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On Markovian Modelling of Vehicular Traffic Flow in Gwagwalada Metropolis, Nigeria

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Abstract Analysis of queue performance parameters is inevitable for efficient and effective congestion control and traffic free networking system. This study assesses the efficiency and effectiveness of road traffic systems, using the Atlas Hotel road intersection, Gwagwalada, Abuja Nigeria as a case study. Queue theory analytic methodologies are applied and parameters such as average arrival and service rate are calculated based on the data obtained at the said junction on daily basis for a period of one week. Inter-arrival time is 10 seconds and M/M/3 queue model was found appropriate. Considering the 15278 vehicles observed, major arrivals and departure were on the SDP leg, while the least arrivals and departure was witnessed on the Town-Hall leg, the El-Rufai and NITEL legs had varying degrees of arrivals and departure. Based on the data gathered, it was observed that the evenings record the highest magnitude of vehicular traffic at the intersection while mornings had the least. It was concluded that with reference to the intersection studied, road traffic system performance within the Gwagwalada metropolis is stable as Level of Service is A with Average Control Delay (seconds/vehicle) $\leq$ 10 seconds per vehicle. Modernization and re-design of roadways, traffic education, as well as introduction of road traffic utilities were recommended.

Keywords Traffic Flow; Queuing System; Arrival and Departure Rates; Gwagwalada

2010 Mathematics Subject Classification 60K25, 60K30

1 Introduction

Analysis of queuing and service times is essential for designing effective congestion control at any service point, (Ogunfiditimi and Oguntade, [1]). As nations and countries of the world strive for growth and development in both the human and infrastructures, congestions and delays are inevitable and they have become an integral part of life .We encounter queues in School, Hospitals, Banks, Restaurants, Bars, Shopping Mall, Filling Stations, as well as traffic congestion on roads especially at the road intersections.

The problem of traffic congestion is foreseeable as long as the number of vehicles in the metros is near / beyond the existing road capacity. In a road network, the intersection is a major cause of bottlenecks thus contributing to congestion. Various types of intersection are at-grade intersection, signalized and unsignalized intersection, and roundabouts.

However, the work seeks to find possible solution to some of the nagging road intersection problems especially for the unsignalized case like that of Atlas Junction in Gwagwalada metropolis FCT Nigeria. A necessary tool for this process is the analysis of queues; often called Queuing Theory. Queuing theory is the mathematical study of waiting lines, or the act of joining a line (queues), (János, [2]). In queuing theory a model is constructed so that queue lengths and waiting times can be predicted. The issue of queuing has been a subject of scientific debate, for there is no known society that is not confronted with the problem of queuing. Wherever there is competition for limited resource queuing is likely to occur (Taha [3]; Ogunfiditimi and Oguntade, [1]; Aderamo and Atomode, [4]; Martin *et al.* [5], Odunukwe, [6]. Congestion and queues are notable features in the Federal Capital Territory of Nigeria and within the Gwagwalada metropolis in particular.

In the light of the above problem(s), the study seeks to assess various measures put in place by agencies of government such as Federal Road Safety Corps (FRSC) and possibly proffers solution to some of these problems through the application of queue theory in the assessment of the efficiency and effectiveness of vehicular traffic at the Atlas Hotel road intersections.

2 Theoretical Framework

There are so many literatures on the application queuing of theory to real life situations where congestions, queue, much traffic, delays, time wasting, loss of money , valuables and sometimes lost of precious life are order of the day all of which emanated from the novel work of Erlang [7], the principal pioneer of queuing theory. Erlang, [7] studied problems of telephone congestion. That is, the waiting times of subscribers in a manually operated system, the average waiting time and the chance that a subscriber will obtain service immediately without waiting and he examined how much the waiting time will be affected if the number of operators is altered or conditions are changed in any other way. Other notable authors with novel work include Phillips *et al.* [8] and Taha, [3]. Kendall, [9] introduced an A/B/C queuing notation and Irene [10] used Simulation approach to design a Single-Server Queuing System.

