MATEMATIKA, 2016, Volume 32, Number 2, 133–140

On mild solutions of a semilinear mixed Volterra-Fredholm functional integrodifferential evolution nonlocal problem in Banach spaces

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Abstract The existence, uniqueness and continuous dependence on initial data of mild solutions of nonlocal mixed Volterra-Fredholm functional integrodifferential equations with delay in Banach spaces has been discussed and proved in the present paper. The results are established by using the semigroup theory and modified version of Banach contraction theorem.

Keywords Mixed Volterra-Fredholm functional integrodifferential equation; fixed point; semigroup theory; nonlocal condition.

2010 Mathematics Subject Classification 47H10, 45J05, 47B38.

1 Introduction

Byszewski and Acka [1] established the existence, uniqueness and continuous dependence of a mild solution of a semilinear functional differential equation with nonlocal condition of the form

$$\frac{du(t)}{dt} + Au(t) = f(t, u_t), t \in [0, a],$$
$$u(s) + \left[g\left(u_{t_1}, \ldots, u_{t_p}\right)\right](s) = \phi(s), s \in [-r, 0],$$

where $0 < t_1 < \ldots < t_p \leq a \ (p \in N), -A$ is the infinitesimal generator of a C_0 semigroup of operators on a general Banach space, f, g and ϕ are given functions and $u_t(s) = u(t+s)$ for $t \in [0, a], s \in [-r, 0]$. The problems of existence, uniqueness and other qualitative properties of solutions for semilinear differential equations in Banach spaces has been studied extensively in the literature for last many years, see [2-5], [6], [7-9], [10].

Theorem about the existence, uniqueness and stability of solutions of differential, integrodifferential equations and functional differential abstract evolution equations with nonlocal conditions were studied by Byszewski [1, 11], Balachandran and Chandrasekaran [12], Lin and Liu [13] and Balachandran and Park [14].

In the present paper, we consider semilinear mixed Volterra-Fredholm functional integrodifferential equations of the form

$$\frac{dx(t)}{dt} = Ax(t) + f\left(t, x_t, \int_0^t k(t, s, x_s)\, ds, \int_0^T h(t, s, x_s)\, ds\right), t \in [0, T], \tag{1}$$

$$x(t) + \left(g\left(x_{t_1}, \ldots, x_{t_p}\right)\right)(t) = \phi(t), t \in [-r, 0], \tag{2}$$

where A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $T(t), t \geq 0$ on X, f, k, h, g and ϕ are given functions satisfying some assumptions and $x_t(\theta) = x(t+\theta)$, for $\theta \in [-r, 0]$ and $t \in [0, T]$. The objective of this paper is to improve the results of [2] and also generalize the results of Jain and Dhakne [15]. We are finding the results with less restriction by using modified version of Banach contraction principle.

The rest of the paper is organised as follows: In section 2 we give preliminaries and hypotheses. In section 3 we prove existence and uniqueness of solutions. In section 4 we deal with continuous dependence of initial data of mild solutions. Finally in section 5 we give application to illustrate the theory.

2 Preliminaries and hypotheses

Let X be Banach space with the norm $\|.\|$. Let $C = C([-r, 0], X), 0 < r < \infty$ be the Banach space of all continuous functions $\psi : [-r, 0] \to X$ endowed with the supremum norm

$$\|\psi\|_C = \sup\{\|\psi(t)\| : -r < t < 0\}.$$

Let $B = C([-r, T], X), T > 0$ be the Banach space of all continuous functions $x : [-r, T] \to X$ with the supremum norm $\|x\|_B = \sup\{\|x(t)\| : -r \le t \le T\}$. For any $x \in B$ and $t \in [0, T]$, we denote x_t the element of C given by $x_t(\theta) = x(t + \theta)$, for $\theta \in [-r, 0]$ and ϕ is given element of C.

In this paper, we assume that there exists positive constant $M \ge 1$ such that $\|T(t)\| \le M$, for every $t \in [0, T]$.

