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Three-Dimensional Cylindrical Model for Single-Row Dynamic Routing

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Abstract Single-row routing is a technique to route pairs of pins aligned in a single axis into non-crossing nets to form a least congested network. The technique has its main application in the printed circuit board design where the nets between the pins are drawn statically, that is, in a fixed manner. In the dynamic single-row routing, the nets from the pairs of pins in the network are allowed to change according to the requirement. This paper proposes a new dynamic single-row routing model for the switching of pins based on the cylindrical design. In the model, a cylinder has unlimited number of planes that are formed by traversing its circular cross-section. Each plane houses a network of single-row pins that share the same pins. As the planes do not overlap, the nets in the networks do not cross as they are placed in different planes. This makes it possible to allow the configuration of the nets to change according to pin connection requirements for forming the dynamic model. The single-row routings in each network are produced optimally using our earlier model called ESSR (Enhanced Simulated annealing for Single-row Routing). This suggests that the cylindrical model is optimal and suitable for adoption into problems requiring massive pin connections such as in switching.

Keywords Single-row Routing; Dynamic Configuration; ESSR; Induction formula

1 Introduction

A VLSI integrated-circuit is formed by combining thousands number of transistors into a single chip. In a real large systems the number of interconnections between the microscopic components may exceeds thousands or millions. Due to its importance and pervasive applications in the industries, a significant demand increase for digital designers to optimize the number of wire routing and interconnections in this circuit. Hence, various routing techniques have been introduced in the literature to help in the designs. This involves single-row routing, maze routing, channel routing, line probe routing and many more.

Routing is a process of selecting paths in order to perform given task. Generally, routing can be divided into two types, namely, static and dynamic routing. In static routing, the routes are configured manually by the administrator and these routes will not change after the configuration unless a human change them. Static routes are typically used in small networks. The major advantages of static routing are reduced routing protocol overhead and network traffic. However, it requires a manual reconfiguration and this cannot be done automatically. Therefore, it will be difficult to use static routing in complex network.

The opposite of static routing is dynamic routing. This routing protocol usually supported by software applications which dynamically set up the configuration for each routes. Unlike static routing, dynamic routing enables routers to select paths according to real-time logical network layout. To perform routing in complex network, dynamic routing should be very useful. The router itself will dynamically choose different or better router if a link goes down.

Routing problem occurs in many kinds of job. In travelling salesman problem (TSP), the salesman needs to perform a complete tour starting from a point and need to determine the shortest path to cover all the identified stations, passing each station only once and come back to starting point while minimizing the distance travelled [1]. Routing method also widely used to solve problems in printed circuit boards (PCB's) design [2], VLSI design [3], circuit switching and transportation network.

In [2], Salleh *et al.* proposed enhanced simulated annealing for single-row routing problem (ESSR). The strategy is by performing slow cooling, so that the nets will align themselves to a configuration in the lowest energy. The total energy in a net list is given as follows:

$$E_L = \sum_{i=1}^{m_i} \sum_{j=1}^{m_j} | h_{i,j} |,$$

where m_i is the number of segments in the net N_i, for $i = 1,2,3,...,m$. The absolute value of the height $h_{i,j}$ is the segment relative to the node axis. In order to speed up the convergence, the value of street congestion is pivoted while having the energy drops directly proportional to the number of doglegs. As the energy drops, the number of doglegs can be minimized. This simulated annealing is finally proven efficient to produce small number of street congestion and doglegs especially in large number of nets.

In very large system, the number of interconnections may exceed tens of thousands; therefore an efficient design that optimizes wire routing is needed. Several methods are used in this research to find the best design for this problem such as Kernighan-Lin algorithm, traveling salesman problem, simulated annealing algorithm and single row routing problem. A model consists of 160 pins has been developed to illustrates the problem. The result has shown that the system 'freezes' and no further changes occur at certain temperature [3].

In [4], dynamic single-row routing technique is applied on channel assignment problem according to the given requirements. Here, we are motivated to extend the use of dynamic routing technique by applying the concept into single-row network and later transform it into a cylindrical model.