Ogunfiditimi and Oguntade [1] compared queuing and service patterns in major key service departments of a teaching hospital. In another study, Odunukwe [6] applied queuing models to customers management in banking industry while study on road traffic congestion and the quest for effective transportation was carrried out in the works of Gerlough and Schuhl, [11], Aworemi *et al.* [12], Aderamo and Atomode, [4], and Agbonika, [13]. Gerlough and Schuhl, [11] used Poisson distribution and theory of probability to model high way traffic in California .Other studies on queing theory can be found in Abolnikov *et al.* [14],Taha, [3] and Adan *et al.* [15].

3 Methods

3.1 The Design of Experiment

The data for this study were collected from the popular Atlas Hotel road intersection which is situated in Gwagwalada Area Council of FCT (See the appendix for the map of Gwagwalada). The intersection has four road legs. They are the legs that send/bring traffic to/from the SDP junction, NITEL axis, Town Hall axis and the El-Rufai Park axis as display in Figure 1. Before a vehicle could go through the intersection it must join a queue (May not if queue does not exist) in order to be passed.

The number of arrivals (that is, the number of vehicles) from each leg was taken at regular time interval. Vehicles queue along the various legs of the intersection and move successively until they get to the imaginary arrival line (with respect to this study they are designated as having arrived when vehicles cross the imaginary line) from which, they then continue to move (if they have to go), until they get to the imaginary service line (which signifies that service has started); this service is only assumed to have ended when the vehicle crosses the imaginary departure line (this corresponds to the service line for the leg the vehicle is approaching). Data were collected on Monday to Sunday (seven days); from the hours of 7:00 am - 9:00 am, 11:00 am - 1:00 pm and 3:00 pm - 5:00 pm as shown in Table 3.

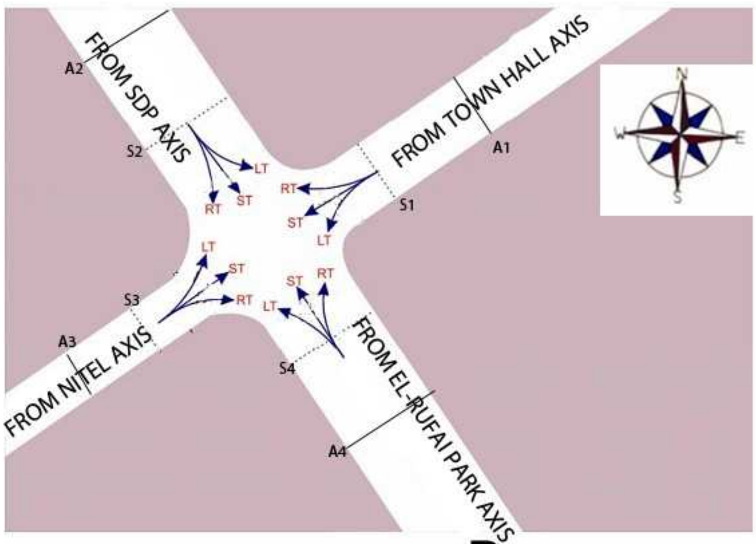

Figure 1: State of The Intersection With Representation of Points Arrivals Begin (A1, A2, A3, A4), Points Service Begins (S1, S2, S3, S3)

3.2 Method of Data Analysis

First we use the chi-square distribution to study the pattern and reactions to change of the system i.e. identification of vehicles, server and queue characteristics that are apparent in the system. After which we estimate queue parameters using the data collected. In classical queuing models, it is possible for the system to operate on Poisson arrivals and Negative-Exponential service distributions for queues (M/M/c). It is not always the case for all systems that arrivals and service distributions follow the Poisson arrivals and Negative Exponential service rate. For more application of Poisson processes see Gerlough and Schuhl, [11], and Whittle, [16] and the references therein.

The M/G/c and G/G/c amongst others are also possible where M/G/c is a Poisson arrival distribution and General Service distribution, while G/G/c is a General distributed arrival and service; (in general here could be any of the known distributions). Therefore, as is appropriate, assertions about parameter distributions will only be made after the data collected have been analysed.

