MATEMATIKA, 2018, Volume 34, Number 1, 23–30

MHD Flow and Heat Transfer in a Moving Conducting Inclined Channel

Mekonnen Shiferaw Ayno

Department of Mathematics, University of Swaziland
M201 Kwalusini, Swaziland
mekk_aya@yahoo.com

Article history
Received: 1 July 2016
Received in revised form: 6 June 2017
Accepted: 24 October 2017
Published on line: 1 June 2018

Abstract The problem of unsteady laminar, incompressible, electrically conducting micropolar fluid between inclined channel of rectangular cross-section was studied. It is assumed the flow is under the influence of transverse magnetic field and the walls of the channel have constant temperatures and finite conductivity. The numerical solutions was obtained for the velocity, magnetic field profile, microrotation and temperature fields for various parametric conditions. These results are illustrated graphically to illustrate the effects of the physical parameters governing the flow. It is found that velocity, magnetic field profile and microrotations promote the motion of the fluid with increase the wall conductance. It is also found that both velocity and magnetic field decreases with magnetic parameter increases.

Keywords Magnetohydrodynamics (MHD); unsteady; micropolar fluid; wall conductance.

Mathematics Subject Classification 76W99, 76U05.

1 Introduction

The study of flow through channels with applied magnetic field and heat transfer is receiving considerable interest in the literature because of its wide applicabilities. Among many applications: solar technology, safety aspects of gas cooled reactors, accelerators and crystal growth in liquids, etc can be mentioned. The study of Couette flow in a channel of an electrically conducting fluid under the action of a transversely applied magnetic field has some important applications in transpiration cooling in turbojet and rocket engines, like combustion chamber walls, exhaust nozzles and gas turbine blades.

Remarkable attempts have been made to study the influence of MHD on various flow geometries. Flow between two inclined planes with/ without considering Magnetohydrodynamics (MHD) effect studied by different researchers. Malashetty and Umavathi [1] and Malashetty *et al.* [2] studied MHD two-fluid flow models in an inclined channels. Sanyal and Sanyal [3] analyzed the hydromagnetic slip flow with heat transfer in an inclined channel. Aiyesimi *et al.* [4]

studied the combine effects of magnetic field on the MHD flow of a third grade fluid through inclined channel in the presence of a uniform magnetic field by considering heat transfer. The mixed convection heat transfer in an open ended inclined channels with reversal was studied by Rheault and Bilgen [5]. The Soret effects due to natural convection between heated inclined plates with magnetic field was analyzed by Raju *et al.* [6]. Daniel and Daniel [7] studied convective flow of two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient. The fully developed mixed convection flow between inclined infinite parallel porous plates filled with porous medium was discussed by Cimpean [8]. Chang and Lundgren [9] considered the effects of perfectly conducting walls on the flow in a rectangular duct. Roy and DAS [10] examine the effects of wall conduction for MHD flow with heat transfer in and inclined plane.

A micropolar fluid is the fluid with internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. The classical theories of continuum mechanics are inadequate to explain the microscopic manifestations of such complex hydrodynamic behavior. The heat transfer in micropolar fluids is also important in various applications. The theory of micropolar fluid formulated by Eringen [11] can be used to analyze the behavior of many of the fluids involved in technical processes and engineering applications, polymers, flows of exotic lubricants, animal bloods and real fluids with suspensions. An excellent review of micropolar fluids and their applications was given by Ariman et al. [12].

In this problem magnetohydrodynamic unsteady heat transfer flow of microplar fluid between inclined channels taking into account the induced magnetic field and moving wall conductance is considered. The resulting governing equations are solved numerically for non-dimensional velocity, magnetic field, microrotation and temperature profiles presented for values of the parameters characterizing the flow.

2 Mathematical Formulation of the Problem

Let us consider an unsteady incompressible and electrically conducting micropolar fluid through an incline channel with inclination α flows in porous channels rectangular cross-section with the thickness of lower plate h_1 and that of the upper plate is h_2. The distance between the porous walls is considered d. Assume a uniform magnetic field strength B_0 is applied along the y-axis. It is assumed also fluid injected in one of the wall and an equal rate suction in the other, the constant temperature, conductivity and magnetic permeability of the lower plate is of T_1, σ_1 and μ_1 respectively and for the upper plate T_2, σ_2 and μ_2. Let conductivity and magnetic permeability of the fluid be σ_3 and μ_3 respectively. The flow is assumed unidirectional and fully developed, the Boussinesq approximation is employed for the density variation, and driven by the constant pressure gradient $(-\frac{\partial p}{\partial x})$ further the upper plate moves with a constant velocity U_0 while the lower plate is kept stationary.