Definition 2.1 A function $x \in B$satisfying the equations:

$$\begin{aligned} x(t) = T(t)\left[\phi(0) - \left(g\left(x_{t_1}, ..., x_{t_p}\right)\right)(0)\right] \\ + \int_0^t T(t-s) f\left(s, x_s, \int_0^s k(s, \tau, x_\tau)\, d\tau, \int_0^T h(s, \tau, x_\tau)\, d\tau\right) ds,\ t \in [0, T] \\ x(t) + \left(g\left(x_{t_1}, ..., x_{t_p}\right)\right)(t) = \phi(t), -r \le t \le 0, \end{aligned}$$

is called a mild solution of the initial value problem (1) – (2).

Our results are based on the modified version of Banach contraction principle.

Lemma 2.1 [16, p.196] *Let X be a Banach space. Let D be an operator which maps the elements of X into itself for which D^r is a contraction, where r is a positive integer. Then D has a unique fixed point.*

We make the following assumptions:

(A_1) Let $f : [0, T] \times C \times X \times X \to X$ such that for every $w \in B, x, y \in X$ and $t \in [0, T], f(., w_t, x, y) \in B$ and there exists a constant $L > 0$ such that

$$\|f(t, \psi, x, y) - f(t, \phi, u, v)\| \le L(\|\psi - \phi\|_C + \|x - u\| + \|y - v\|), \quad \phi, \psi \in C,\ x, y, u, v \in X.$$

(A_2) Let $k : [0, T] \times [0, T] \times C \to X$ such that for every $w \in B$ and $t \in [0, T], k(., ., w_t) \in B$ and there exists a constant $K > 0$ such that

$$\|k(t, s, \psi) - k(t, s, \phi)\| \le K(\|\psi - \phi\|_C), \phi, \psi \in C.$$

(A_3) Let $h : [0, T] \times [0, T] \times C \to X$such that for every $w \in B$ and $t \in [0, T], h(., ., w_t) \in B$and there exists a constant $H > 0$ such that

$$\|h(t, s, \psi) - h(t, s, \phi)\| \le H(\|\psi - \phi\|_C), \phi, \psi \in C.$$

(A_4) Let $g : C^p \to C$ such that there exists a constant $G \geq 0$ such that

$$\left\| \left(g\left(x_{t_1},...,x_{t_p}\right)\right)(t) - \left(g\left(y_{t_1},...,y_{t_p}\right)\right)(t)\right\| \leq G\left(\|x-y\|_B\right), t \in [-r,0].$$

3 Existence of mild solution

Theorem 3.1 *Consider that the assumptions $(A_1)-(A_4)$ are satisfied. Then the initial value problem $(1)-(2)$ has a unique mild solution x on $[-r,T]$.*

Proof: Let $x(t)$ be a mild solution of the problem $(1)-(2)$. Then it satisfies the equivalent integral equation

$$\begin{aligned} x(t) &= T(t)\phi(0) - T(t)\left(g\left(x_{t_1},...,x_{t_p}\right)\right)(0) \\ &\quad + \int_0^t T(t-s) f\left(s, x_s, \int_0^s k(s,\tau,x_\tau)\,d\tau, \int_0^T h(s,\tau,x_\tau)\,d\tau\right) ds, t \in [0,T], \quad (3) \\ x(t) &+ \left(g\left(x_{t_1},...,x_{t_p}\right)\right)(t) = \phi(t), -r \leq t \leq 0. \quad (4) \end{aligned}$$

Now we rewrite solution of initial value problem $(1)-(2)$ as follows: For $\phi \in C$, define $\widehat{\phi} \in B$ by

$$\widehat{\phi}(t) = \begin{cases} \left[\phi(t) - \left(g\left(x_{t_1},...,x_{t_p}\right)\right)(t)\right], \text{ if } -r \leq t \leq 0, \\ T(t)\left[\phi(0) - \left(g\left(x_{t_1},...,x_{t_p}\right)\right)(0)\right], \text{ if } 0 \leq t \leq T. \end{cases}$$