The intersection (as shown in Figure 1) allows three possible routes for which vehicles on any leg at any one time may go. Though the intersection is unsignalized, there is the existence of a traffic warden. This kind of a system is called a multiple-server system with multiple singe queues. The single-Queue Multiple-Server model (when one leg is considered along with all the possible routes of exist) is chosen for this study with the following assumptions made:

- No vehicle leaves the queue without being served.
- Infinite capacity feeds of vehicles in queuing system (i.e. no limit for queue capacity).
- FIFO (First-In-First-Out) or FCFS (First-Come-First-Serve).

3.2.1 Mean Arrival Rate and Time

$\lambda$is the mean arrival rate, n the number of vehicles that entered the system between the three two (2) hour periods. Also h is the number of hours within these periods. Then, the mean arrival rate is given by the formula, $\lambda_{\text{leg}_i} = (n_{\text{arrivals}})/h$ arrivals per hour.

The mean arrival time across each leg is given as $1/\lambda$.

3.2.2 Mean Service Rate

μ is the mean service rate and n the number of vehicles that were served within the three, two (2) hour periods. Also, let h be the number of hours within these periods. Then, the mean service rate is given by the formula, $\mu_{\text{leg}_i} = (n_{\text{served}})/h$, services per hour.

3.2.3 Poisson Distribution

Poisson's law, applying to rare events, has thus far been the chief theoretical instrument for dealing with problems of vehicular traffic on two or three lane highways, especially the problem of determining the distribution of such traffic both in time and in space, (Gerlough and Schuhl [11]).

The Poisson distribution is used as a model for the number of discrete occurring events (such as number of vehicle arrivals at an intersection) in a specific time period. λ is the parameter which indicates the average number of events in the given time interval.

$$P(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ for } x = 0, 1, 2, \ldots. \tag{1}$$

3.2.4 Negative Exponential Distribution

The most common stochastic queuing models assume that inter-arrival times and service times are exponentially distributed. The density of an exponential distribution with parameter $\lambda > 0$ is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, \text{ if } x \geq 0, \\ 0, \text{ otherwise.} \end{cases} \tag{2}$$

An exponential random variable can be used to model the time until the next vehicle arrives at an intersection. Exponential distribution is a family of Gamma distribution which is a subclass of Erlang family of density, (For more elaborate details on probability distributions, see Whittle, [16], Spiegel *et al.* [17] and Awogbemi and Oguntade, [18].

3.2.5 Markov Process

If sequence $X_1, X_2...X_t$ is independent, then (X_t) has the Markov property, that is, the distribution of X_{t+1} given $X_t, X_{t-1}, ..., X_1$ is the same as the distribution of X_{t+1} given X_t.This is a property possessed by many physical systems provided we include sufficiently many components in the specification of the state X_t.For instance we may choose the state vector in such a way that X_t includes components of X_{t-1} for each t. (See Brockwell and Davis,[19]).

In a general language, Markovian property implies that given the present state of a dynamical process, the future state of the process is independent of its past states. It means that all the past information are embedded in the present state which is the only information needed by the future of the process. (For further details see Oguntade and Akano,[20] and references therein)

These are processes whose future behaviour cannot be accurately predicted from its past behaviour (except the current or present behaviour) and which involves random chance or probability. Flow of traffic is an example of Markov processes.

3.2.6 Chi-Square Distribution

This test is designed to investigate the agreement of a set of observed frequency and expected frequency on the assumption of a theoretical model for the phenomenon being studied. The test is used for investigating dependency or independency of two attributes of classification. χ^2 statistic given by

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \tag{3}$$

where O_i is observed frequency of the ith observation and E_i is expected frequency of the ith observation

3.2.7 Level of Service

The Level of Service (LOS) of an intersection is a qualitative measure of capacity and operating conditions and is directly related to vehicle delay. LOS is given a letter designation from A to F as shown in Table 1, with LOS A representing very short delays and LOS F representing very long delays. As a practical consideration, LOS D is considered the limit of acceptable operation in an urban environment. LOS C is the desirable condition. LOS conditions for unsignalized intersections is shown in table .

4 Data Analysis and Results

The analysis of data collected on number of arrivals and their service time at the Atlas Hotel road intersection were carried out using Minitab 16 and Queue Theory Software (QTS) and Microsoft Excel.

