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Convex ordered Γ-semi hypergroups associated to strongly regular relations

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Abstract In this study, we introduce and investigate the notion of convex ordered Γ-semihypergroups associated to strongly regular relations. Afterwards, we prove that if σ is a strongly regular relation on a convex ordered Γ-semihypergroup $(S, \Gamma, \leq)$, then the quotient set $(S/\sigma, \Gamma_\sigma, \preceq)$ is an ordered Γ_σ-semigroup. Also, some results on the product of convex ordered Γ-semihypergroups are given. As an application of the results of this paper, the corresponding results of ordered semihypergroups are also obtained by moderate modifications.

Keywords Algebraic hyperstructure, Convex ordered Γ-semihypergroup, Strongly regular relation, Pseudoorder

AMS mathematics subject classification 16Y99.

1 Introduction

Hyperstructure theory was born in 1934 when Marty [1] defined hypergroups based on the notion of hyperoperation. Since then, hundreds of papers and several books have been written on this topic, for example, see [2–5].

Nowadays, the algebraic hyperstructures are studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics such as, geometry, topology, cryptography and code theory, graphs and hypergraphs, probability theory, binary relations, theory of fuzzy and rough sets, automata theory, economy, etc. (see [3,6,7]). A semihypergroup is an algebraic structure similar to a semigroup, but the composition of two elements is a non-empty set. As a reference for more definitions on semihypergroups we refer to [2]. One of motivations for the study of hyperstructures comes from chemical reactions. In [8], Davvaz and Dehghan-Nezhad provided examples of hyperstructures associated with chain reactions. In [9], Davvaz et al. introduced examples of weak hyperstructures associated with dismutation reactions. In [10], Davvaz et al. investigated the examples of hyperstructures and weak hyperstructures associated with redox reactions. Also, see [4, 11, 12] for more applications of hyperstructures in Chemistry. Another motivation for the study of hyperstructures comes from physical phenomenon as the nuclear fission. This motivation and the results were presented by S. Hošková, J. Chvalina and P. Račková (see [13], [14]).

The concept of ordering hypergroups was investigated by Chvalina [15] in 1994 as a special class of hypergroups. Many researchers are concerned about ordered hyperstructures because of nice connection between ordered structures and ordered hyperstructures. The concept of ordered semihypergroups is a generalization of the concept of ordered semigroups. To the best of my knowledge, the concept of ordered semihypergroups appeared for the first time in the work of Heidari and Davvaz [16]. As we know, an *ordered semihypergroup* $(S, \circ, \leq)$ is a semihypergroup $(S, \circ)$ together with a partial order relation $\leq$ that is compatible with the semihypergroup hyperoperation. Furthermore, Davvaz et al. [17] introduced pseudoorders in ordered semihypergroups, by extending the corresponding notions of ordered semigroup theory that was first studied by Kehayopulu and Tsingelis [18,19]. They investigated the properties of ordered semigroups derived from ordered semihypergroups. In [20], Gu and Tang introduced the notion of ordered regular equivalence relations of an ordered semihypergroup and studied some of their basic properties. In [20] the authors answered to the open problem given by Davvaz et al. [17]. In [21], Davvaz and Omidi defined the notion of an ordered semihyperring as a generalization of an ordered semiring.

A partial order on a set is an order which is reflexive, antisymmetric and transitive. A set with a partial order is called a partially ordered set. A preorder on an arbitrary nonempty set X is a binary relation on X which is reflexive and transitive. An antisymmetric preorder is said to be an order. In [22], Ganesamoorthy and Karpagavalli introduced and analyzed congruence relations in ordered sets. Sen and Saha [23] introduced Γ-semigroups as a generalization of semigroups. The notion of an ordered Γ-semigroup was first introduced by Sen and Seth [24] in 1993. Many authors studied different aspects of Γ-semigroups and ordered Γ-semigroups, for instance, Chinram and Tinpun [25], Hedayati [26], Hila [27], Kwon and Lee [28], and many others. The notion of Γ-H_ν-semiring introduced by Hedayati and Davvaz in [29]. In [30], Heidari et al. introduced the notion of a Γ-semihypergroup as a generalization of a semigroup, a generalization of a semihypergroup and a generalization of a Γ-semigroup. For examples and other basic properties of Γ-semihypergroups, we refer [30–33].

Theorem 1 [32] *Let σ be an equivalence relation on a Γ-semihypergroup S. Then σ is a strongly regular relation if and only if S/σ is a Γ_σ-semigroup.*

2 Convex ordered Γ-semihypergroups and some of their basic properties

The concept of ordered Γ-semihypergroups is a generalization of the concept of ordered Γ-semigroups. Omidi *et al.* [34] studied quasi-Γ-hyperideals and hyperfilters in ordered Γ-semihypergroups. For the convenience of the reader we repeat the relevant definitions from [34], thus making our paper self-contained. Throughout this paper, unless otherwise mentioned, S will denote an ordered Γ-semihypergroup.

Definition 1 *An algebraic hyperstructure $(S,\Gamma,\leq)$ is said to be an ordered Γ-semihypergroup if (S,Γ) is a Γ-semihypergroup and $(S,\leq)$ is an ordered set such that: for any $x,y,z \in S$ and $\gamma \in \Gamma$, $x \leq y$ implies $z\gamma x \preceq z\gamma y$ and $x\gamma z \preceq y\gamma z$. Here, $A \preceq B$ means that for any $a \in A$, there exists $b \in B$ such that $a \leq b$, for all non-empty subsets A and B of S.*

Let $(S,\circ,\leq)$ be an ordered semihypergroup and $\Gamma = \{\circ\}$. Then, $(S,\Gamma,\leq)$ is an ordered Γ-semihypergroup. Thus every ordered semihypergroup is an ordered Γ-semihypergroup.