Within the framework of the above-noted assumptions, the appropriate conservation equations can be described following Roy [10] and Sutton [13] as

$$(\mu+\kappa)\frac{\partial^2 u}{\partial y^2}+\kappa\frac{\partial \Gamma}{\partial y}+\rho g\beta(T-T_2)\sin\alpha+\frac{B_0}{\mu_0}\frac{\partial B}{\partial y}-\frac{dp}{dx}=\rho\frac{\partial u}{\partial t}+\rho v_0\frac{\partial u}{\partial y} \tag{1}$$

$$\frac{1}{\mu_0\sigma}\frac{\partial^2 B}{\partial y^2}+B_0\frac{\partial u}{\partial y}=\frac{\partial B}{\partial t}+v_0\frac{\partial B}{\partial y} \tag{2}$$

$$\gamma\frac{\partial^2\Gamma}{\partial y^2}-\kappa\frac{\partial u}{\partial y}-2\kappa\Gamma=\rho\ j\frac{\partial\Gamma}{\partial t}+\rho\ jv_0\frac{\partial\Gamma}{\partial y} \tag{3}$$

$$k_f\frac{\partial^2T}{\partial y^2}+(2\mu+\kappa)\big(\frac{\partial u}{\partial y}\big)^2+\gamma\frac{\partial\Gamma}{\partial y}\big)^2+2\kappa\big(\frac{1}{2}\frac{\partial u}{\partial y}-\Gamma\big)^2+\frac{1}{\mu^2\sigma}\big(\frac{\partial B}{\partial y}\big)^2=\ \rho C_p\frac{\partial T}{\partial t}+\rho v_0\frac{\partial T}{\partial y}. \tag{4}$$

Initial conditions:

$$u(y,0)=B(y,0)=\Gamma(y,0)=T(y,0)=0. \tag{5}$$

The boundary conditions:

$$\begin{aligned}u(0,t)=\Gamma(0,t)=\phi_1B(0,t)+\frac{\partial B}{\partial y}(0,t)=0,\quad T(0,t)=T_1,\qquad t>0,\\ u(h,t)=U_0,\quad \Gamma(h,t)=\phi_2B(h,t)+\frac{\partial B}{\partial y}(0,t)=0,\quad T(h,t)=T_2,\qquad t>0.\end{aligned} \tag{6}$$

where $\phi_1=\frac{\sigma_3\mu_3}{\sigma_2\mu_2}$, $\phi_2=\frac{\sigma_3\mu_3}{\sigma_1\mu_1}$, ρ is density, u is fluid velocity, B is magnetic field, Γ is microrotation, μ is the dynamic viscosity, κ is the gyroviscosity, j is the microinertia, k_f is the thermal conductivity of the substance, γ is material constant and C_p is the specific heat.

Introduce the non dimensional variables through

$$\begin{aligned}\hat{u}=\frac{u}{U_0},\quad \hat{\Gamma}=\frac{\Gamma d}{U_0},\quad \hat{B}=\frac{B}{U_0},\\ \theta=\frac{T-T_1}{T_2-T_1},\quad \hat{t}=\frac{tU_0}{d},\quad \hat{y}=\frac{y}{d}.\end{aligned} \tag{7}$$

Substituting equation (7) into equations (1-4), after dropping the hat we get the following non-dimensional equations

$$\frac{1}{(1-C)Re}\frac{\partial^2u}{\partial y^2}+\frac{C}{(1-C)Re}\frac{\partial\Gamma}{\partial y}+\frac{Gr}{Re^2}\sin\alpha\ \theta+S\frac{\partial B}{\partial y}-P=\frac{\partial u}{\partial t}+\frac{R}{Re}\frac{\partial u}{\partial y} \tag{8}$$

$$\frac{1}{R_m}\frac{\partial^2B}{\partial y^2}+\frac{\partial u}{\partial y}=\frac{\partial B}{\partial t}+\frac{R}{Re}\frac{\partial B}{\partial y} \tag{9}$$