2 Problem Background

Single-row routing problem is actually two-dimensional in nature. In this paper, a method to transform the problem into a dynamic three-dimensional model is proposed. In the dynamic single-row routing, the nets from the pairs of pins in the network are allowed to change according to the requirement. This new dynamic single-row routing model for the switching of pins is based on the cylindrical design. We model this transformation into a cylindrical model mainly because a cylinder is symmetrical along an infinite number of planes due to the fact that its cross sectional area is that of a circle. A circle always has infinite lines of symmetry. This differs with other shapes which has limited symmetrical axis as shown in Figure 1. Therefore, due to this fact, we will be able to generate unlimited number of planes inside a cylinder. This makes it possible to allow the configuration of the nets to change according to pin connection requirements for forming the dynamic model.

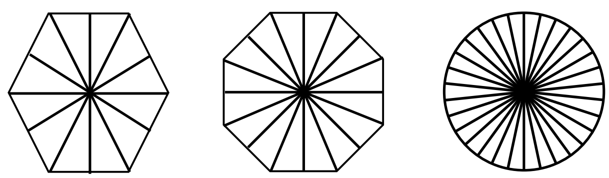


Figure 1: A regular hexagon has 6 lines of symmetry, a regular octagon has 8 lines of symmetry but a circle has unlimited symmetrical axis.

Each plane houses a network of single-row pins that share the same pins. As the planes do not overlap, the nets in the networks do not cross as they are placed in different planes at different angles. The angle can be determined by $2\pi / L_{\max}$ where $L_{\max}$ is the maximum number of configurations. Overall radius for the cylindrical model will be $r = Q_{\max}$ where Q_i is the congestion level for $i = \{1, 2, ..., n\}$. This is illustrated in Figure 2.

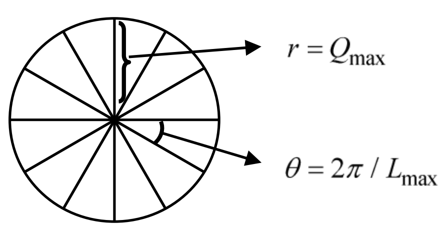


Figure 2: The components of cylindrical model.

The single-row routings in each network are produced optimally using our earlier model called ESSR (Enhanced Simulated annealing for Single-row Routing) [2].

3 Dynamic Single-row Routing

In this problem, we are given a set of n evenly spaced nodes, sometimes called terminals or pins, on a domain as illustrated in Figure 3. The nodes $P = \{P_i\}$ for $i = 1, 2, ..., n$ are arranged horizontally from left to right along single-row axis. The problem is to construct $n/2$ nets from list $L = N_k$ for $k = 1, 2, ..., n$ that connect all nodes pair by pair in such a way that it obeys all the rules of single row routing as below [5]:

(i) $N_i \cap N_j = \phi, i \neq j$

(ii) $\cup N_i = \{1, 2, ..., n\}$

(iii) The nets are to be drawn from left to right, while reverse direction is not allowed.

(iv) The path is made up of horizontal and vertical segments.

(v) The path should not cross.

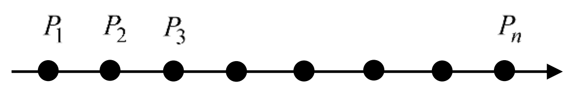


Figure 3: Node axis

Another point of concern is to minimize the number of street congestion Q and doglegs D. Single-row routing (SRR) is a combinatorial optimization problem which is known to be NP-complete [5]. This classical technique is use to solve major problem arises in layout design such as conductor routing in PCB's. In this problem, a set of n evenly spaced pins representing terminals were given and is drawn horizontally from left to right. The path joining two successive pins is called net, wiring tracks or conductor path. This path must not cross each other and made up of horizontal and vertical segments only. The movement of the path is also in forward direction while the reverse is not allowed. Each net in the single-row consist of two terminals, v_x and v_{x+1} with a unit interval x and $x+1$. The nets also must satisfy the following conditions [6]:

(i) $N_i \cap N_j = \phi, i \neq j$

(ii) $\cup N_i = \{1, 2, ..., n\}$

The area above the single-row axis is called upper street, while below is the lower street. The number of wiring tracks in the upper street is called upper street congestion and denoted by Q_u while Q_l is the number of horizontal wiring tracks in the lower street and yet called lower street congestion. The overall street congestion of a realization is the maximum number of net covering a terminal. In mathematical formulation, Q is expressed as $Q = \max(Q_u, Q_l)$

The interstreet crossing in nets or often called doglegs D is a vertical line crossing the terminals axis. Its number will greatly determine the congestion level in PCBs layout. These interstreet crossings in single-row routing problem are allowed in order to prevent the path from crossing each other. However, its number need to be minimized therefore the overall length of the track will be shortened and make it more compact by reducing the space taken. Thus, the communication cost between terminals will be reduced and the performance is improved [7]. Figure 4 illustrates the terminologies of single-row routing technique.