Example 1 Let $S = [0,1]$ and $\Gamma = \mathbb{N}$. For every $x,y \in S$ and $\gamma \in \Gamma$, we define $\gamma : S \times \Gamma \times S \to \mathcal{P}^*(S)$ by

$$x\gamma y = \left[0, \frac{xy}{\gamma}\right].$$

Then, γ is a hyperoperation. For every $x,y,z \in S$ and $\alpha,\beta \in \Gamma$, we have

$$(x\alpha y)\beta z = \left[0, \frac{xyz}{\alpha\beta}\right] = x\alpha(y\beta z).$$

This means that S is a Γ-semihypergroup [31]. Consider S as a poset with the natural ordering. Thus, $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup. Here, $x\gamma z \preceq y\gamma z$ means that

$$\Big[0, \frac{xz}{\gamma}\Big] \subseteq \Big[0, \frac{yz}{\gamma}\Big],$$

for all $x, y, z \in S$, $\gamma \in \Gamma$ and $x \leq y$. In fact, we define a relation $\leq$ in S by $x \leq y$ if and only if

$$\Big[0, \frac{xz}{\gamma}\Big] \subseteq \Big[0, \frac{yz}{\gamma}\Big]$$

for all $x, y, z \in S$ and $\gamma \in \Gamma$. With respect to this relation S becomes an ordered Γ-semihypergroup.

Strongly regular relations are important in order to study the quotient structures. Let $\sigma \subseteq S \times S$ be an equivalence relation on an ordered Γ-semihypergroup S. If A and B are non-empty subsets of S, then $A\overline{\overline{\sigma}}B$ means that for all $a \in A$ and for all $b \in B$, we have $a\sigma b$. The equivalence relation σ is called *strongly regular* if for all $a, b, x \in S$ and $\gamma \in \Gamma$, from $a\sigma b$, it follows that $(a\gamma x)\overline{\overline{\sigma}}(b\gamma x)$ and $(x\gamma a)\overline{\overline{\sigma}}(x\gamma b)$. Let σ be an equivalence relation on S. Let $[a]_\sigma$ be the equivalence class of a with respect to σ and $S/\sigma = \{[a]_\sigma \mid a \in S\}$ be the quotient set. For every $\gamma \in \Gamma$, we define γ_σ on S/σ as follows:

$$[a]_\sigma \gamma_\sigma [b]_\sigma = \{[c]_\sigma \mid c \in a\gamma b\}.$$

Now, we set $\Gamma_\sigma = \{\gamma_\sigma \mid \gamma \in \Gamma\}$.

Example 2 Let $S = \{a, b, c, d\}$ and $\Gamma = \{\gamma, \beta\}$ be the sets of binary hyperoperations defined as follows:

γ	a	b	c	d
a	a	$\{a, b\}$	$\{c, d\}$	d
b	$\{a, b\}$	b	$\{c, d\}$	d
c	$\{c, d\}$	$\{c, d\}$	c	d
d	d	d	d	d

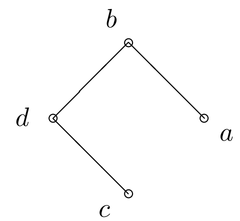
β	a	b	c	d
a	a	$\{a, b\}$	$\{c, d\}$	d
b	$\{a, b\}$	$\{a, b\}$	$\{c, d\}$	d
c	$\{c, d\}$	$\{c, d\}$	c	d
d	d	d	d	d

Clearly, S is a Γ-semihypergroup. We have $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup where the order relation $\leq$ is defined by:

$$\leq := \{(a, a), (a, b), (b, b), (c, b), (c, c), (c, d), (d, b), (d, d)\}.$$

The covering relation and the figure of S are given by:

$$\prec = \{(a, b), (c, d), (d, b)\}.$$



Suppose that σ is the relation on S defined as follows:

$$\sigma = \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,a),(c,d),(d,c)\}.$$

Clearly, σ is a strongly regular relation on S. Thus, $S/\sigma = \{u_1, u_2\}$, where $u_1 = \{a,b\}$ and $u_2 = \{c,d\}$. We define a relation $\preceq$ on S/σ by

$$\preceq := \{([x]_\sigma, [y]_\sigma) \in S/\sigma \times S/\sigma \mid \exists x_1 \in [x]_\sigma, \exists y_1 \in [y]_\sigma \text{ such that } (x_1, y_1) \in \leq\}.$$

Since $(d,b) \in \leq$, we obtain $u_2 \preceq u_1$. It is not difficult to verify that $(S/\sigma, \Gamma_\sigma, \preceq)$ is an ordered Γ_σ-semigroup, where γ_σ and β_σ are defined in the following tables:

γ_σ	u_1	u_2
u_1	u_1	u_2
u_2	u_2	u_2

β_σ	u_1	u_2
u_1	u_1	u_2
u_2	u_2	u_2

and $\preceq = \{(u_1,u_1),(u_2,u_1),(u_2,u_2)\}$.

Let σ be a strongly regular relation on an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. Then the quotient order relation defined on S/σ by setting

$$\preceq := \{([x]_\sigma, [y]_\sigma) \in S/\sigma \times S/\sigma \mid \exists x_1 \in [x]_\sigma, \exists y_1 \in [y]_\sigma \text{ such that } (x_1, y_1) \in \leq\}$$

is not necessarily an order on $(S/\sigma, \Gamma_\sigma)$. This is shown by the following example.