$$\frac{2-C}{M^2\ a_j\ Re}\frac{\partial^2\Gamma}{\partial y^2}-\frac{1}{(1-C)a_j\ Re}\big(-2\Gamma-\frac{\partial u}{\partial y}\big)=\frac{\partial\Gamma}{\partial t}+\frac{R}{Re}\frac{\partial\Gamma}{\partial y} \tag{10}$$

$$\begin{aligned}\frac{1}{P_r\ Re}\frac{\partial^2\theta}{\partial y^2}+E_c\ Re(2-C)\Big[\frac{1}{1-C}\big(\frac{\partial u}{\partial y}\big)^2+\frac{1}{M^2}\big(\frac{\partial\Gamma}{\partial y}\big)^2\Big]\\ +\frac{2Re\ E_c}{1-C}\big(\frac{1}{2}\frac{\partial u}{\partial y}-\Gamma\big)^2+\frac{E_c\ S}{R_m}\big(\frac{\partial B}{\partial y}\big)^2=\ \frac{\partial\theta}{\partial t}+\frac{R}{Re}\frac{\partial\theta}{\partial y}\end{aligned} \tag{11}$$

where

$$C = \frac{\kappa}{\mu + \kappa} \text{ (coupling number)},$$
$$Re = \frac{\rho U_0 d}{\mu} \text{ (Reynolds number)},$$
$$G_r = \frac{d^3 g\beta(T_1 - T_2)}{\upsilon^2} \text{ (Grashof number)},$$
$$S = \frac{B_0^2}{\rho\mu_0 U_0^2} \text{ (magnetic force number)},$$
$$P = \frac{\partial p}{\partial x}\frac{d}{\rho U_0^2} \text{ (nondimensional pressure gradient)},$$
$$R = \frac{\rho v_0 d}{\mu} \text{ (suction parameter)},$$
$$R_m = \mu_0 \sigma d U_0 \text{ (magnetic Reynolds number)},$$
$$M = \frac{d^2\kappa(2\mu + \kappa)}{\gamma(\mu + \kappa)} \text{ (micropolar parameter)},$$
$$a_j = \frac{j}{d^2} \text{ (micro-inertia density parameter)},$$
$$P_r = \frac{\mu C_p}{k_f} \text{ (Prandtl number)},$$
$$E_c = \frac{\mu^2}{\rho^2 C_p d^2 (T_2 - T_1)} \text{ (Eckert number)}.$$

Initial conditions:

$$u(y,0) = B(y,0) = \Gamma(y,0) = \theta(y,0) = 0. \tag{12}$$

The boundary conditions:

$$u(0,t) = 1, \quad \Gamma(0,t) = 0, \quad \phi_1 B(0,t) + \frac{\partial B}{\partial y}(0,t) = 0, \quad \theta(0,t) = 1, \qquad t > 0,$$
$$u(1,t) = 0, \quad \Gamma(1,t) = 0, \quad \phi_2 B(1,t) + \frac{\partial B}{\partial y}(1,t) = 0, \quad \theta(1,t) = 0, \qquad t > 0. \tag{13}$$

3 Results and Discussion

The non linear equations (8)- (11) together with the initial and boundary conditions (12) and (13) are solved numerically using Matlab pdepe. Since the problem involves many parameters, the parameters $P_r = 0.01, Re = 1, R = -2.0, Ec = 0.02, R_m = 0.5, C = 0.5, \alpha = 30^0, G_r = 1.0, M = 3, a_j = 0.1$ are fixed to analysis the effects of the other parameters on velocity, magnetic field, microrotation and the temperature. Figure 1 shows the profile of velocity u, magnetic field B, microrotations Γ and the temperature distribution θ for $\phi_1 = -1.0, \phi_2 = 100, S = 5$. The microrotation show the symmetric effect (Eringen [11]).

Figure 2(a) presented the effects of ϕ_2 on velocity for $\phi_1 = -2.0$. It is seen that velocity increases with increase in ϕ_2. Similar result is shown for velocity in Figure 2(b) for $\phi_1 = -1.0$.