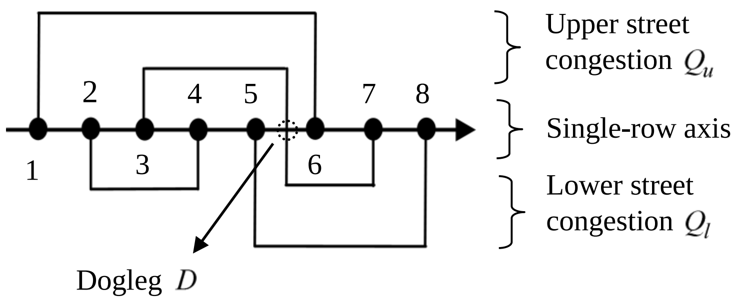


Figure 4: The single-row routing terminologies.

Figure 5 shows four nets in the order $L = \{N_1, N_4, N_3, N_2\}$ formed from the following intervals: $N_1 = (1,6), N_2 = (2,4), N_3 = (5,8)$ and $N_4 = (3,7)$. The net ordering in the figure gives a street congestion value $Q = 2$, as P_6 has 2 nets covering from below $(Q_l = 2)$ and P_4 has 2 nets covering above $(Q_u = 2)$. In this figure, it is clearly seen the number of doglegs for this realization is 1 which is present in the interval $(3,7)$.

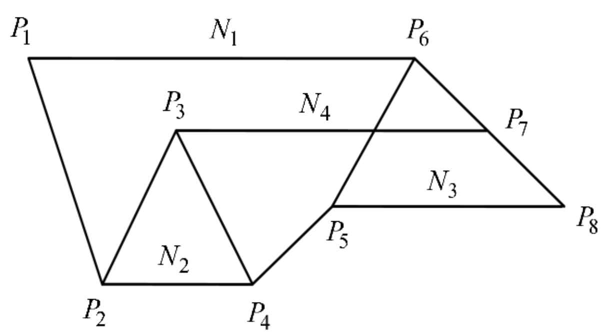


Figure 5: Energy level diagram for net ordering $L = \{N_1, N_4, N_3, N_2\}$

Figure 6 shows the graphical realization corresponding to the net ordering $L = \{N_1, N_4, N_3, N_2\}$ from Figure 5.

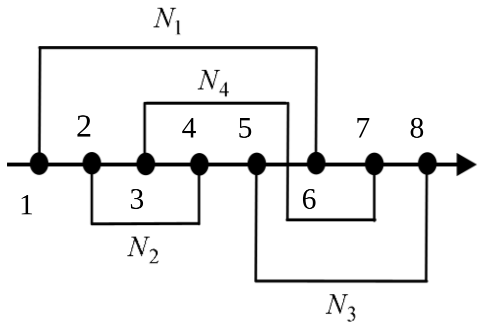


Figure 6: Realization for net ordering $L = \{N_1, N_4, N_3, N_2\}$.

From the graphical realization, it is easy to determine the number of doglegs and the level of congestion for the respective net ordering. In general, it is difficult to determine an optimal realization due to the large number of interacting variables in the problem, especially when the number of nets is large. A feasible realization that is the one that approximates the solution close to its optimal value is often accepted in many cases. A model then has been developed by [2] to visualize optimal single-row routing problem by taking into account various necessary and sufficient conditions for optimal routing. This model makes use of enhanced simulated annealing (SA) technique.

SA method is a probabilistic method which is first simulated by Metropolis *et al.* and Kirkpatrick *et al.* ([8], [9]). This algorithm now has become a very useful tool in solving a variety of combinatorial optimization problems. SA makes use of iterative improvement procedure starts with an initial state. SA generates new solution at each temperature while the temperature is lowered gradually, until it met the stopping criteria. SA avoids being trapped at local minima by accepting sometimes uphill moves. This acceptance is determined by using Boltzmann probability

$$P(\Delta E) = e^{\frac{-\Delta E}{T_i}} ,$$

where ΔE is the difference costs between the current solution and previous one while T_i is current temperature.