Example 3 Let $S = \{a,b,c,d,e\}$ and $\Gamma = \{\gamma, \beta\}$ be the sets of binary hyperoperations defined below:

γ	a	b	c	d	e
a	$\{a,b\}$	$\{b,c\}$	c	$\{d,e\}$	e
b	$\{b,c\}$	c	c	$\{d,e\}$	e
c	c	c	c	$\{d,e\}$	e
d	$\{d,e\}$	$\{d,e\}$	$\{d,e\}$	d	e
e	e	e	e	e	e

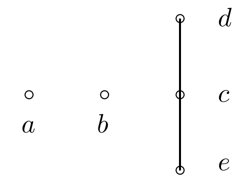
β	a	b	c	d	e
a	$\{b,c\}$	c	c	$\{d,e\}$	e
b	c	c	c	$\{d,e\}$	e
c	c	c	c	$\{d,e\}$	e
d	$\{d,e\}$	$\{d,e\}$	$\{d,e\}$	d	e
e	e	e	e	e	e

Clearly, S is a Γ-semihypergroup [33]. We have $(S,\Gamma,\leq)$ is an ordered Γ-semihypergroup where the order relation $\leq$ is defined by:

$$\leq:=\{(a,a),(b,b),(c,c),(d,d),(e,e),(c,d),(e,c),(e,d)\}.$$

The covering relation and the figure of S are given by:

$$\prec=\{(c,d),(e,c)\}.$$



Suppose that σ is the relation on S defined as follows:

$$\begin{aligned}\sigma &= \{(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(b,a),\\ &\quad (a,c),(c,a),(b,c),(c,b),(d,e),(e,d)\}.\end{aligned}$$

Clearly, σ is a strongly regular relation on S. Thus, $S/\sigma=\{u_1,u_2\}$, where $u_1=\{a,b,c\}$ and $u_2=\{d,e\}$. We have

$$u_1\preceq u_2 \text{ since } c\in u_1, d\in u_2, (c,d)\in\leq,$$

$$u_2\preceq u_1 \text{ since } e\in u_2, c\in u_1, (e,c)\in\leq .$$

Since $u_1\neq u_2$, it follows that $\preceq$ is not an order relation on S/σ.

We say that a preorder relation is a relation which satisfies conditions reflexivity and transitivity. We continue this section with the following theorem.

Theorem 2 *Let σ be a strongly regular equivalence relation on a preordered Γ-semihypergroup $(S,\Gamma,\leq)$. Then, $(S/\sigma,\Gamma_\sigma,\preceq)$ is a preordered Γ_σ-semigroup with respect to the following hyperoperation on the quotient set S/σ as follows:*

$$[a]_\sigma\gamma_\sigma[b]_\sigma=\{[c]_\sigma \mid c\in a\gamma b\},$$

where for all $[a]_\sigma,[b]_\sigma\in S/\sigma$ the preorder relation $\preceq$ is defined by the rule:

$$\preceq:=\{([a]_\sigma,[b]_\sigma)\in S/\sigma\times S/\sigma \mid \forall a_1\in[a]_\sigma, \exists b_1\in[b]_\sigma \text{ such that } (a_1,b_1)\in\leq\}.$$

Proof By Theorem 1, $(S/\sigma,\Gamma_\sigma)$ is a Γ_σ-semigroup. First, we show that the binary relation $\preceq$ is a preorder relation on S/σ. Since $a\leq a$, we have $[a]_\sigma\preceq[a]_\sigma$ for every $a\in S$. Thus $\preceq$ is reflexive. To prove transitivity, consider the relations $[a]_\sigma\preceq[b]_\sigma$ and $[b]_\sigma\preceq[c]_\sigma$ in S/σ. Take any $a_1\in[a]_\sigma$; then there exists $b_1\in[b]_\sigma$ such that $a_1\leq b_1$. For this $b_1\in[b]_\sigma$ there exists $c_1\in[c]_\sigma$ such that $b_1\leq c_1$. Hence for every $a_1\in[a]_\sigma$ there exists some $c_1\in[c]_\sigma$ such that $a_1\leq c_1$. This implies that $[a]_\sigma\preceq[c]_\sigma$. This proves the transitivity of the relation $\preceq$ in S/σ. Hence S/σ is a preordered set.

Suppose that $[a]_\sigma,[b]_\sigma,[x]_\sigma\in S/\sigma$ such that $[a]_\sigma\preceq[b]_\sigma$. If $[t]_\sigma=[x]_\sigma\gamma_\sigma[a]_\sigma$, then for every $t_1 \in [t]_\sigma$ there exist $x_1 \in [x]_\sigma$ and $a_1 \in [a]_\sigma$ such that $t_1 \in x_1\gamma a_1$. Since $a_1 \in [a]_\sigma \preceq [b]_\sigma$ there exists $b_1 \in [b]_\sigma$ such that $a_1 \leq b_1$. Hence $x_1\gamma a_1 \leq x_1\gamma b_1$. Thus there exists $s_1 \in x_1\gamma b_1$ such that $t_1 \leq s_1$. Therefore, $[t]_\sigma = [t_1]_\sigma \preceq [s_1]_\sigma = [x]_\sigma\gamma_\sigma[b]_\sigma$. Therefore, $(S/\sigma, \Gamma_\sigma, \preceq)$ is a preordered Γ_σ-semigroup. □

In the following, we introduce the notion of convex ordered Γ-semihypergroups associated to strongly regular relations and present some examples of them. We will also discuss the link between convex ordered Γ-semihypergroups and ordered Γ_σ-semigroups.