If $y \in B$ and $x(t) = y(t) + \widehat{\phi}(t), t \in [-r,T]$, then it is easy to see that y satisfies

$$y(t) = 0; \ \ t \in [-r,0] \quad (5)$$

and

$$y(t) = \int_0^t T(t-s) f\left(s, y_s + \widehat{\phi_s}, \int_0^s k\left(s,\tau,y_\tau + \widehat{\phi_\tau}\right) d\tau, \int_0^T h\left(s,\tau,y_\tau + \widehat{\phi_\tau}\right) d\tau\right) ds, t \in [0,T] \quad (6)$$

if and only if $x(t)$ satisfies the equations $(3)-(4)$.

We define the operator $F : B \to B$, by

$$(Fy)(t) = \begin{cases} 0, -r \leq t \leq 0, \\ \int_0^t T(t-s) f\left(s, y_s + \widehat{\phi_s}, \int_0^s k\left(s,\tau,y_\tau + \widehat{\phi_\tau}\right) d\tau, \int_0^T h\left(s,\tau,y_\tau + \widehat{\phi_\tau}\right) d\tau\right) ds, \text{ if } 0 \leq t \leq T. \end{cases} \quad (7)$$

From the definition of an operator F defined by the equation (7), it is to be noted that the equation $(5)-(6)$ can be written as

$$y = Fy.$$

Now we show that F^n is a contraction on B for some positive integer n. Let $y, w \in B$ and using assumptions $(A_1) - (A_4)$, we get