In this study, the single-row routing is assumed to be dynamic which means all the pins will be configured dynamically according to the constantly changing intervals between the pairs of pins and routes themselves by taking into account all the conditions for optimal routing. Unlike the two-dimensional static single-row networks, the cylindrical model allows any combination of pairs of pins suitable for dynamic routing. In our model, pin x is not necessary to be connected with pin y all the time. At different time slots, it can be connected to any other different pins.

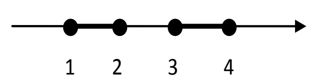
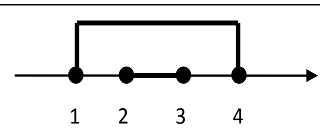
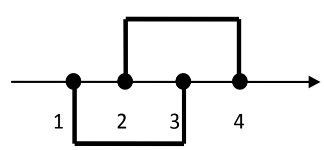
4 Multi Connection of Pins Using the Cylindrical Model

In this problem, the connection for each net need to be established by obeying the rules of SRR as stated in Section 2. Our model proposes the task of transforming dynamical single-row routing configuration into a cylindrical model. One of the important things is to identify how many axes a cylindrical model will have for any n pins together with its radius. Therefore, the very first step is to determine all possible net ordering or list L for every n.

4.1 Maximum Possible Net Ordering

The maximum possible net ordering or list L can be calculated using few steps below. Let us begin with the smallest number of pins we can have as an illustration for the problem description above which is $n = 4$. Table 1 illustrates all possible ways to route these pins.

Table 1: All possible routings for $n = 4$.

Order	Realization	Q	D
1		0	0
2		1	0
3		2	0

From Table 1, it can be seen that each of the realization obeys all conditions for a successful routing. For this example, the most optimal route is the first order while the third order is the most expensive one since it has highest number of congestions level. The optimal route from this illustration forms the list $L = \{(1,2),(3,4)\}$. Next, we continue with $n = 6$.

Table 2: Net Ordering for $n = 6$.

Fixed net	Remaining pins	Net Ordering
(1,2)	3,4,5,6	$L_1 = \{(1,2),(3,4),(5,6)\}$ $L_2 = \{(1,2),(3,5),(4,6)\}$ $L_3 = \{(1,2),(3,6),(4,5)\}$
(1,3)	2 ,4,5,6	$L_4 = \{(1,3),(2,4),(5,6)\}$ $L_5 = \{(1,3),(2,5),(4,6)\}$ $L_6 = \{(1,3),(2,6),(4,5)\}$
(1,4)	2,3,5,6	$L_7 = \{(1,4),(2,3),(5,6)\}$ $L_8 = \{(1,4),(2,5),(3,6)\}$ $L_9 = \{(1,4),(2,6),(3,5)\}$
(1,5)	2,3,4,6	$L_{10} = \{(1,5),(2,3),(4,6)\}$ $L_{11} = \{(1,5),(2,4),(3,6)\}$ $L_{12} = \{(1,5),(2,6),(3,4)\}$
(1,6)	2,3,4,5	$L_{13} = \{(1,6),(2,3),(4,5)\}$ $L_{14} = \{(1,6),(2,4),(3,5)\}$ $L_{15} = \{(1,6),(2,5),(3,4)\}$
		: :
	TOTAL	$L = \{L_1, L_2, L_3, ..., L_{15}\}$

Note: $(a,b) = (b,a)$

Table 2 summarizes the all possible routing for $n = 6$. It can be clearly seen there are 15 different net orderings can be form. For larger n, more net orderings can be generated this job is quite computationally expensive. Therefore, an induction formula is derived to compute all possible routings and the result is summarized in Table 3.

