambda)B_1(3-s^2-st-t^2)}\left[-1+\frac{B_2}{B_1}+B_1\left(\frac{s+t}{2-s-t}\right)\right]$$
$$\eta_2^*=\frac{2}{3}\frac{(3-\lambda)(2-s-t)^2}{(2-\lambda)B_1(3-s^2-st-t^2)}\left[1+\frac{B_2}{B_1}+B_1\left(\frac{s+t}{2-s-t}\right)\right]$$

The result is sharp.

Remark 1 For $s = 1$ in aforementioned Theorems 1, 2, 3, Corollaries 2, 3, 4 and Examples 1, 2, we arrive at the results obtained by Goyal and Goswami [2] and for $s = 1, t = -1$ in aforementioned Theorems 1, 2, 3, Corollaries 2, 3, 4 and Examples 1, 2, we arrive at the results obtained by Shanmugam *et al.* [4].

3 Conclusion

We have established Feket-Szegö inequalities for generalized Sakaguchi type functions defined through fractional derivatives. The applications of the above inequalities for subclass of functions defined by convolution with a normalized analytic functions are also derived.

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