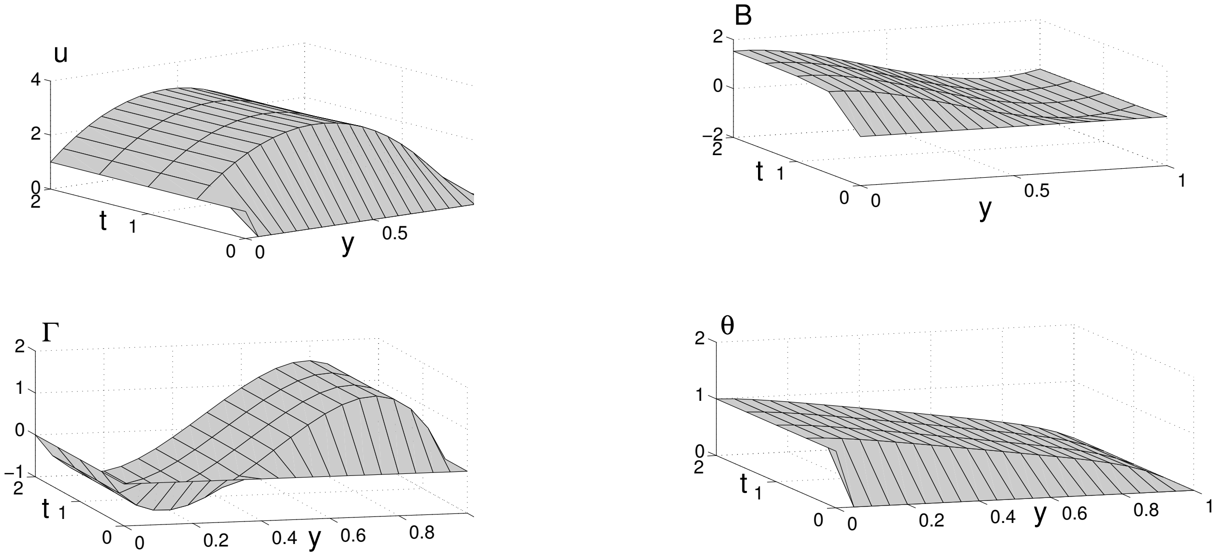


Figure 1: The Profile of Velocity(u), Magnetic Field (B), Microrotation (Γ), Temperature (θ) for $\phi_1 = -1, \phi_2 = 100, S = 5$.

Comparing Figure 2(a) and Figure 2(b) it is seen that for fixed ϕ_2 the velocity increase with decrease in ϕ_1. Increasing the values of ϕ_2 increases the effect of the magnetic field for fixed value of $\phi_1 = -1$ as seen in Figure 2(c), similar effect is observed when $\phi_1 = -2$ from Figure 2(d). It can be seen from Figure 2(c) and Figure 2(d) when ϕ_2 is fixed the magnetic filed decreases with increase in ϕ_1.

The effect of ϕ_2 on microrotation is depicted in Figure 3(a) for $\phi_1 = -2$. Microrotation in magnitude increase with increase in ϕ_2 for $\phi_1 = -2$. Similar result is seen in Figure 3(b) for $\phi_1 = -1$. In both figures the microrotaion symmetric effect is seen at the center (Eringen [11]). The effect of the magnetic force number, S, is shown in Figure 4(a) and Figure 4(b). Increasing the magnetic force number decreases the velocity as depicted in Figure 4(a). The magnetic field decrease with the increase in the magnetic force number towards the lower plate and similar effect is seen in magnitude for B towards the other end.

4 Conclusion

This article provides MHD effect on micropolar fluid flow between inclined moving plates taking into account the induced magnetic field between inclined plates of permeable and conducting walls. The flow characteristics are studied for velocity, magnetic field, microrotaions and temperature. The effect of wall parameters (ϕ_1, ϕ_2) on velocity, magnetic field and micro rotation studied and it is found that both velocity and magnetic field increases with increase in these parameters. It is also observed a similar effect in magnitude on microrotaion. Velocity decreases with magnetic force number increases. The magnetic field, B, decreases in magnitude with increase in magnetic force number.