4.1 Arrival Rate and Probability Distribution

Table 2 shows how the fitting of the Poisson distribution to the experimental data was carried out in tabular form as displayed above. Other related results from the table are as

Table 1: The Level of Service for Unsignalized Intersection

Level-of-Service (LOS)	Average Control Delay (seconds/vehicle)	Description
A	≤10.0	No delays at intersections with continuous flow of traffic. Uncongested operations: high frequency of long gaps available for all left and right turning traffic. No observable queues.
B	10.1 to 15.0	Same as LOS A
C	15.1 to 25.0	Moderate delays at intersections with satisfactory to good traffic flow. Light congestion; infrequent backups on critical approaches.
D	25.1 to 35.0	Increased probability of delays along every approach. Significant congestion on critical approaches, but intersection functional. No standing long lines formed.
E	35.1 to 50.0	Heavy traffic flow condition. Heavy delays probable. No available gaps for cross-street traffic or main street turning traffic. Limit of stable flow.
F	>50.0	Unstable traffic flow. Heavy congestion. Traffic moves in forced flow condition. Average delays greater than one minute highly probable. Total breakdown.

follows:

DF	Chi-Sq	P-Value
18	25.9197	0.102

Mean of frequency distribution is

$$\frac{\sum F_i X_i}{\sum F} = \frac{15278}{252} = 60.63.$$

Thus the mean arrival rate per 10 minutes duration is $60.6 \cong 61$ vehicles;

The mean arrival rate per unit time is then given as

$$\lambda = \frac{60.6}{10} = 6.06 \cong \text{ vehicles per minute.}$$

This is confirmed by

Arrival rate $(\lambda) = \dfrac{\text{number of arrival}}{\text{Time taken}}$.

Number of vehicles that arrived = 15278.

Time within which vehicles arrived = 151200 seconds = 2520 minutes.

Table 2: Probability Distribution of Numbers of Arrivals in Every Ten Minutes

Number of Arrivals in 10 Minutes	Observed	Poisson Probability	Theoretical Freq.	Contribution to Chi Sq.
≤ 43	8	0.010864	2.7378	10.1145
44	1	0.004817	1.2138	0.0377
45-46	2	0.015042	3.7905	0.8458
47-48	2	0.024967	6.2917	2.9274
49-50	4	0.038146	9.6129	3.2773
51-52	12	0.053826	13.5642	0.1804
53-54	16	0.070356	17.7297	0.1687
55-56	25	0.085427	21.5275	0.5601
57-58	25	0.096607	24.3449	0.0176
59-60	24	0.101999	25.7038	0.1129
61-62	27	0.100774	25.3951	0.1014
63-64	28	0.093365	23.5281	0.8500
65-66	23	0.081278	20.4822	0.3095
67-68	21	0.066608	16.7853	1.0583
69-70	17	0.051477	12.9723	1.2506
71-72	6	0.037580	9.4701	1.2715
73-74	4	0.025956	6.5408	0.9870
75-76	4	0.016986	4.2805	0.0184
77-78	2	0.010548	2.6580	0.1629
$\geq$79	1	0.013377	3.3709	1.6676

Thus

$$\lambda = \frac{15278}{2520} = 6.06 \cong 6 \text{ vehicles per minutes.}$$

Inter-arrival time $= \frac{1}{\lambda} = \frac{1}{6.06} = 0.165$ minutes $= 10$ seconds.

And $P(x) = \frac{(60.63)^x \ell^{-60.63}}{\text{x!}}$.

Critical Value

$\chi^2{}_{k-1-m.1-\alpha} = \chi^2{}_{18.095} = 28.9000$ at 5% level of significance.

Decision Rule: H_0 is accepted if $\chi^2_{calculated} < \chi^2_{tabulated}$.

Thus since $\chi^2_{calculated} = 25.9197 < \chi^2_{tabulated} = 28.9000, H_0$ is accepted and it is concluded that arrivals at the Atlas hotel road intersection follows the null hypothesis of Poisson distribution. The process is thus Markovian (M) in nature. That is, the observed data can be constructed as a sample coming from the postulated theoretical distribution. It implies there is no evidence to indicate that the true theoretical distribution differs from the postulated distribution.