Definition 2 *Let σ be a strongly regular equivalence relation on an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. We say that S is a convex ordered Γ-semihypergroup associated to σ if the following correlation takes place:*

$$\forall x, y, z \in S,\ (x, z) \in \sigma \text{ and } x \leq y \leq z \Rightarrow (x, y) \in \sigma.$$

Remark 1 Consider the identity relation $\sigma_{id} = \{(a, b) \in S \times S \mid a = b\}$ on an ordered Γ-semihypergroup S. If our ordered Γ-semihypergroup is an ordered Γ-semigroup, i.e., our hyperoperations are binary operations, we conclude that S is a convex ordered Γ-semigroup associated to σ_{id}. However, this is not true for ordered Γ-semihypergroups, as shown in the following example. Note that the identity relation of an ordered Γ-semihypergroup is generally not strongly regular.

Example 4 Let $(S, \Gamma, \leq)$ be the ordered Γ-semihypergroup defined as in Example 3. Set $\sigma_{id} = \{(a, a), (b, b), (c, c), (d, d), (e, e)\}$. Clearly, σ_{id} is not a strongly regular relation of S. This implies that S is not a convex ordered Γ-semihypergroup associated to σ_{id}.

In the following, we present some examples of convex ordered Γ-semigroups.

Example 5 Let $S = \{a, b, c, d, e\}$ and $\Gamma = \{\gamma\}$. We define

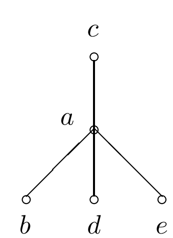
γ	a	b	c	d	e
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$	e
b	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$	e
c	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$	e
d	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$	e
e	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$	e

Then S is a Γ-semihypergroup. We have $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup where the order relation $\leq$ is defined by:

$$\begin{aligned} \leq \ := \{&(a, a), (b, b), (c, c), (d, d), (e, e), (a, c), (b, a), \\ &(b, c), (d, a), (d, c), (e, a), (e, c)\}. \end{aligned}$$

The covering relation and the figure of S are given by:

$$\prec = \{(a, c), (b, a), (d, a), (e, a)\}.$$



Let σ be strongly regular relation on S define as follows:

$$\sigma = \{(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(b,a), (a,c),(c,a),(a,d),(d,a),(b,c),(c,b),(b,d), (d,b),(c,d),(d,c)\}.$$

Then, $S/\sigma = \{u_1, u_2\}$, where $u_1 = \{a,b,c,d\}$ and $u_2 = \{e\}$. It is not difficult to see that S is a convex ordered Γ-semihypergroup.

Example 6 Let $S = \{a,b,c,d,e\}$ and $\Gamma = \{\gamma, \beta\}$ be the sets of binary hyperoperations defined below:

γ	a	b	c	d	e
a	$\{a,b\}$	$\{b,e\}$	c	$\{c,d\}$	e
b	$\{b,e\}$	e	c	$\{c,d\}$	e
c	c	c	c	c	c
d	$\{c,d\}$	$\{c,d\}$	c	d	$\{c,d\}$
e	e	e	c	$\{c,d\}$	e

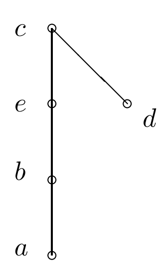
β	a	b	c	d	e
a	$\{b,e\}$	e	c	$\{c,d\}$	e
b	e	e	c	$\{c,d\}$	e
c	c	c	c	c	c
d	$\{c,d\}$	$\{c,d\}$	c	d	$\{c,d\}$
e	e	e	c	$\{c,d\}$	e

Clearly S is a Γ-semihypergroup [35]. We have $(S,\Gamma,\leq)$ is an ordered Γ-semihypergroup where the order relation $\leq$ is defined by:

$$\leq \; := \{(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(a,c), (a,e),(b,c),(b,e),(d,c),(e,c)\}.$$

The covering relation and the figure of S are given by:

$$\prec = \{(a,b),(b,e),(d,c),(e,c)\}.$$



Let σ be strongly regular relation on S define as follows:

$$\sigma = \{(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(b,a), (a,e),(e,a),(b,e),(e,b),(c,d),(d,c)\}.$$

Then, $S/\sigma = \{u_1, u_2\}$, where $u_1 = \{a,b,e\}$ and $u_2 = \{c,d\}$. We can easily verify that S is a convex ordered Γ-semihypergroup.

Example 7 Let $S = \{1, 2, 3, 4, 5, \cdots\}$ and $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_k\}$ for some $k \in \mathbb{N}$. For every $x, y \in S$ and $\gamma_i \in \Gamma$, we define $\gamma_i : S \times \Gamma \times S \to \mathcal{P}^*(S)$ by $x\gamma_i y = \{z \in S \mid z \leq \min\{x, i, y\}\}$. Then, γ_i is a hyperoperation. Clearly, S is a Γ-semihypergroup. Consider S as a poset with the natural ordering. Thus, $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup. Here, $x\gamma_i z \preceq y\gamma_i z$ means that $\{u \in S \mid u \leq \min\{x, i, z\}\} \subseteq \{v \in S \mid v \leq \min\{y, i, z\}\}$, for all $x, y, z \in S$, $\gamma_i \in \Gamma$ and $x \leq y$. In fact, we define a relation $\leq$ in S by $x \leq y$ if and only if $\{u \in S \mid u \leq \min\{x, i, z\}\} \subseteq \{v \in S \mid v \leq \min\{y, i, z\}\}$ for all $x, y, z \in S$ and $\gamma_i \in \Gamma$. With respect to this relation S becomes an ordered Γ-semihypergroup. Consider

$$\sigma = \{(a, b) \in S \times S \mid a = b\} \cup \{(1, 2), (2, 1)\}.$$

By routine checking, we can easily verify that σ is a strongly regular relation on S. It is not difficult to verify that S is a convex ordered Γ-semihypergroup.