Table 3: Inductive relationship for orders of pins up to $n = 20$.

n	Inductive Relationship	$L_{\max}$
4	$(n-1)(n-3)$	3
6	$(n-1)(n-3)(n-5)$	15
8	$(n-1)(n-3)(n-5)(n-7)$	105
10	$(n-1)(n-3)(n-5)(n-7)(n-9)$	945
12	$(n-1)(n-3)(n-5)(n-7)(n-9)(n-11)$	10 395
14	$(n-1)(n-3)(n-5)(n-7)(n-9)(n-11)(n-13)$	135 135
16	$(n-1)(n-3)(n-5)(n-7)(n-9)(n-11)(n-13)(n-15)$	2 027 025
18	$(n-1)(n-3)(n-5)(n-7)(n-9)(n-11)(n-13)(n-15)$ $(n-17)$	34 459 425
20	$(n-1)(n-3)(n-5)(n-7)(n-9)(n-11)(n-13)(n-15)$ $(n-17)(n-19)$	654 729 075

For $n \geq 2$, it can be summarized in a form of equation as below. Let $L_{\max}$ be the maximum number or net ordering for n pins, therefore

$$L_{\max} = \prod[n-(2r-1)]; (2r-1) < n \text{ for } r = 1, 2, 3, \ldots \qquad (1)$$

5 Dynamic Switching Model

In this paper, we propose a new technology for single-row routing problem. Previous research only focuses on two-dimensional static single-row network. However, in this paper, we extend this problem into a three-dimensional cylindrical model of dynamic single-row routing. A cylindrical model is a three-dimensional model in cylinder shape which having unlimited number of planes inside that are formed by traversing its circular cross-section. Each realization or routing will form an axis and the radius of the cylinder is proportional to the maximum street congestion for all routings.

By using Equation (1), $L_{\max}$ for $n = 6$ is 15. The realizations for each network in L is produced optimally using our earlier model called ESSR. Such realization obeys all the rules for single-row routing while minimizing both congestion level Q and number of doglegs D. Each realization will form a plane in the cylinder. Therefore, for $n = 6$, 15 planes at different angle will be form and is drawn at $\theta = 2\pi/15$.

The height of each plane is the axis of the cylinder and is proportional to the number of street congestion of the respected L. Overall radius for the cylindrical model will be $r = Q_{\max}$ and its total length depends on number of pins along the single-row axis. Figure 7 demonstrates a cylindrical model having six planes. Notice that the single-row axis will be the intersection lines for all planes. Each plane houses a network. As the planes do not overlap, the nets in the networks do not cross as they are placed in different planes.

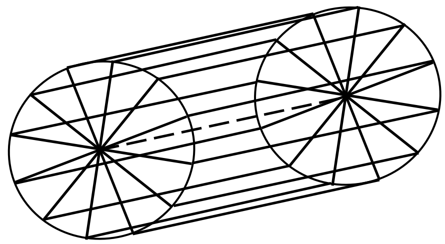


Figure 7: A six planes cylindrical model

A clear front view of this model for $L_{\max}$ up to 15 is illustrated as in Figure 8 while Figure 9 shows the cross section of the model that illustrates the position of the respective plane for L_7, L_9 and L_{12}.

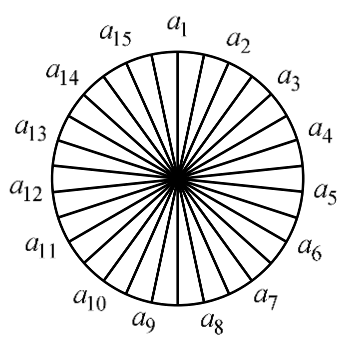


Figure 8: Front view of the cylindrical model showing the axis a_i for all 15 planes.

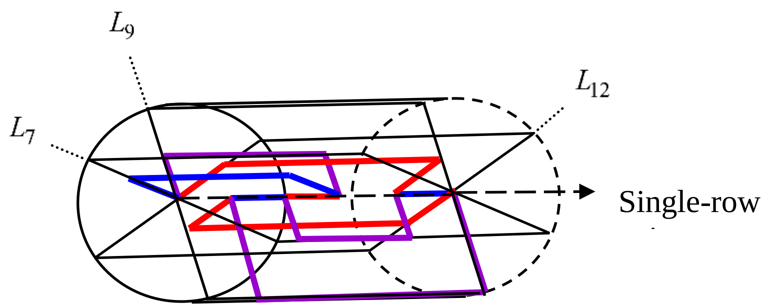


Figure 9: Cross section of the cylindrical model illustrating the position of three planes at different angle.

6 Simulation and Results

A random list L is form at random time t. The list at each time slot may be different or repeated from the previous one. Then, we seek for its optimal realization. Assuming for ten different time slots, the respected list is as in the Table 4.