The Figure 2 shows the charts of both the observed and calculated frequencies as displayed in Table 2. The chart in Figure 3 show the arrival time in ten minutes with the

Table 3: Number of Vehicles Arriving at the Intersection through Specified Legs

DAY	TIME FRAME	SDP	EL-RUFAI	NITEL	TOWNHALL	TOTAL
DAY 1	MORNING	334	130	93	124	681
	AFTERNOON	290	119	188	127	724
	EVENING	276	132	186	132	726
	TOTAL	900	381	467	383	2131
DAY 2	MORNING	277	128	185	126	716
	AFTERNOON	285	128	189	135	737
	EVENING	300	134	201	141	776
	TOTAL	862	390	575	402	2229
DAY 3	MORNING	279	124	175	113	691
	AFTERNOON	287	133	186	120	726
	EVENING	300	144	194	129	767
	TOTAL	866	401	555	362	2184
DAY 4	MORNING	261	137	182	130	710
	AFTERNOON	278	132	190	131	731
	EVENING	307	140	190	140	777
	TOTAL	846	409	562	401	2218
DAY 5	MORNING	281	133	176	121	711
	AFTERNOON	286	133	186	125	730
	EVENING	285	131	183	133	732
	TOTAL	852	397	545	379	2173
DAY 6	MORNING	259	134	179	108	680
	AFTERNOON	280	153	186	119	738
	EVENING	288	145	201	116	750
	TOTAL	827	432	566	343	2168
DAY 7	MORNING	283	132	144	105	664
	AFTERNOON	252	140	202	139	733
	EVENING	279	153	207	138	777
	TOTAL	814	425	553	382	2174

corresponding frequency. Figure 4 to Figure 6 display multiple bar chats of arrivals for the four legs at Atlas intersection for morning, afternoon and evening for seven days respectively.

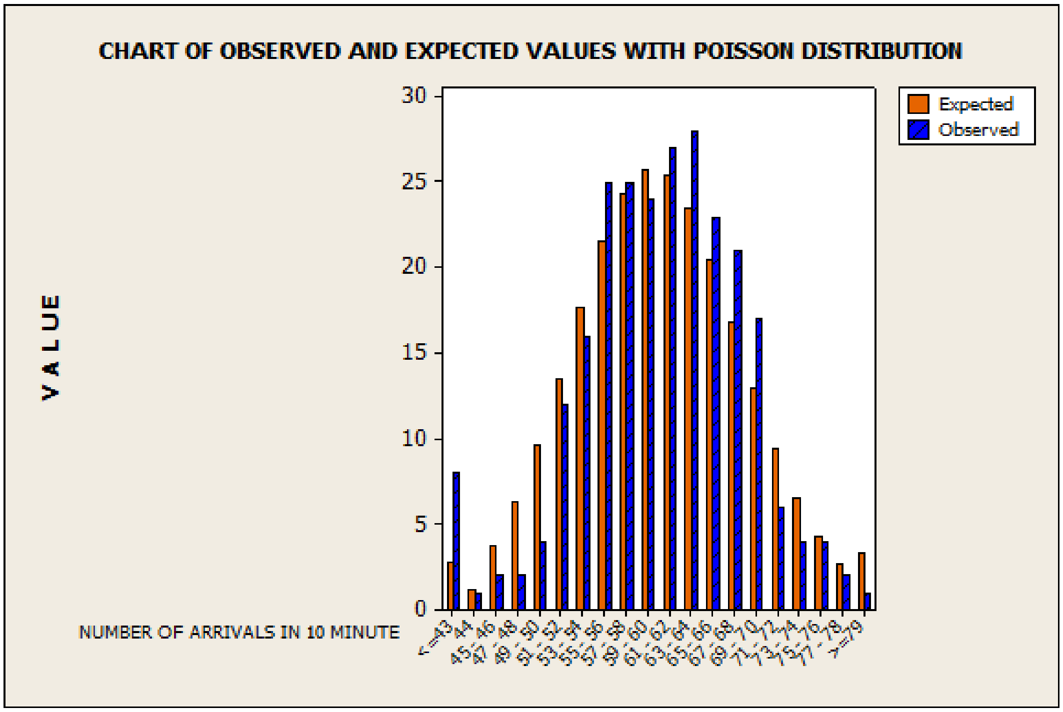

Figure 2: Chart of Observed and Expected Values with Poisson Distribution

Figure 7 is a bar chart of all the arrivals per axis for the period under consideration. The highest arrival statistics was observed on SDP axis while Town Hall axis has the lowest value of arrivals.