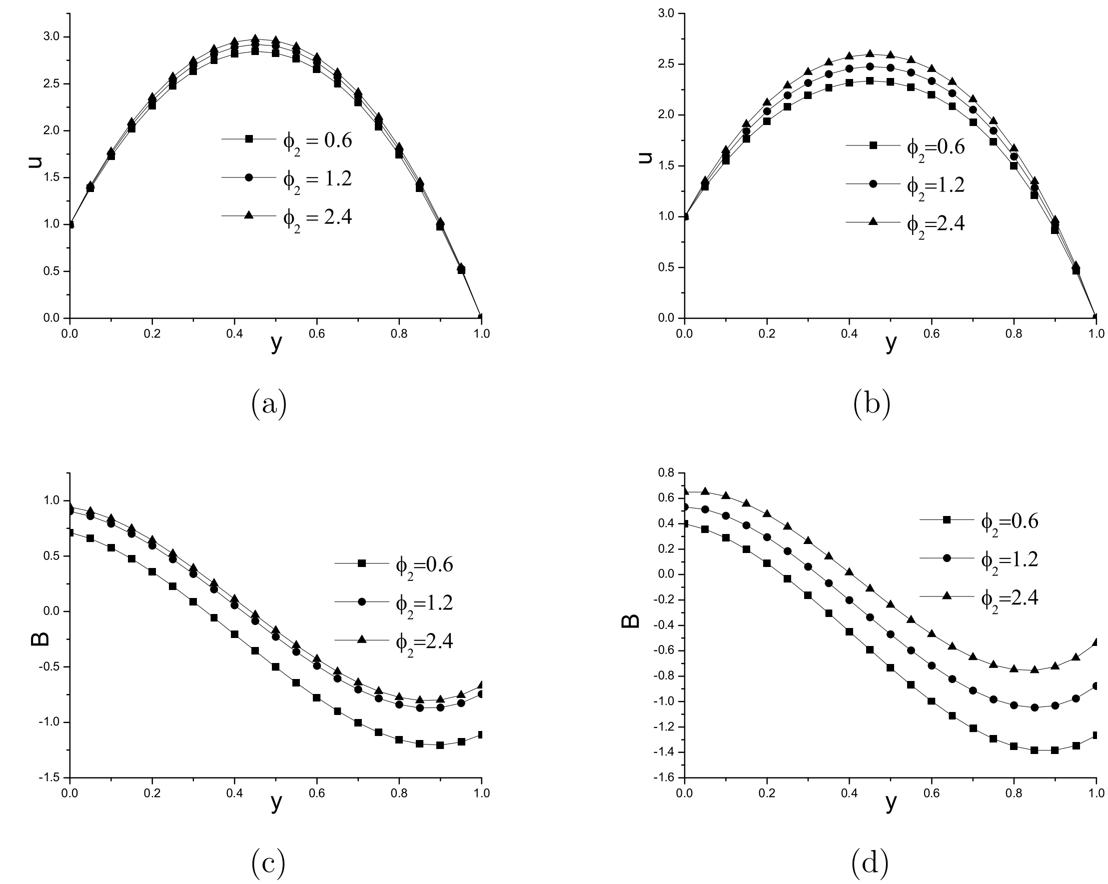


Figure 2: Effect of ϕ_2 on Velocity (a) for $\phi_1 = -2$, (b) for $\phi_1 = -1$ on Magnetic Field (c) for $\phi_1 = -2$, (d) for $\phi_1 = -1$ for $S = 5$.