Now, we present an example of an ordered Γ-semihypergroup which is not a convex ordered Γ-semihypergroup.

Example 8 Assume that we want to consider the ordered Γ-semihypergroup S given in Example 3. By routine calculations, $(S, \Gamma, \leq)$ is not a convex ordered Γ-semihypergroup. Indeed:

$$(e, d) \in \sigma \text{ and } e \leq c \leq d \text{ but } (e, c) \notin \sigma.\text{e}$$

Using Theorem 2, we obtain the main result of this paper.

Theorem 3 *Let σ be a strongly regular equivalence relation on a convex ordered Γ-semihypergroup $(S, \Gamma, \leq)$. Then, $(S/\sigma, \Gamma_\sigma, \preceq)$ is an ordered Γ_σ-semigroup with respect to the following hyperoperation on the quotient set S/σ as follows:*

$$[a]_\sigma \gamma_\sigma [b]_\sigma = \{[c]_\sigma \mid c \in a\gamma b\},$$

where for all $[a]_\sigma, [b]_\sigma \in S/\sigma$ the order relation $\preceq$ is defined by the rule:

$$\preceq := \{([a]_\sigma, [b]_\sigma) \in S/\sigma \times S/\sigma \mid \forall a_1 \in [a]_\sigma, \exists b_1 \in [b]_\sigma \text{ such that } (a_1, b_1) \in \leq\}.$$

Proof By Theorem 2, $(S/\sigma, \Gamma_\sigma, \preceq)$ is a preordered Γ_σ-semigroup. It remains to prove only that $\preceq$ is antisymmetric. To see this, let $[a]_\sigma \preceq [b]_\sigma$ and $[b]_\sigma \preceq [a]_\sigma$ in S/σ. Take $a \in [a]_\sigma$; then there exists $b_1 \in [b]_\sigma$ such that $a \leq b_1$. For this $b_1 \in [b]_\sigma$ there exists $a_1 \in [a]_\sigma$ such that $b_1 \leq a_1$. This implies that $a \leq b_1 \leq a_1$. Since S is a convex ordered Γ-semihypergroup, it follows that $[a]_\sigma = [b]_\sigma$. Hence the proof is completed. □

Example 9 We come back to Example 5 and consider convex ordered Γ-semihypergroup $(S, \Gamma, \leq)$. By routine checking, we can easily verify that $(S/\sigma, \Gamma_\sigma, \preceq)$ is an ordered Γ_σ-semigroup, where γ_σ is defined in the following table:

γ_σ	u_1	u_2
u_1	u_1	u_2
u_2	u_1	u_2

and $\preceq = \{(u_1, u_1), (u_2, u_1), (u_2, u_2)\}$.

Example 10 Let $(S, \Gamma, \leq)$ be the convex ordered Γ-semihypergroup defined as in Example 6. It is not difficult to verify that $(S/\sigma, \Gamma_\sigma, \preceq)$ is an ordered Γ_σ-semigroup, where γ_σ and β_σ are defined in the following tables:

γ_σ	u_1	u_2
u_1	u_1	u_2
u_2	u_2	u_2

β_σ	u_1	u_2
u_1	u_1	u_2
u_2	u_2	u_2

and $\preceq = \{(u_1, u_1), (u_1, u_2), (u_2, u_2)\}$.

Definition 3 *Let $(S, \Gamma, \leq_S)$ and $(T, \Gamma, \leq_T)$ be two ordered Γ-semihypergroups. The mapping $\varphi : S \to T$ is called a homomorphism if for all $a, b \in S$ and $\gamma \in \Gamma$, we have*

(1) $\varphi(a\gamma b) \subseteq \varphi(a)\gamma\varphi(b)$*;*

(2) φ *is isotone, that is, for any $a, b \in S$, $a \leq_S b$ implies $\varphi(a) \leq_T \varphi(b)$.*

If the condition (1) holds for the equality instead of the inclusion, φ is said to be a good homomorphism. An *isomorphism* from S into T is a bijective good homomorphism from S onto T. If S is isomorphic to T, then it is denoted by $S \cong T$.

We will consider ordered Γ-semihypergroups that satisfy the following condition: If $a \leq b$ and $(a, a_1) \in \sigma$, then there is an element $b_1 \in S$ such that $(b, b_1) \in \sigma$ and $a_1 \leq b_1$. The natural map of S onto S/σ is $\pi : S \to S/\sigma$ given by $\pi(x) = [x]_\sigma$. Let $a \leq b$. Then, by the previous condition, for given $a_1 \in [a]_\sigma$, there is a $b_1 \in [b]_\sigma$ such that $a_1 \leq b_1$. So, we have $[a]_\sigma \preceq [b]_\sigma$. Hence, the mapping π is isotone.

Example 11 Let $S = \{a, b, c, d, e\}$ and $\Gamma = \{\gamma\}$. We define

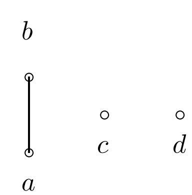
γ	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	$\{a, b\}$	a
d	a	a	$\{a, b\}$	$\{a, b\}$

Then S is a Γ-semihypergroup. We have $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup where the order relation $\leq$ is defined by:

$$\leq \quad := \{(a, a), (b, b), (c, c), (d, d), (a, b)\}.$$

The covering relation and the figure of S are given by:

$$\prec = \{(a, b)\}.$$



Let σ be strongly regular relation on S define as follows:

$$\begin{aligned}\sigma &= \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,a),(a,c),\\ &\quad (c,a),(b,c),(c,b)\}.\end{aligned}$$

Then, $S/\sigma = \{u_1, u_2\}$, where $u_1 = \{a,b,c\}$ and $u_2 = \{d\}$. It is easy to see that the natural map of S onto S/σ is isotone.