Table 4: Generating list L for $n = 20$ at ten different time slots.

t_i	L_i
t_0	$L_0 = \{(1,20),(2,15),(3,10),(4,5),(6,9),(7,19),(8,17),(11,13),(12,16),(14,18)\}$
t_1	$L_1 = \{(1,20),(2,15),(3,10),(4,5),(6,9),(7,14),(8,9),(11,18),(12,13),(16,17)\}$
t_2	$L_2 = \{(1,3),(2,18),(8,17),(14,19),(11,13),(4,16),(5,9),(6,7),(10,12),(15,20)\}$
t_3	$L_3 = \{(1,7),(2,11),(3,9),(4,19),(5,15),(6,17),(10,18),(8,12),(13,14),(16,20)\}$
t_4	$L_4 = \{(1,10),(2,19),(3,17),(4,7)5,9),(6,12),(8,11),(13,18),(14,15),(16,20)\}$
t_5	$L_5 = \{(1,4),(2,10),(3,9),(11,17),(5,8),(6,12),(7,13),(14,18),(15,20),(16,19)\}$
t_6	$L_6 = \{(1,11),(2,12),(3,10),(4,9),(5,17),(6,19),(7,20),(8,13),(16,18),(10,15)\}$
t_7	$L_7 = \{(1,15),(2,11),(3,10),(4,9),(5,13),(6,14),(7,20),(8,17),(12,19),(16,18)\}$
t_8	$L_8 = \{(1,9),(2,8),(3,10),(4,11),(5,7),(6,12),(13,17),(14,18),(15,20),(16,19)\}$
t_9	$L_9 = \{(1,5),(2,10),(3,9),(4,20),(6,12),(7,11),(8,19),(13,17),(14,16),(15,18)\}$

Next step is to identify the optimal configuration for each L_i. Obviously, to complete this step manually is very computationally expensive and almost impossible. However, this step can be done easily using our previous model, ESSR. Figures 10-12 present the optimization of energy level for each plane in the cylindrical model while minimizing congestion level and number of doglegs.

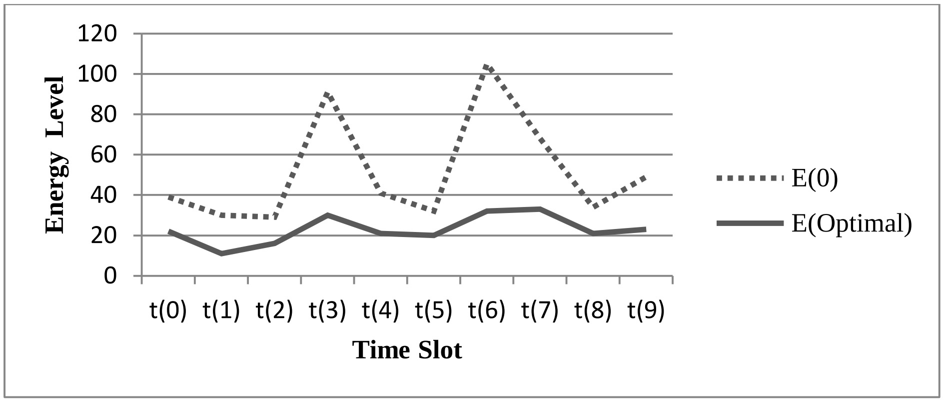


Figure 10: The optimization of energy level at each time slot.

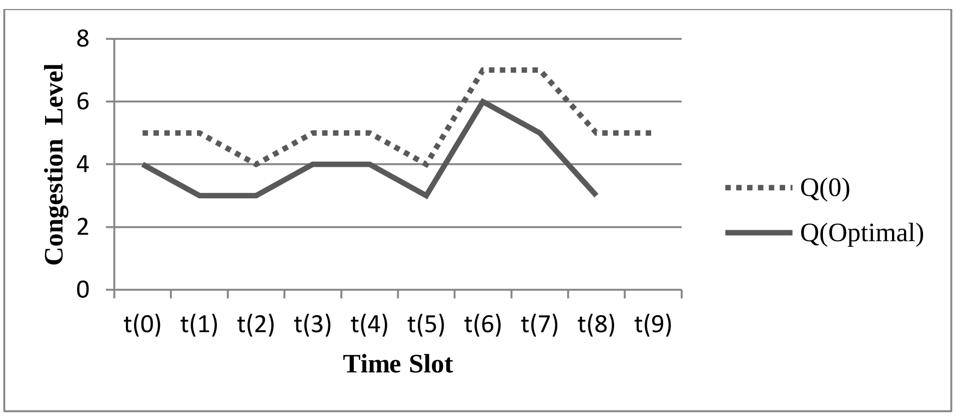


Figure11: The minimization of congestion level at each time slot.