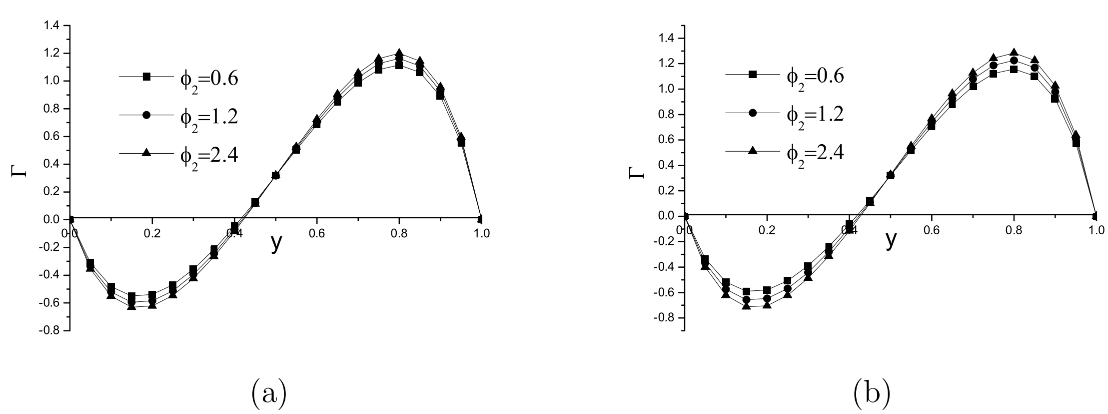
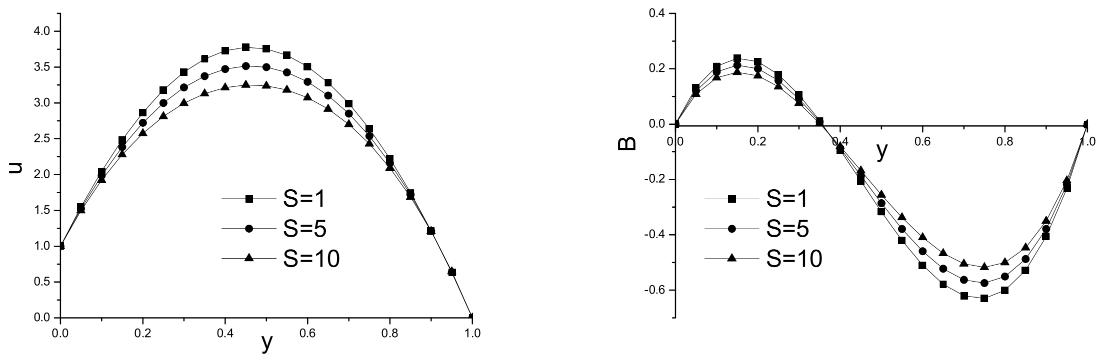


Figure 3: Effect of ϕ_2 on Microrotation (a) with $\phi_1 = -2$, (b) with $\phi_1 = -1$ for $S = 5$.



(a) (b)

Figure 4: Effect of Magnetic Force Number, S, on Velocity (a), Magnetic Field (b) for $\phi_1 = -1$ and $\phi_2 = -2$.

Acknowledgement

The author would like to express his gratitude to the anonymous referee for the comments and suggestions that improved this paper.

References

[1] Malashetty, M. S. and Umavathi, J. C., Two phase magnetohydrodynamic flow and heat transfer in an inclined channel. *Int. J. Multiphas Flow*. 1997. 23: 545–560.

[2] Malashetty, M. S., Umavathi, J. C. and Prathap Kumar, J. Convective magneto-hydrodynamic two fluid flow and heat transfer in an inclined channel. *Heat Mass Transfer*. 2001. 37: 259–264.

[3] Sanyal, D. C. and Sanyal, M. K. Hydromagnetic slip flow with heat transfer in an inclined channel. *Czech J Phys*. 1989. 39: 529–536.

[4] Aiyesimi, Y. M., Okedayo, G. T. and Lawal, O. W. Effects of magnetic field on the MHD flow of a third grade fluid through inclined channel with ohmic heating. *J Appl Computat Math* . 2014. 10.4172: 2168–9679.

[5] Rheault, S. and Bilgen, E. Mixed convection heat transfer in an open ended inclined channels with reversal. *Heat Mass Transfer*. 1993. 29:107–116.

[6] Raju, M. C. and Varma, S. V. K. and Sumon, S. Soret effects due to natural convection between heated inclined plates with magnetic field. *J Mech Eng*. 2008. 39: 65–70.

[7] Daniel, S. and Daniel, Y. S. Convective flow two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient. *Int J Eng Sci*. 2013. 2(4):12–18.

[8] Cimpean, D., Pop I. , Inghamand, D. B. Merkin, J. H. Fully developed mixed convection flow between inclined parallel porous plates filled with a porous medium. *Transport Porous Med*. 2009. 77: 87–102.

[9] Chang, C. C. and Lundgren, T. S. Duct flow in magneto hydrodynamics. *Z Angew Math Phys*. 1961. 12: 100–114.

[10] Roy, A. and Das, A. K. MHD flow with heart transfer in an inclined channel under transverse magnetic field and wall conductivity. *Bull Cal Math Soc*. 1972. 13: 193–204.

[11] Eringen, A. C. Theory of Micropolar Fluids. *J Math Mech* 1966. 16: 1–18.

[12] Ariman, T., Turk, M.A., Sylvester, N.D. Review article-applications of microcontinuum fluid mechanics. *Int J Eng Sci*. 1974. 12: 273.

[13] Sutton, W. G. and Sherman, A. *Engineering magnetohydrodynamics*. McGraw-Hill Inc. 1965.