$$
\begin{aligned}
&\|(Fy)(t) - (Fw)(t)\| \\
&\leq \int_0^t \|T(t-s)\| \left\| \begin{array}{l} f\left(s, y_s + \widehat{\phi_s}, \int_0^s k\left(s,\tau, y_\tau + \widehat{\phi_\tau}\right) d\tau, \int_0^T h\left(s,\tau, y_\tau + \widehat{\phi_\tau}\right) d\tau\right) \\ -f\left(s, w_s + \widehat{\phi_s}, \int_0^s k\left(s,\tau, w_\tau + \widehat{\phi_\tau}\right) d\tau, \int_0^T h\left(s,\tau, w_\tau + \widehat{\phi_\tau}\right) d\tau\right) \end{array} \right\| ds \\
&\leq \int_0^t ML \left[\left\| \left(y_s + \widehat{\phi_s}\right) - \left(w_s + \widehat{\phi_s}\right) \right\|_C + \int_0^s K(\left\| \left(y_\tau + \widehat{\phi_\tau}\right) - \left(w_\tau + \widehat{\phi_\tau}\right) \right\|_C) \, d\tau \right. \\
&\qquad \left. + \int_0^T H\Big(\left\| \left(y_\tau + \widehat{\phi_\tau}\right) - (w_\tau + \widehat{\phi_\tau}) \right\|_C \Big) d\tau \right] ds \\
&\leq ML \int_0^t \|y_s - w_s\|_C \, ds + ML \int_0^t K \int_0^s \|y_\tau - w_\tau\|_C d\tau ds + ML \int_0^t H \int_0^T \|y_\tau - w_\tau\|_C d\tau \, ds \\
&\leq ML \int_0^t \|y - w\|_B \, ds + ML \int_0^t K \int_0^s \|y - w\|_B d\tau ds + ML \int_0^t H \int_0^T \|y - w\|_B d\tau \, ds \\
&\leq ML \|y - w\|_B \, t + MLK \|y - w\|_B \frac{t^2}{2} + MLH \|y - w\|_B \frac{t^2}{2} \\
&\leq ML \|y - w\|_B \, t + MLKT \|y - w\|_B \frac{t}{2} + MLHT \|y - w\|_B \frac{t}{2} \\
&\leq ML \|y - w\|_B \, t + MLKT \|y - w\|_B \, t + MLHT \|y - w\|_B \, t. \\
&\leq ML (1 + KT + HT) \|y - w\|_B \, t. \\
&\left\| \left(F^2 y\right)(t) - \left(F^2 w\right)(t) \right\| = \|(F(Fy))(t) - (F(Fw))(t)\| = \|(F(y_1))(t) - (F(w_1))(t)\| \\
&\leq \int_0^t \|T(t-s)\| \left\| \begin{array}{l} f\left(s, y_{1s} + \widehat{\phi_s}, \int_0^s k\left(s,\tau, y_{1\tau} + \widehat{\phi_\tau}\right) d\tau, \int_0^T h\left(s,\tau, y_{1\tau} + \widehat{\phi_\tau}\right) d\tau\right) \\ -f\left(s, w_{1s} + \widehat{\phi_s}, \int_0^s k\left(s,\tau, w_{1\tau} + \widehat{\phi_\tau}\right) d\tau, \int_0^T h\left(s,\tau, w_{1\tau} + \widehat{\phi_\tau}\right) d\tau\right) \end{array} \right\| ds \\
&\leq \int_0^t ML \left[\left\| \left(y_{1s} + \widehat{\phi_s}\right) - \left(w_{1s} + \widehat{\phi_s}\right) \right\|_C + \int_0^s K\Big(\left\| \left(y_1\tau + \widehat{\phi_\tau}\right) - \left(w_{1\tau} + \widehat{\phi_\tau}\right) \right\|_C \Big) d\tau \right. \\
&\qquad \left. + \int_0^T H\Big(\left\| \left(y_{1\tau} + \widehat{\phi_\tau}\right) - \left(w_{1\tau} + \widehat{\phi_\tau}\right) \right\|_C \Big) d\tau \right] ds \\
&\leq \int_0^t ML \|y_{1s} - w_{1s}\|_C \, ds + ML \int_0^t K \int_0^s \|y_{1\tau} - w_{1\tau}\|_C d\tau ds \\
&\qquad + ML \int_0^t H \int_0^T \|y_{1\tau} - w_{1\tau}\|_C d\tau ds \\
&\leq ML \int_0^t \|y_1 - w_1\|_{C([-r,s],X)} \, ds + ML \int_0^t K \int_0^s \|y_1 - w_1\|_{C([-r,\tau],X)} d\tau \, ds \\
&\qquad + ML \int_0^t H \int_0^T \|y_1 - w_1\|_{C([-r,\tau],X)} d\tau \, ds \\
&\leq ML \int_0^t \sup_{\tau \in [-r,s]} \|y_1(\tau) - w_1(\tau)\| \, ds + MLK \int_0^t \int_0^s \sup_{\eta \in [-r,\tau]} \|y_1(\eta) - w_1(\eta)\| \; d\tau \, ds \\
&\qquad + MLH \int_0^t \int_0^T \sup_{\eta \in [-r,\tau]} \|y_1(\eta) - w_1(\eta)\| \; d\tau \, ds \\
&\leq ML \int_0^t \sup_{\tau \in [-r,s]} \|Fy(\tau) - Fw(\tau)\| \, ds + MLK \int_0^t \int_0^s \sup_{\eta \in [-r,\tau]} \|Fy(\eta) - Fw(\eta)\| \, d\tau \, ds
\end{aligned}
$$