A key property of the product of ordered Γ-semihypergroups is depicted in the following.

Theorem 4 *Let $(S_i, \Gamma_i, \leq_i)$ be an ordered Γ_i-semihypergroup for all $i \in I$. Then, $\prod_{i\in I} S_i = \{(x_i)_{i\in I} \mid x_i \in S_i\}$ is an ordered $\prod_{i\in I}\Gamma_i$-semihypergroup.*

Proof Define $\odot : (\prod_{i\in I} S_i) \times (\prod_{i\in I}\Gamma_i) \times (\prod_{i\in I} S_i) \to \mathcal{P}^*(\prod_{i\in I} S_i)$ by $(x_i)_{i\in I} \odot (\gamma_i)_{i\in I} \odot (y_i)_{i\in I} = \{(z_i)_{i\in I} \mid z_i \in x_i\gamma_i y_i\}$, for all $(x_i)_{i\in I}, (y_i)_{i\in I} \in \prod_{i\in I} S_i$ and $(\gamma_i)_{i\in I} \in \prod_{i\in I}\Gamma_i$. It is easy to verify that $\odot$ is well defined. The order defined on $\prod_{i\in I} S_i$ as follows:

$$(x_i)_{i\in I} \leq (y_i)_{i\in I} \text{ if and only if, for all } i \in I,\ x_i \leq_i y_i.$$

Now, we prove that $(\prod_{i\in I} S_i, \prod_{i\in I}\Gamma_i, \leq)$ is an ordered $\prod_{i\in I}\Gamma_i$-semihypergroup. We have

$$\begin{aligned}
&(x_i)_{i\in I} \odot (\alpha_i)_{i\in I} \odot ((y_i)_{i\in I} \odot (\beta_i)_{i\in I} \odot (z_i)_{i\in I})\\
&= (x_i)_{i\in I} \odot (\alpha_i)_{i\in I} \odot \{(u_i)_{i\in I} \mid u_i \in y_i\beta_i z_i\}\\
&= \{(v_i)_{i\in I} \mid v_i \in x_i\alpha_i u_i, u_i \in y_i\beta_i z_i\}\\
&= \{(v_i)_{i\in I} \mid v_i \in x_i\alpha_i(y_i\beta_i z_i)\}\\
&= \{(v_i)_{i\in I} \mid v_i \in (x_i\alpha_i y_i)\beta_i z_i\}\\
&= \{(v_i)_{i\in I} \mid v_i \in w_i\beta_i z_i, w_i \in x_i\alpha_i y_i\}\\
&= \{(w_i)_{i\in I} \mid w_i \in x_i\alpha_i y_i\} \odot (\beta_i)_{i\in I} \odot (z_i)_{i\in I}\\
&= ((x_i)_{i\in I} \odot (\alpha_i)_{i\in I} \odot (y_i)_{i\in I}) \odot (\beta_i)_{i\in I} \odot (z_i)_{i\in I}.
\end{aligned}$$

Now, suppose that $(x_i)_{i\in I} \leq (y_i)_{i\in I}$ for $(x_i)_{i\in I}, (y_i)_{i\in I} \in \prod_{i\in I} S_i$. If $(t_i)_{i\in I} \in (a_i)_{i\in I} \odot (\gamma_i)_{i\in I} \odot (x_i)_{i\in I}$, where $(a_i)_{i\in I} \in \prod_{i\in I} S_i$ and $(\gamma_i)_{i\in I} \in \prod_{i\in I}\Gamma_i$, then $t_i \in a_i\gamma_i x_i$. Since $(x_i)_{i\in I} \leq (y_i)_{i\in I}$, it follows that $x_i \leq_i y_i$ for all $i \in I$. By hypothesis, we have $t_i \in a_i\gamma_i x_i \leq_i a_i\gamma_i y_i$. So, there exists $s_i \in a_i\gamma_i y_i$ such that $t_i \leq_i s_i$. Thus we have $(t_i)_{i\in I} \leq (s_i)_{i\in I}$. This implies that $(a_i)_{i\in I} \odot (\gamma_i)_{i\in I} \odot (x_i)_{i\in I} \leq (a_i)_{i\in I} \odot (\gamma_i)_{i\in I} \odot (y_i)_{i\in I}$. Therefore, $\prod_{i\in I} S_i$ is an ordered $\prod_{i\in I}\Gamma_i$-semihypergroup and the proof is completed. □

In view of the Theorem 4, we have the following:

Theorem 5 *Let $(S_i, \Gamma_i, \leq_i)$ be a convex ordered Γ_i-semihypergroup for all $i \in I$. Then, $\prod_{i\in I} S_i$ is a convex ordered $\prod_{i\in I}\Gamma_i$-semihypergroup.*

Proof Let σ_i be a strongly regular relation on S_i for all $i \in I$. Then σ is a strongly regular relation on $\prod_{i\in I} S_i$ where $(x_i)_{i\in I}\sigma(y_i)_{i\in I}$ if and only if $x_i\sigma_i y_i$ for all $x_i, y_i \in S_i$ and $i \in I$. Indeed: If $(x_i)_{i\in I}\sigma(y_i)_{i\in I}$, then $x_i\sigma_i y_i$ for all $x_i, y_i \in S_i$. Hence, $(x_i\gamma_i z_i)\overline{\overline{\sigma_i}}(y_i\gamma_i z_i)$ for all $z_i \in S_i$, $\gamma_i \in \Gamma_i$ and $i \in I$. Thus, $((x_i)_{i\in I} \odot (\gamma_i)_{i\in I} \odot (z_i)_{i\in I})\overline{\overline{\sigma}}((y_i)_{i\in I} \odot (\gamma_i)_{i\in I} \odot (z_i)_{i\in I})$.