4.2 Service Time and Probability Distribution

Table 4 above shows the frequency distribution of service time with the corresponding frequencies as observed at the Atlas intersection, Gwagwalada.

Mean of Service Distribution

Mean $= \frac{\sum F_i X_i}{\sum F} = \frac{97351}{15278} = 6.73.$

Number of vehicles served $= 15278$.

Time within which vehicles where served $= 2600$ (43 hours and 20 minutes).

Thus service rate $(\mu) = \frac{15278}{2600} = 5.88 \cong 6$ vehicles were served per minute.

Therefore; inter-departure time $= \frac{1}{\mu} = \frac{1}{5.88} = 0.17$ minutes or 10 seconds.

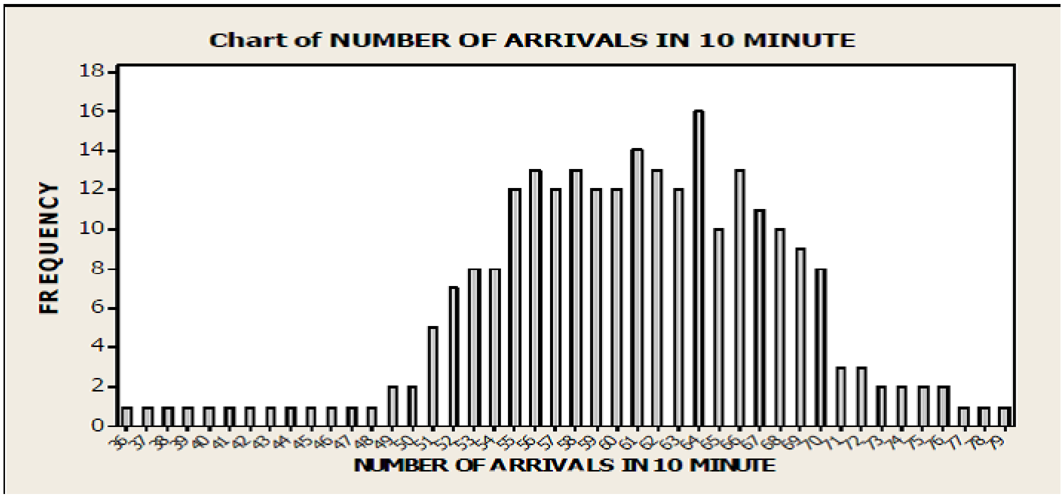

Figure 3: Plot of Individual Arrivals

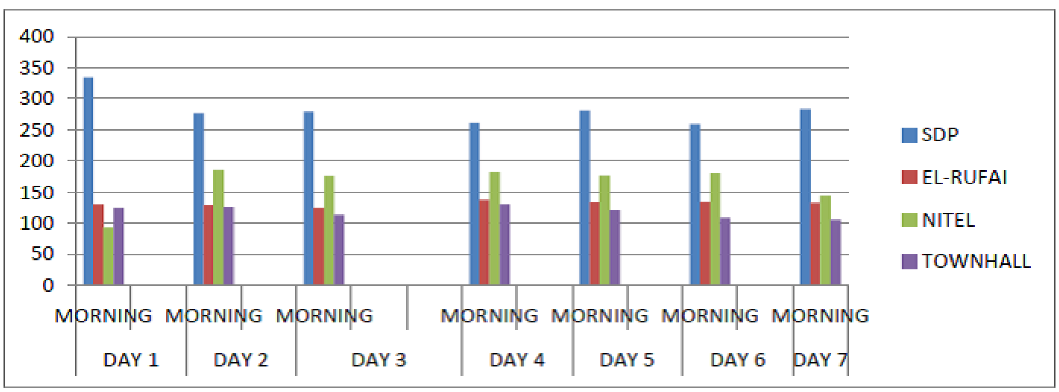

Figure 4: Morning Arrivals for Seven Days

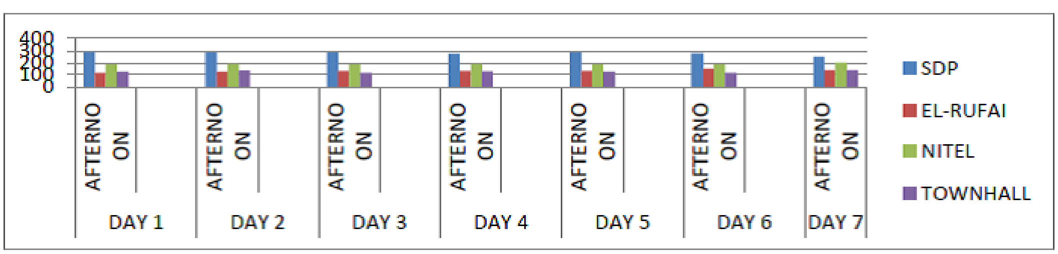

Figure 5: Afternoon Arrivals for Seven Days

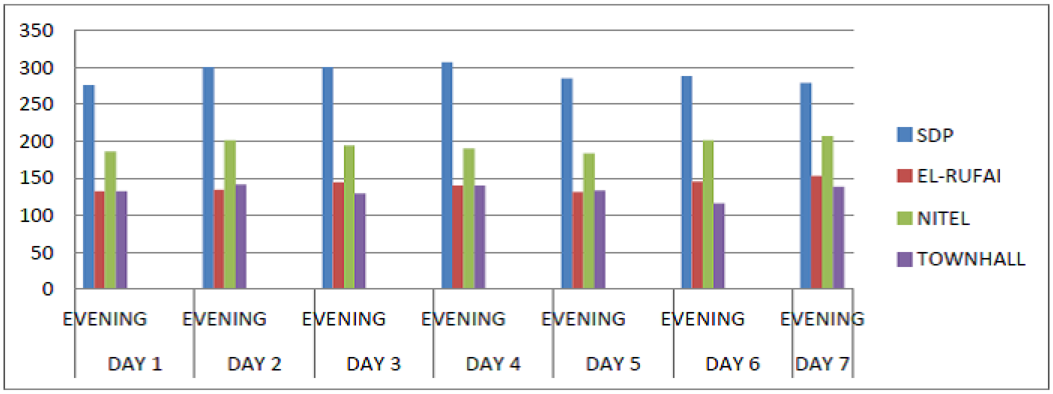

Figure 6: Evening Arrivals for Seven Days

The service time is fitted to the theoretic exponential distribution in order to assess goodness of fit.