$$
\begin{aligned}
&\quad + MLH \int_0^t \int_0^T \sup_{\eta \in [-r,\tau]} \| Fy(\eta) - Fw(\eta) \| \, d\tau \, ds \\
&\le ML \int_0^t \sup_{\tau \in [-r,s]} \left(ML(1+KT+HT) \|y-w\|_B \, \tau \right) ds \\
&\quad + MLK \int_0^t \int_0^s \sup_{\eta \in [-r,\tau]} \left(ML(1+KT+HT) \|y-w\|_B \, \eta \right) d\tau \, ds \\
&\quad + MLH \int_0^t \int_0^T \sup_{\eta \in [-r,\tau]} \left(ML(1+KT+HT) \|y-w\|_B \, \eta \right) d\tau ds \\
&\le M^2L^2 (1+KT+HT) \|y-w\|_B \Bigg[\int_0^t \left(\sup_{\tau \in [-r,s]} \tau \right) ds + \int_0^t K \int_0^s \left(\sup_{\eta \in [-r,\tau]} \eta \right) d\tau ds \\
&\qquad + \int_0^t H \int_0^T \left(\sup_{\eta \in [-r,\tau]} \eta \right) d\tau \, ds \Bigg] \\
&\le M^2L^2 (1+KT+HT) \|y-w\|_B \left[\int_0^t s ds + \int_0^t K \int_0^s \tau \, d\tau \, ds + \int_0^t H \int_0^T \tau \, d\tau \, ds \right] \\
&\le M^2L^2 (1+KT+HT) \|y-w\|_B \left[\frac{t^2}{2} + K \frac{t^3}{3!} + H \frac{t^3}{3!} \right] \\
&\le M^2L^2 (1+KT+HT) \|y-w\|_B \left[\frac{t^2}{2} + KT \frac{t^2}{3!} + HT \frac{t^2}{3!} \right] \\
&\le M^2L^2 (1+KT+HT) \|y-w\|_B \left[\frac{t^2}{2!} + KT \frac{t^2}{2!} + HT \frac{t^2}{2!} \right] \\
&\le M^2L^2 (1+KT+HT)^2 \|y-w\|_B \frac{t^2}{2!}.
\end{aligned}
$$

Continuing in this way, we get

$$
\| (F^n y)(t) - (F^n w)(t) \| \le \frac{\left[ML(1+KT+HT)t \right]^n}{n!} \|y-w\|_B .
$$

For n large enough,

$$
\frac{\left[ML(1+KT+HT)t \right]^n}{n!} < 1.
$$

Thus there exists a positive integer $n$such that F^n is a contraction in B. By virtue of Lemma 2.1, the operator Fhas a unique fixed point $\widetilde{y}$ in B. Then $\widetilde{x} = \widetilde{y} + \widehat{\phi}$ is solution of the initial value problem (1) – (2). This completes the proof of the theorem (3.1). □

4 Continuous dependence of a mild solution

Theorem 4.1 *Suppose that the function f, k, h and $g$satisfies the assumption $(A_1) - (A_4)$. Then for each $\phi_1, \phi_2 \in C$and for the corresponding mild solutions x_1, x_2 of the problems*

$$
\frac{dx(t)}{dt} = Ax(t) + f\left(t, x_t, \int_0^t k(t,s,x_s) \, ds, \int_0^T h(t,s,x_s) \, ds \right), t \in [0,T], \tag{8}
$$

$$
x(t) + \left(g\left(x_{t_1}, \ldots, x_{t_p} \right) \right)(t) = \phi_i(t), t \in [-r, 0], (i = 1, 2). \tag{9}
$$

The following inequality

$$\|x_1 - x_2\|_B \leq \left[\|\phi_1 - \phi_2\|_C + \left(G + LHT^2\right)\|x_1 - x_2\|_B\right] Me^{MLT(1+KT)}$$

is true.

Additionally, if $M\left(G + LHT^2\right)e^{MLT(1+KT)} < 1$ then

$$\|x_1 - x_2\|_B \leq \frac{Me^{MLT(1+KT)}}{1 - M\left(G + LHT^2\right)e^{MLT(1+KT)}}\|\phi_1 - \phi_2\|_C .$$

Proof was given in paper [17], so we omit details here.