Similarly, $((z_i)_{i\in I}\odot(\gamma_i)_{i\in I}\odot(x_i)_{i\in I})\overline{\overline{\sigma}}((z_i)_{i\in I}\odot(\gamma_i)_{i\in I}\odot(y_i)_{i\in I})$. Therefore, σ is a strongly regular relation on $\prod_{i\in I} S_i$. Now, let $((x_i)_{i\in I},(z_i)_{i\in I})\in\sigma$ and $(x_i)_{i\in I}\leq(y_i)_{i\in I}\leq(z_i)_{i\in I}$. Then, $(x_i,z_i)\in\sigma_i$ and $x_i\leq_i y_i\leq_i z_i$. Since S_i is a convex ordered Γ_i-semihypergroup for all $i\in I$, it follows that $(x_i,y_i)\in\sigma_i$. So, we obtain $((x_i)_{i\in I},(y_i)_{i\in I})\in\sigma$. This completes the proof. □

Let $(S,\Gamma,\leq)$ be an ordered Γ-semihypergroup. A relation σ on S is called a *pseudoorder* on S if (1) $\leq\subseteq\sigma$; (2) $a\sigma b$ and $b\sigma c$ imply $a\sigma c$ for all $a,b,c\in S$; (3) $a\sigma b$ implies $a\gamma c\overline{\overline{\sigma}}b\gamma c$ and $c\gamma a\overline{\overline{\sigma}}c\gamma b$ for all $a,b,c\in S$ and $\gamma\in\Gamma$. In the following, we investigate the behavior of pseudoorders on the product of ordered Γ-semihypergroups.

Lemma 1 *Let σ_i be a pseudoorder on the ordered Γ_i-semihypergroup $(S_i,\Gamma_i,\leq_i)$ for all $i\in I$. Then, σ is a pseudoorder on $\prod_{i\in I} S_i$ where $(x_i)_{i\in I}\sigma(y_i)_{i\in I}$ if and only if $x_i\sigma_i y_i$ for all $x_i,y_i\in S_i$ and $i\in I$.*

Proof Let $((x_i)_{i\in I},(y_i)_{i\in I})\in\leq$. Then $(x_i,y_i)\in\leq_i\subseteq\sigma_i$ for all $i\in I$. So, we have $((x_i)_{i\in I},(y_i)_{i\in I})\in\sigma$. Thus, $\leq\subseteq\sigma$. Let $((x_i)_{i\in I},(y_i)_{i\in I})\in\sigma$ and $((y_i)_{i\in I},(z_i)_{i\in I})\in\sigma$. Then, $(x_i,y_i)\in\sigma_i$ and $(y_i,z_i)\in\sigma_i$ for all $i\in I$. Hence, $(x_i,z_i)\in\sigma_i$ and so $((x_i)_{i\in I},(z_i)_{i\in I})\in\sigma$. Now, let $((x_i)_{i\in I},(y_i)_{i\in I})\in\sigma$ and $(z_i)_{i\in I}\in\prod_{i\in I} S_i$. Then, $(x_i,y_i)\in\sigma_i$ for all $i\in I$. Since σ_i is pseudoorder on S_i, we obtain $(x_i\gamma_i z_i)\overline{\overline{\sigma_i}}(y_i\gamma_i z_i)$ for all $\gamma_i\in\Gamma_i$ and $i\in I$. Hence, for all $u_i\in x_i\gamma_i z_i$ and for all $v_i\in y_i\gamma_i z_i$, we have $(u_i,v_i)\in\sigma_i$. This implies that $((u_i)_{i\in I},(v_i)_{i\in I})\in\sigma$. So, we have

$$(x_i)_{i\in I}\odot(\gamma_i)_{i\in I}\odot(z_i)_{i\in I}\overline{\overline{\sigma}}(y_i)_{i\in I}\odot(\gamma_i)_{i\in I}\odot(z_i)_{i\in I}.$$

By a similar argument, we obtain

$$(z_i)_{i\in I}\odot(\gamma_i)_{i\in I}\odot(x_i)_{i\in I}\overline{\overline{\sigma}}(z_i)_{i\in I}\odot(\gamma_i)_{i\in I}\odot(y_i)_{i\in I}.$$

Therefore, σ is a pseudoorder on $\prod_{i\in I} S_i$. □

Theorem 6 *Let $(S,\Gamma,\leq)$ be an ordered Γ-semihypergroup and σ a pseudoorder on S. Then, there exists a strongly regular equivalence relation $\sigma^*=\{(a,b)\in S\times S\mid a\sigma b \text{ and } b\sigma a\}$ on S such that $(S/\sigma^*,\Gamma_{\sigma^*},\preceq_{\sigma^*})$ is an ordered Γ_{σ^*}-semigroup, where $\preceq_{\sigma^*}:=\{(\sigma^*(x),\sigma^*(y))\in S/\sigma^*\times S/\sigma^*\mid \exists a\in\sigma^*(x),\exists b\in\sigma^*(y) \text{ such that } (a,b)\in\sigma\}$.*

Proof See Omidi *et al.* [34]. □

Now, we are in the position to end this paper as follows:

Theorem 7 *Let σ_i be a pseudoorder on ordered Γ_i-semihypergroup $(S_i,\Gamma_i,\leq_i)$ for all $i\in I$ and σ the pseudoorder on $\prod_{i\in I} S_i$ mentioned in Lemma 1. Then,*

$$\Big(\prod_{i\in I} S_i\Big)/\sigma^*\cong\prod_{i\in I}(S_i/\sigma_i^*).$$

Proof Let σ_i be a pseudoorder on ordered Γ_i-semihypergroup $(S_i,\Gamma_i,\leq_i)$ for all $i\in I$, respectively. On $\prod_{i\in I} S_i$ we define

$$(x_i)_{i\in I}\sigma(y_i)_{i\in I}\Leftrightarrow x_i\sigma_i y_i,\forall x_i,y_i\in S_i.$$

By Lemma 1, σ is a pseudoorder on $\prod_{i\in I} S_i$. By Theorems 6 and 4, $(\prod_{i\in I} S_i)/\sigma^*$ and $\prod_{i\in I}(S_i/\sigma_i^*)$ are ordered $\prod_{i\in I}\Gamma_i$-semigroups. Define $\psi : \prod_{i\in I}(S_i/\sigma_i^*) \to (\prod_{i\in I} S_i)/\sigma^*$ by $\psi((\sigma_i^*(x_i))_{i\in I}) = \sigma^*((x_i)_{i\in I})$ for all $x_i \in S_i$ and $i \in I$. We prove that ψ is an isomorphism. We have

$$\begin{aligned}(\sigma_i^*(x_i))_{i\in I} = (\sigma_i^*(y_i))_{i\in I} \quad & \Leftrightarrow \sigma_i^*(x_i) = \sigma_i^*(y_i), \text{ for all } i \in I \\ & \Leftrightarrow x_i\sigma_i^* y_i, \text{ for all } i \in I \\ & \Leftrightarrow x_i\sigma_i y_i, y_i\sigma_i x_i, \text{ for all } i \in I \\ & \Leftrightarrow ((x_i)_{i\in I}, (y_i)_{i\in I}) \in \sigma, ((y_i)_{i\in I}, (x_i)_{i\in I}) \in \sigma \\ & \Leftrightarrow ((x_i)_{i\in I}, (y_i)_{i\in I}) \in \sigma^* \\ & \Leftrightarrow \sigma^*((x_i)_{i\in I}) = \sigma^*((y_i)_{i\in I}) \\ & \Leftrightarrow \psi((\sigma_i^*(x_i))_{i\in I}) = \psi((\sigma_i^*(y_i))_{i\in I}).\end{aligned}$$

Therefore, ψ is well defined and one to one. Clearly, ψ is onto. Now, we prove that ψ is a homomorphism. Suppose that $(\sigma_i^*(x_i))_{i\in I}$ and $(\sigma_i^*(y_i))_{i\in I}$ are two arbitrary elements of $\prod_{i\in I}(S_i/\sigma_i^*)$. Then,

$$\begin{aligned}&\psi((\sigma_i^*(x_i))_{i\in I} \odot (\gamma_i)_{i\in I} \odot (\sigma_i^*(y_i))_{i\in I}) \\ &= \psi(\{(\sigma_i^*(z_i))_{i\in I} \mid z_i \in \sigma_i^*(x_i) \circ (\gamma_i)_{i\in I} \circ \sigma_i^*(y_i), \forall i \in I\} \\ &= \{\psi((\sigma_i^*(w_i))_{i\in I} \mid w_i \in x_i\gamma_i y_i, \forall i \in I\} \\ &= \{\sigma^*((w_i)_{i\in I}) \mid w_i \in x_i\gamma_i y_i, \forall i \in I\} \\ &= \sigma^*((x_i)_{i\in I}) \odot (\gamma_i)_{i\in I} \odot \sigma^*((y_i)_{i\in I}) \\ &= \psi((\sigma_i^*(x_i))_{i\in I}) \circ (\gamma_i)_{i\in I} \circ \psi((\sigma_i^*(y_i))_{i\in I}).\end{aligned}$$

Now, suppose that $(\sigma_i^*(x_i))_{i\in I} \preceq (\sigma_i^*(y_i))_{i\in I}$. Then, $\sigma_i^*(x_i) \leq_i \sigma_i^*(y_i)$ for all $i \in I$. Thus, $x_i\sigma_i y_i$ for all $i \in I$. Hence, $(x_i)_{i\in I}\sigma(y_i)_{i\in I}$. This means that $\sigma^*((x_i)_{i\in I}) \preceq \sigma^*((y_i)_{i\in I})$ So, we have $\psi((\sigma_i^*(x_i))_{i\in I}) \preceq \psi((\sigma_i^*(y_i))_{i\in I})$ Therefore, ψ is an isomorphism. This completes the proof of the theorem. □

Remark 2 If σ^* is a strongly regular relation mentioned in Theorem 6, then $(S/\sigma^*, \Gamma_{\sigma^*}, \preceq_{\sigma^*}$) becomes a convex ordered Γ_{σ^*}-semigroup. Indeed: Let $x \leq y \leq z$ with $(x, z) \in \sigma^*$. Then $[x]_{\sigma^*} \preceq_{\sigma^*} [y]_{\sigma^*}$ and $[y]_{\sigma^*} \preceq_{\sigma^*} [z]_{\sigma^*} = [x]_{\sigma^*}$. Applying antisymmetry of $\preceq_{\sigma^*}$, we coclude $[x]_{\sigma^*} = [y]_{\sigma^*}$. It follows that $(x, y) \in \sigma^*$. Hence S/σ^* is convex.

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