Figure 8 shows the histogram of the frequency distribution of service time given in Table 4. The plot of the distribution given above confirms that service time follows the exponential distribution. The process is thus Markovian (M) in nature. From the foregoing it can be concluded that the traffic state at the Atlas hotel road intersection follows the M/M/c model; which in this case is the M/M/3.

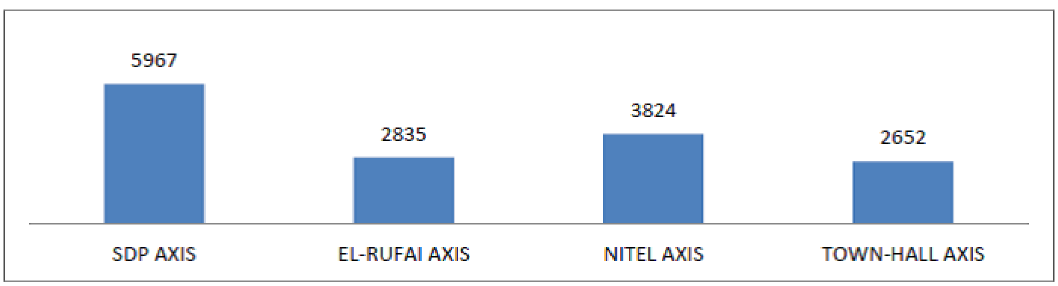

Figure 7: Number of Arrivals Per Axis

5 Conclusion

Based on the results of the analysis, it can be concluded that the level of service for unsignalized intersection at Atlas Hotel Gwagwalada Abuja falls into category A with no excessive delay at intersection and continuous flow of traffic. Traffic at the intersection is thus found to be consistent and ideal except for the identified problems that are peculiar to the study area; like bad road surfaces, absence of modern traffic utilities and the recklessness of motorcyclists and commuters alike.