5 Applications

To illustrate the application of our result in section 3, consider the following semilinear partial functional mixed integrodifferential equation of the form

$$\frac{\partial w(u,t)}{\partial t} = \frac{\partial^2 w(u,t)}{\partial u^2}$$
$$+ F\left(t, w(u, t-r), \int_0^t k_1(t, w(u, s-r))ds, \int_0^T h_1(t, w(u, s-r))\,ds\right), t \in [0,T], \quad (10)$$
$$w(0,t) = w(\pi, t) = 0, 0 \leq t \leq T, \quad (11)$$
$$w(u,t) + \sum_{i=1}^{p} w(u, t_i + t) = \phi(u,t), 0 \leq u \leq \pi, -r \leq t \leq 0, \quad (12)$$

where $0 < t_1 \leq \ldots \leq t_p \leq T$, the function $F : [0,T] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous. We assume that the function F, k_1 and h_1 satisfying the following conditions:

For every $t \in [0,T]$ and $u, v, x_1, x_2, y_1, y_2 \in \mathbb{R}$, there exists a constant

$$|F(t,u,x_1,x_2) - F(t,v,y_1,y_2)| \leq l\left(|u-v| + |x_1 - y_1| + |x_2 - y_2|\right);$$
$$|k_1(t,s,u) - k_1(t,s,v)| \leq p\left(|u-v|\right);$$
$$|h_1(t,s,u) - h_1(t,s,v)| \leq q\left(|u-v|\right).$$

Let us take $X = L^2[0,\pi]$. Define the operator $A : X \to X$ by $A(z) = z''$ with domain $D(A) = \{z \in X : z, z' \text{ are absolutely continuous}, z'' \in X \text{ and } z(0) = z(\pi) = 0\}$.

Then the operator A can be written as

$$Az = \sum_{n=1}^{\infty} -n^2 (z, z_n)\, z_n, \quad z \in D(A)$$

where $z_n(u) = \left(\sqrt{2}/\pi\right)\sin nu, n = 1, 2, \ldots$ is the orthogonal set of eigenvectors of A and A is the infinitesimal generator of an analytic semigroup $T(t), t \geq 0$ and is given by

$$T(t)\,z = \sum_{n=1}^{\infty} \exp\left(-n^2 t\right)(z, z_n)\, z_n, z \in X.$$

Now, the analytic semigroup $T(t)$being compact, there exists constant M such that

$$|T(t)| \leq M, \text{ for each } t \in [0,T].$$

Define the function $f : [0,T] \times C \times X \times X \to X$, as follows

$$\begin{aligned} &f(t,\psi,x,y)(u) = F(t,\psi(-r),x(u),y(u)), \\ &k(t,\phi)(u) = k_1(t,\phi(-r)u), \\ &h(t,\phi)(u) = h_1(t,\phi(-r)u). \end{aligned}$$

For $t \in [0,T], \psi, \phi \in C, x \in X$ and $0 \leq u \leq \pi$. With these choices of the functions the equations (10)-(12) can be formulated as an abstract mixed integrodifferential equation in Banach space X :

$$\begin{aligned} &\frac{dx(t)}{dt} = Ax(t) + f\left(t, x_t, \int_0^t k(t,s,x_s)\,ds, \int_0^T h(t,s,x_s)\,ds\right), t \in [0,T], \\ &x(t) + \big(g\left(x_{t_1}, \ldots, x_{t_p}\right)\big)(t) = \phi(t), t \in [-r,0]. \end{aligned}$$

Since, all the hypotheses of theorem (3.1) are satisfied, the theorem (3.1), can be applied to guarantee the existence of mild solution $w(u,t) = x(t)u, t \in [0,T], u \in [0,\pi],$ of semilinear partial integrodifferential equation (10) − (12).

6 Conclusion

In this paper, the existence, uniqueness and continuous dependence of initial data on a mild solution of semilinear mixed Volterra-Fredholm functional integrodifferential equations with nonlocal conditions in general Banach space are discussed. We apply the concepts of semigroup theory and modified version of Banach contraction theorem. We also give example to illustrate the theory.

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