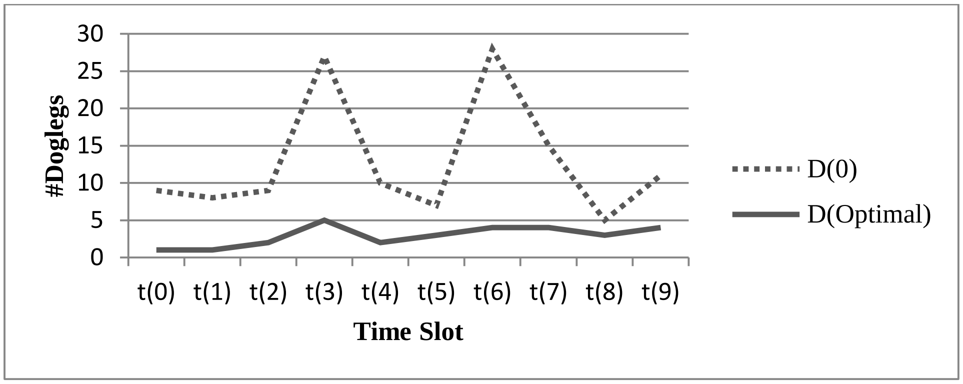


Figure12: The minimization of number of doglegs at each time slot.

This suggests that each plane will have optimal configuration for each pair of pins. Since the networks is configured dynamically according to constantly changing intervals between the pair of pins, our cylindrical model is optimal and suitable for adoption into problems requiring massive pin connections such as in switching. Through our simulation program, we also identify the time taken to achieve optimality for each plane and this is summarized in Figure 13.

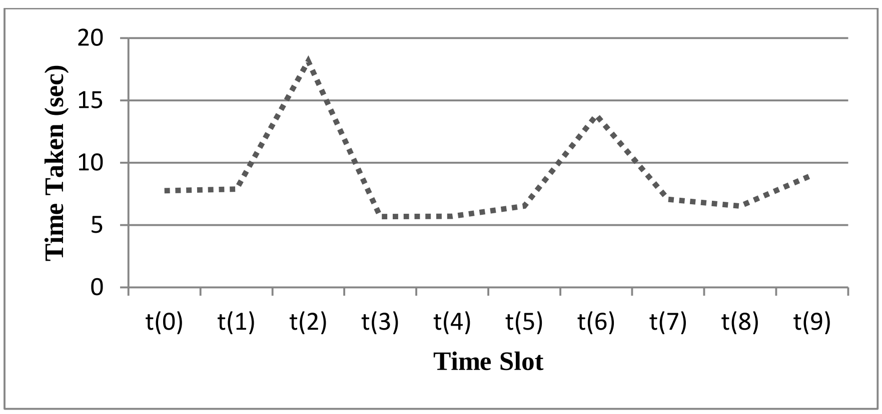


Figure 13: Time taken to achieve optimal arrangement at each time slot.

7 Conclusion

In this paper, we proposed a new technology to single-row routing (SRR) problem. We introduced a method to transform two-dimensional SRR problem into a dynamic three-dimensional problem. This transformation is modeled as a cylinder due to the fact that a cylinder is symmetrical along an infinite number of planes since its cross sectional area is that of a circle. This differs with other shapes which has limited planes of symmetry. Therefore, we will be able to generate unlimited number of planes inside this cylindrical model. First, the single-row routing technique is described clearly and the problem statement is discussed. We applied the concept of combinations in statistics and derived an induction formula in order to identify all possible L can be form for n numbers of pins. Then we seek for optimal realizations for each L that satisfy single-row rules and conditions while minimizing number of street congestion and doglegs. Each realization will form an axis in the cylindrical model and its radius is equal to the maximum number of street congestion for all realizations achieved. From the simulation program, we were able to produce optimal realization for each L_i at reasonable time taken. This suggests that our model is suitable for adoption into problems requiring massive connection of pins.

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