The queue parameters are thus given below with the aid of Queuing Theory Software

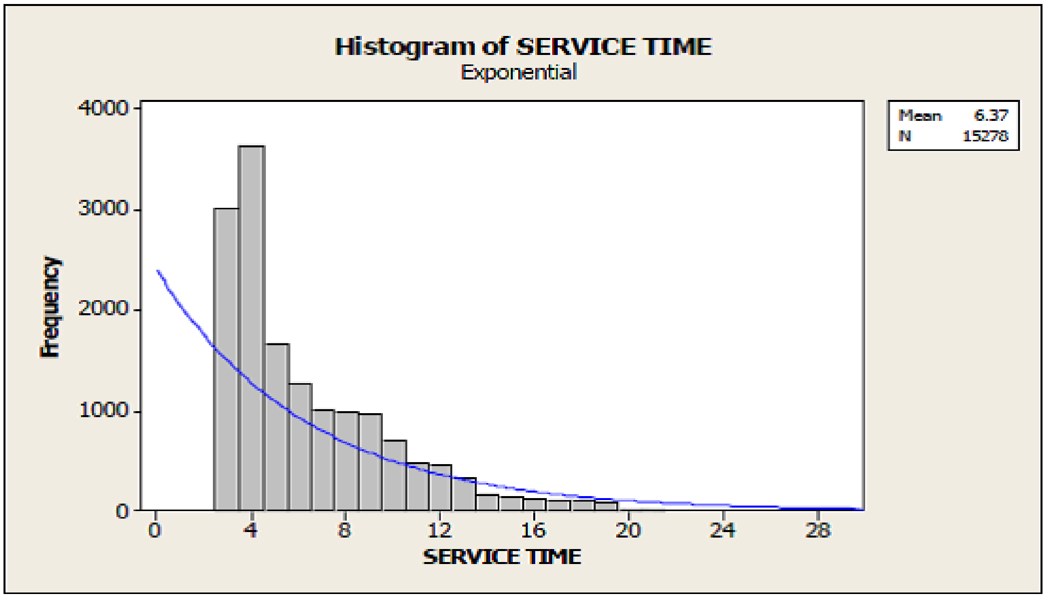

Figure 8: Exponential Distribution of Service Time

Table 4: Analysis of Service Time

Service Time (X)	**Observed Frequency** (F)	(XF)
3	3021	9063
4	3650	14600
5	1662	8310
6	1278	7668
7	1016	7112
8	982	7856
9	969	8721
10	697	6970
11	479	5269
12	461	5532
13	324	4212
14	165	2310
15	142	2130
16	115	1840
17	108	1836
18	100	1800
19	75	1425
20	17	340
21	17	357
TOTAL	15278	97351

(QTS). In Table 5, the estimated results of queue parameters for Poisson arrivals and Exponential service time are shown.

Table 5: M/M/c: Poisson Arrivals to Multiple Exponential Severs

Input Parameters:	
Arrival rate (λ)	6.06
Mean service time ($1/\mu$)	0.17
Number of servers in the system (c)	3
Results:	
Mean inter arrival time ($1/\lambda$)	0.165017
Service Rate (μ)	5.882353
Average # arrivals in mean service time (r)	1.0302
Sever utilization (ρ)	84.34 %
Fraction of time all servers are idle (ρ_0)	0.352313
Mean number of customers in the system (L)	1.081338
Mean number of customers in the queue (L_q)	0.051138
Mean waiting time (W)	0.178439
Mean wait time in the queue (W_q)	0.008439
Probability arriving customer is delayed in queue ($1 - W_{q(0)}$)	0.097778

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Appendix

Figure 9: Aeriel Map of Gwagwalada Metropolis